The third international conference on Quantum Topology

June 20 – 25, 2016

Book of abstracts

Laboratory of Quantum Topology, Chelyabinsk State University
and
Steklov Mathematical Institute of Russian Academy of Sciences

Moscow, 2016.

Program Committee

Vladimir Turaev, *Chelyabinsk State University and Indiana University* (chairman)

Sergey Matveev, *Chelyabinsk State University* (co-chairman)

Andrey Vesnin, *Sobolev Institute of Mathematics, Siberian Branch of Russian Academy of Sciences*

Alexander Gaifullin, *Steklov Mathematical Institute of Russian Academy of Sciences*

Ivan Dynnikov, *Steklov Mathematical Institute of Russian Academy of Sciences*

Iskander Taimanov, *Sobolev Institute of Mathematics, Siberian Branch of Russian Academy of Sciences*

Evgeniy Fominykh, *Chelyabinsk State University*

Speakers

Valeriy Bardakov (Sobolev Institute of Mathematics and Chelyabinsk State University, Russia)

Leonid Chekhov (Steklov Mathematical Institute and Poncelet Lab., Russia)

David Cimasoni (University of Geneva, Switzerland)

Evgeny Fominykh (Chelyabinsk State University, Russia)

Stefan Friedl (Universität Regensburg, Germany)

Alexander Gaifullin (Steklov Mathematical Institute, Russia)

Stavros Garoufalidis (Georgia Institute of Technology, USA)

Nathan Geer (Utah State University, USA)

Eugene Gorsky (UC Davis, USA and National Research University Higher School of Economics, Russia)

Masaharu Ishikawa (Tohoku University, Japan)

Rinat Kashaev (University of Geneva, Switzerland)

Thang Lê (Georgia Institute of Technology, USA)

Vassily Manturov (Bauman Moscow State Technical University and Chelyabinsk State University, Russia)

Gwénaël Massuyeau (University of Strasbourg, France)

Alexander Mednykh (Novosibirsk State University, Sobolev Institute of Mathematics and Chelyabinsk State University, Russia)

Michele Mulazzani (University of Bologna, Italy)

Carlo Petronio (University of Pisa, Italy)

Maxim Prasolov (Moscow State University, Russia)

Alexander Shumakovitch (The George Washington University, USA)

Alexey Sleptsov (Insitute for Theoretical and Experimental Physics and Chelyabinsk State University, Russia)

Iskander Taimanov (Sobolev Institute of Mathematics, Russia)

Vladimir Turaev (Indiana University, USA and Chelyabinsk State University, Russia)

Andrei Vesnin (Sobolev Institute of Mathematics and Chelyabinsk State University)

Victor Vassiliev (Steklov Mathematical Institute, Russia)

Vladimir Vershinin (University of Montpellier, France, Sobolev Institute of Mathematics and Chelyabinsk State University, Russia)

On representations of virtual braid group and groups of virtual links

Valeriy Bardakov
bardakov@math.nsc.ru
Sobolev Institute of Mathematics, Koptyuga 4, Novosibirsk 630090, Russia
Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

We introduce some representation ψ of the virtual braid group VB_n into the automorphism group $Aut(F_{n,2n+1})$ of a free product $F_{n,2n+1} = F_n * \mathbb{Z}^{2n+1}$, where F_n is a free group and $\mathbb{Z}^{2n+1}$ is a free abelian group. This representation generalizes some other representations. In particular, the representation $\varphi_0 : VB_n \longrightarrow Aut(F_n)$ defined in [1]; the representation $\varphi_1 : VB_n \longrightarrow Aut(F_{n+1})$ defined in [2], [3] (see also, [4]); the representation $\varphi_2 : VB_n \longrightarrow Aut(F_{n,n+1})$ defined in [5]; the representation $\varphi_3 : VB_n \longrightarrow Aut(F_{n,2})$ defined in [6]. On the other hand the Artin representation is faithful. It is interesting to construct a representation which is an extension of it.

Theorem 1. *There is a representation $VB_n \longrightarrow Aut(F_{n,n})$ which is an extension of Artin representation and in some sense is equivalent to the representation ψ.*

From the result of O. Chterental [7] follows that for $n > 3$ the representations φ_1, φ_2 and φ_3 have non-trivial kernels. Analogous question for ψ is opened.

Using any of the representation $\psi, \varphi_0, \varphi_1, \varphi_2, \varphi_3$ one can defines a group $G_\psi(L)$, $G_{\varphi_0}(L)$, $G_{\varphi_1}(L)$, $G_{\varphi_2}(L)$, $G_{\varphi_3}(L)$ of a virtual link L. A connection between these groups gives

Theorem 2. *The groups $G_{\varphi_0}(L)$, $G_{\varphi_1}(L)$, $G_{\varphi_2}(L)$, $G_{\varphi_3}(L)$ are homomorphic images of the group $G_\psi(L)$. If L is a virtual knot, then we have isomorphisms $G_\psi(L) \cong G_{\varphi_1}(L) \cong G_{\varphi_2}(L) \cong G_{\varphi_3}(L)$.*

The talk is based on the joint work with M. V. Meshchadim and Yu. A. Mikhalchishina [8]. The author is partially supported by the Laboratory of Quantum Topology of Chelyabinsk State University (Russian Federation government grant 14.Z50.31.0020) and RFBR grant 16-01-00414 and RNF grant 16-41-02006

References:

[1] V. V. Vershinin, On homology of virtual braids and Burau representation. *J. Knot Theory Raminifications* 10 (2001), no. 5, 795–812.

[2] V. O. Manturov, On the recognition of virtual braids. *Zap. Nauchn. Sem. POMI* 299 (2003), 267–286.

[3] V. G. Bardakov, Virtual and welded links and their invariants. *Sib. Elektron. Mat. Izv.* 2 (2005), 196–199.

[4] V. G. Bardakov, P. Bellingeri, Groups of virtual and welded links. *J. Knot Theory Ramifications* 23 (2014), no. 3, 1450014, 23 pp.

[5] D. Silver, S. G. Williams, Alexander groups and virtual links. *J. Knot Theory Ramifications* 10 (2001), no. 1, 151–160.
[6] H. U. Boden, A. I. Gaudreau, E. Harper, A. J. Nicas, L. White, Virtual knot groups and almost classical knots. arXiv:1506.01726.
[7] O. Chterental, Virtual braids and virtual curve diagrams. arXiv:1411.6313.
[8] V. G. Bardakov, Yu. A. Mikhalchishina, M. V. Neshchadim, Representations of virtual braids by automorphisms and virtual knot groups. arXiv:1603.01425.

Quantum cluster algebras and character varieties of $SL(2,R)$-monodromy problem

Leonid Chekhov
chekhov@mi.ras.ru
Steklov Mathematical Institute, Gubkin st 8, Moscow 119991, Russia
Poncelet Lab., Bol. Vlasievskii 11, Moscow 119002, Russia

We identify the Teichmuller space $T_{g,s,n}$ of (decorated) Riemann surfaces $\Sigma_{g,s,n}$ of genus g, with $s > 0$ holes and $n > 0$ bordered cusps located on boundaries of holes uniformized by Poincare with the character variety of $SL(2,R)$-monodromy problem. The effective combinatorial description uses the fat graph structures; observables are geodesic functions of closed curves and λ-lengths of paths starting and terminating at bordered cusps decorated by horocycles. We derive Poisson and quantum structures on sets of observables relating them to quantum cluster algebras of Berenstein and Zelevinsky. A seed of the corresponding quantum cluster algebra corresponds to the partition of $\Sigma_{g,s,n}$ into ideal triangles, λ-lengths of their sides are cluster variables constituting a seed of the algebra; their number $6g-6+3s+2n$ (and, correspondingly, the seed dimension) coincides with the dimension of $SL(2,R)$-character variety given by $[SL(2,R)]^{2g+s+n-2}/\prod_{i=1}^{n} B_i$, where B_i are Borel subgroups associated with bordered cusps. Moreover, using the explicit parameterization of monodromy elements we can evaluate the Poisson and quantum algebras of monodromy matrices generated by the Poisson and quantum algebras of λ-lengths and show that these algebras are quadratic quasi-Poisson, or quasi-quantum, algebras. These algebras are invariant w.r.t. mutations of cluster algebras, which correspond to MCG transformations, and can be therefore lifted from $T_{g,s,n}$ to the moduil space $M_{g,s,n}$. Complexifying the cluster variables we obtain the character variety of $SL(2,C)$-monodromy problem.

The talk is based on the joint works with with M. Mazzocco and V. Roubtsov [1, 2, 3].

References:

[1] L. Chekhov and M. Mazzocco, Colliding holes in Riemann surfaces and quantum cluster algebras. arXiv:1509.07044.

[2] L. Chekhov, M. Mazzocco, and V. Roubtsov, PainlevP№ monodromy manifolds, decorated character varieties and cluster algebras. arXiv:1511.03851.
[3] L. Chekhov, M. Mazzocco, and V. Roubtsov, Decorated character varieties of monodromy manifolds and quantum cluster algebras. in preparation.

Splitting numbers and signatures

David Cimasoni
david.cimasoni@unige.ch
University of Geneva, 2-4 rue du Lièvre, Geneva 1211, Switzerland

The splitting number of a link is the minimal number of crossing changes between different components required to convert it into a split link. This invariant was studied by Batson-Seed [1] using Khovanov homology, by Cha-Friedl-Powell [2] using the Alexander polynomial and covering link calculus, and by Borodzik-Gorsky [3] using Heegaard-Floer homology.

In this talk, I will prove a new lower bound on the splitting number in terms of the (multivariable) signature and nullity of [4]. Although very elementary and easy to compute, this bound turns out to be suprisingly efficient. In particular, I will show that it compares very favorably to the methods mentioned above.

The talk is based on the joint work [5] with A. Conway and K. Zaharova. The author is partially supported by Swiss National Science Foundation.

References:
[1] J. Batson, C. Seed, A link-splitting spectral sequence in Khovanov homology, *Duke Math. J.*, Vol. 164 (2015), no. 5, 801–841.
[2] J. C. Cha, S. Friedl, M. Powell, Splitting numbers of links, *Proc. Edinb. Math. Soc. (2)*, to appear.
[3] M. Borodzik, E. Gorsky, Immersed concordances of links and Heegaard Floer homology, preprint.
[4] D. Cimasoni, V. Florens, Generalized Seifert surfaces and signatures of colored links, *Trans. Amer. Math. Soc.*, Vol. 360 (2008), no. 3, 1223–1264.
[5] D. Cimasoni, A. Conway, K. Zacharova, Splitting numbers and signatures, *Proc. Amer. Math. Soc.*, to appear.

Turaev-Viro invariants and complexity of virtual 3-manifolds

Evgeny Fominykh
efominykh@gmail.com
Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

Virtual 3-manifolds were introduced by S. Matveev in 2009 as a natural generalization of the classical 3-manifolds. In this talk we define the complexity of virtual 3-manifolds and calculate it for virtual 3-manifolds defined by special polyhedra with one, two and three 2-components. As a corollary, we establish the exact values of complexity for infinite families of hyperbolic 3-manifolds with geodesic boundary.

A part of the talk is based on a joint work with V. Turaev and A. Vesnin.

The Thurston norm of 3-manifolds with a 2-generator fundamental group

Stefan Friedl
sfriedl@gmail.com
Fakultät für Mathematik, Universität Regensburg, Germany

We will give a straightforward algorithm for computing the Thurston norm of a 3-manifold if the 3-manifold admits a presentation with two generators. The talk is based on the joint work with W. Lück, K. Schreve and S. Tillmann.

Realisation of cycles and small covers over graph-associahedra

Alexander Gaifullin
agaif@mi.ras.ru
Steklov Mathematical Institute, Gubkin st 8, Moscow 119991, Russia

A classical question by Steenrod (late 1940s) was whether it is possible to realize an integral homology class of a topological space by a continuous image of the fundamental class of an oriented smooth closed manifold. (Homology classes satisfying this condition are called realizable.) This question was answered by

Thom (1954) who showed that there exist non-realizable homology classes but a certain multiple of any homology class is realizable.

In 2007 the speaker found an explicit combinatorial procedure that, for a given singular cycle in a topological space, constructs a manifold realizing a multiple of the homology class representing by this cycle. Moreover, this construction allowed us to prove that, for every n, there exists an oriented smooth closed manifold M^n that satisfy the following *Universal Realization of Cycles property* (or the URC-property): A multiple of any n-dimensional integral homology class of any topological space can be realized by an image of the fundamental class of a non-ramified finite-sheeted covering over M^n. Several series of examples of URC-manifolds (i.e. manifolds satisfying the URC-property) were found by the speaker in 2013. The simplest of them was the so-called Tomei manifold, which is a *small cover* of a special simple polytope called the permutohedron.

In the talk we shall present a modification of the explicit procedure for the realization of cycles that will allow us to find URC-manifolds that are even simpler than the Tomei manifolds. In particular, for an important class of simple polytopes called graph-associahedra, we shall show that small covers over them are also URC-manifolds. In particalar, all small covers of a well known Stacheff associahedra are URC-manifolds.

The Kashaev invariant, Nahm sums and modularity

Stavros Garoufalidis

stavros@math.gatech.edu

School of Mathematics, Georgia Institute of Technology, Atlanta GA 30332-1060, USA

The Kashaev invariant of a knot can be extended to a complex valued function on the set of complex roots of unity on the unit circle. Nahm sums are special q-hypergeometric sums defined inside the unit circle. Both are conjectured to have modular properties, and both properties are linked to an explicit map from the Bloch group, that we will discuss.

Joint work with Frank Calegar and Don Zagier. The author is supported in part by the National Science Foundation DMS-1406419.

Modified traces on quantum $sl(2)$ and logarithmic invariants

Nathan Geer
nathan.geer@gmail.com
Utah State University, Mathematics & Statistics, Logan, Utah 84322, USA

In this talk I will discuss a link invariant arising from the restricted quantum of $sl(2)$ at a $2p$-th root of unity. In particular, I will show how we can use a modified trace on the quantum group itself to define a Logarithmic invariant of colored links. Using the integer on the quantum group, this invariant can be extend to an invariant of colored links in a 3-manifold. We expect this 3-manifold invariant to lead to a TQFT. This is joint work with Anna Beliakova and Christian Blanchet.

Homology of Jucys-Murphy elements and the flag Hilbert scheme

Eugene Gorsky
egorskiy@math.ucdavis.edu
Department of Mathematics, UC Davis, One Shields Avenue, Davis CA 95616, USA
International Laboratory of Representation Theory and Mathematical Physics, National Research University Higher School of Economics, Vavilova 7, Moscow, Russia

The Jucys-Murphy elements are known to generate a maximal commutative subalgebra in the Hecke algebra. They can be categorified to a family of commuting complexes of Soergel bimodules. I will describe a relation between the category generated by these complexes and the category of sheaves on the flag Hilbert scheme of points on the plane, using the recent work of Elias and Hogancamp on categorical diagonalization. As an application, I will give an explicit conjectural description of the Khovanov-Rozansky homology of generalized torus links.

The talk is based on the joint work in progress with Andrei Negut and Jacob Rasmussen.

Stable maps and branched shadows of 3-manifolds

Masaharu Ishikawa
ishikawa@m.tohoku.ac.jp
Mathematical Institute, Tohoku University, Sendai 980-8578, Japan

As used in a paper of Costantino and D. Thurston, Turaev's shadow can be regarded locally as the Stein factorization of a stable map. In [1], we introduced the notion of stable map complexity for a compact orientable 3-manifold bounded by (possibly empty) tori counting, with some weights, the minimal number of singular fibers of codimension 2 of stable maps into the real plane, and proved that this number equals its branched shadow complexity. In consequence, we see that the hyperbolic volume is bounded from above and below by the stable map complexity, which is a direct corollary of an observation of Costantino and Thurston and an inequality obtained by Futer, Kalfagianni and Purcell.

This is a joint work with Yuya Koda in Hiroshima University. Partially supported by the Grant-in-Aid for Scientific Research (C), JSPS KAKENHI Grant Number 16K05140.

References:

[1] M. Ishikawa, Y. Koda, Stable maps and branched shadows of 3-manifolds. arXvi:math/1403.0596.

Pachner's 3-3 relations and Hopf algebras

Rinat Kashaev
Rinat.Kashaev@unige.ch
Université de Genève, Section de mathématiques, 2-4 rue du Lièvre Case postale 64 1211 Genève 4, Switzerland

I will discuss a particular construction of solutions to Pachner's 3-3 relation in 4 dimensions by using the structure maps of Hopf algebras and duality pairings. In the particular case of self-dual bi-commutative Hopf algebras there are solutions carrying the full symmetry of the regular 4-dimensional simplex.

Triangular decomposition of skein algebras and quantum Teichmüller spaces

Thang Lê
letu@math.gatech.edu
School of Mathematics, Georgia Institute of Technology, Atlanta GA 30332, USA

We show how to decompose the Kauffman bracket skein algebra of a surface into elementary blocks corresponding to the triangles in an ideal triangulation of the surface. This gives an easy proof of the existence of the quantum trace map of Bonahon and Wong. We also explain the relation between the Kauffman bracket skein algebra and the quantum Teichmüller space.

The author is partially supported by the NSF.

On Knot Theoretical Counterpart of the Groups G_n^k

Vassily Manturov
vomanturov@yadex.ru
Bauman Moscow State Technical University, 2nd Bauman St., 5/1, Moscow 105005, Russia
Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

In 2015, [1], the author initiated the study of groups denoted by G_n^k, depending on two natural parameters, n and k and formulated the main principle:

if a dynamical system describing a motion of n particles, is in general position with respect to some property regulated by n particles, then it has topological invariants valued in G_n^k.

The main examples calculated explicitly [2] led to homomorphisms from the pure braid groups to the groups G_n^3 and G_n^4.

The group G_n^k were found to have close connections with Coxeter groups [3], braid groups and other groups.

In the present talk, we address the question:

what happens if the number of particles is not constant but at some moments of time some two particles can get born or get cancelled?

The main example here is the extension of topological invariants from braids to knots. A braid can be consiered as a motion of n distinct particles on the plane, whence a knot can be represented by a collection of sections by horizontal planes.

In general position, there are finitely many moments, where number of particles changes by two; otherwise, the knot behaves like a braid.

What is the "knot" counterpart of the groups G_n^k-groups considered as analogs of "braids"?

The most naïve approach suggests to consider elements of G_n^k as 1-dimensional braid-like objects ("braid" diagrams modulo moves) and to pass to analogous "knot-like" (closed) objects modulo the same moves. However, this approach fails because besides the usual "braid" moves, one should also require some "cobordism-like" moves which make the whole picture almost trivial.

The right approach (at least for G_n^3) is related not to 1-dimensional formalism, but rather, with a 2-dimensional formalism.

Then diagrams corresponding to our dynamical system will look like 2-knot diagrams, and their moves will look like Roseman moves [4].

The first step of this approach is sketched in [5].

This allows one to take the pull-back of invariants of "2-knot like objects" as toplogical invariants of dynamical systems of this sort.

The author is partially supported by the Laboratory of Quantum Topology of Chelyabinsk State University (Russian Federation government grant 14.Z50.31.0020)

References:

[1] V.O. Manturov, Non-Reidemeister knot theory and its applications in dynamical systems, geometry and topology. http://arxiv.org/abs/1501.05208.

[2] V.O. Manturov, I.M. Nikonov,On Braids and Groups G_n^k. *J. Knot Theory and its Ramifications* 24 (2015), no. 13, 1541009, 16 pp.

[3] V.O. Manturov, On Groups G_n^2 and Coxeter Groups. arXiv:1512.09273.

[4] D. Roseman, Reidemeister-type moves for surfaces in four-dimensional space. *Knot Theory, Banach Center Publications* 42 (1998), Polish Academy of Sciences, Warsaw, 347–380.

[5] V.O. Manturov, A Note on a Map from Knots to 2-Knots. arXiv:1604.06597.

Self-intersection of curves in surfaces and Drinfeld associators

Gwénaël Massuyeau

massuyeau@math.unistra.fr

Institut de Recherche Mathématique Avancée, Université de Strasbourg & CNRS, 7 rue René Descartes, Strasbourg 67084, France

Turaev introduced in the seventies two fundamental operations on the algebra $\mathbb{Q}[\pi]$ of the fundamental group π of a surface with boundary [1]. The first operation is binary and measures the intersection of two oriented curves on the surface, while the second operation is unary and computes the self-intersection of an oriented curve. It is already known that Turaev's intersection pairing has a simple algebraic description when the I-adic completion of the group algebra $\mathbb{Q}[\pi]$ is appropriately identified to the degree-completion of the tensor algebra $T(H)$ of $H := H_1(\pi; \mathbb{Q})$.

We will show that Turaev's self-intersection map has a similar description in the case of a disk with p punctures. In this special case, we will consider those identifications between the completions of $\mathbb{Q}[\pi]$ and $T(H)$ that arise from the Kontsevich integral by embedding π into the pure braid group on $(p+1)$ strands [2, 3]. As a matter of fact, our algebraic description involves a formal power series which is explicitly determined by the Drinfeld associator Φ entering into the definition of the Kontsevich integral; this series is essentially Enriquez' Γ-function of Φ [4]. If time allows, we will also discuss the case of higher-genus surfaces. (This talk is based on the preprint [5].)

References:

[1] V. Turaev, Intersections of loops in two-dimensional manifolds. (Russian) *Mat. Sb.* 106(148) (1978), no. 4, 566–588. English translation: *Math. USSR–Sb.* 35 (1979), 229–250.

[2] N. Habegger, G. Masbaum, The Kontsevich integral and Milnor's invariants. *Topology* 39 (2000), no. 6, 1253–1289.

[3] A. Alekseev, B. Enriquez, C. Torossian, Drinfeld associators, braid groups and explicit solutions of the Kashiwara–Vergne equations. *Publ. Math. Inst. Hautes Études Sci.* 112 (2010), 143–189.

[4] B. Enriquez, On the Drinfeld generators of $\mathfrak{grt}_1(\mathbf{k})$ and Γ-functions for associators. *Math. Res. Lett.* 3 (2006), no. 2-3, 231–243.

[5] G. Massuyeau, Formal descriptions of Turaev's loop operations. Preprint (2015), arXiv:1511.03974.

Jacobians of circulant graphs

Alexander Mednykh

smedn@mail.ru

mednykh@math.nsc.ru

Novosibirsk State University, Pirogova 2, Novosibirsk 630090, Russia

Sobolev Institute of Mathematics, Koptyuga 4, Novosibirsk 630090, Russia

Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

The notion of Jacobian of a graph, also known as the Picard group, the critical group, the sandpile group or the dollar group, was independently introduced by many authors ([1], [2], [3], [4]). Given a graph one can define a Jacobian as the maximum Abelian group generated by flows satisfying the first and the second Kirchhoff's laws. It is a crucial invariant of a finite graph. Its order coincides with the number of spanning trees of the graph. It is also can be considered as a discrete version of the Jacobian of a Riemann surface. The complete structure of the Jacobian is known only for a few families of graphs. For instance, for the wheel graphs, the prism graphs, the Moebius ladders, the complete graphs and some others. The aim of this talk is to provide a general method to determine the structure of Jacobian for an infinite family of circulant graphs.

The author is partially supported by the Laboratory of Quantum Topology of Chelyabinsk State University (Russian Federation government grant 14.Z50.31.0020) and RFBR grants 15-01-07906 and 16-31-50009.

References:

[1] R. Cori, D. Rossin, On the sandpile group of dual graphs. *European J. Combin.* 21 (2000), no. 4, 447–459.

[2] B. Baker, S. Norine, Harmonic morphisms and hyperelliptic graphs. *Int. Math. Res. Notes* 15 (2009), 2914–2955.

[3] N. L. Biggs, Chip-firing and the critical group of a graph. *J. Algebraic Combin.* 9 (1999), no. 1, 25–45.

[4] R. Bacher, P. de la Harpe, T. Nagnibeda, The lattice of integral flows and the lattice of integral cuts on a finite graph. *Bull. Soc. Math. France.* 125 (1997), 167–198.

4-colored graphs and complements of knots and links

Michele Mulazzani

michele.mulazzani@unibo.it

Department of Mathematics, University of Bologna, Piazza di Porta San Donato, Bologna 5 - 40126, Italy

A representation for compact 3-manifolds with non-empty non-spherical boundary via 4-colored graphs (i.e. regular 4-valent graphs endowed by a proper edge-coloration with four colors) has been introduced in [1], where an initial tabulation/classification of such manifolds has been obtained, up to 8 vertices of the representing graphs.

Computer experiments show that the number of graphs/manifolds grows very rapidly with the increasing of the vertices. As a consequence we focused our attentions on the case of 3-manifolds which are the complements of knots or links in the 3-sphere. In this context we obtained the classification of these 3-manifolds, up to 12 vertices of the representing graphs, showing the type of the links involved (they are exactly 22).

For the particular case of knot complements, the classification has been recently extended up to 16 vertices: there are exactly two complements of knots in the 3-sphere, the trivial knot (6 vertices) and the trefoil knot (16 vertices).

All these results are contained in [2], which will soon appear on the arXiv.

Joint work with P. Cristofori, E. Fominykh and V. Tarkaev.

References:

[1] P. Cristofori, M. Mulazzani, "Compact 3-manifolds via 4-colored graphs", *RACSAM*, published online: 24 July 2015. arXiv:1304.5070.

[2] P. Cristofori, E. Fominykh, M. Mulazzani, V. Tarkaev, 4-*colored graphs and knot/link complements*, Preprint, 2016.

On the Hurwitz existence problem for branched covers between surfaces

Carlo Petronio

petronio@dm.unipi.it

Dipartimento di Matematica, Università di Pisa Largo B. Pontecorvo 5, Pisa I-56127, Italy

Given a branched cover $p : \widetilde{\Sigma} \to \Sigma$ beteween closed orientable surfaces, the famous Riemann-Hurwitz formula relates the Euler characteristics of $\widetilde{\Sigma}$ and Σ, the total degree d of p, the number n of branch points in Σ and the sum of the lengths of the partitions $\left((d_{i,j})_{j=1}^{m_i}\right)_{i=1}^{n}$ of d given by the local degrees of p at the preimages of the branch points. The Hurwitz existence problem asks whether a given combinatorial datum

$$\left(\widetilde{\Sigma}, \Sigma, d, n, \left((d_{i,j})_{j=1}^{m_i}\right)_{i=1}^{n}\right)$$

satisfying the Riemann-Hurwitz formula is actually realized by a branched cover $p : \widetilde{\Sigma} \to \Sigma$. The answer is now known to be always in the affirmative when Σ has positive genus, but not when Σ is the Riemann sphere. I will report on recent progress on the problem based on a connection with the geometry of 2-orbifolds.

The talk is based on the joint papers with with M. A. Pascali [1] and [2].

References:

[1] M. A. Pascali, C. Petronio, Surface branched covers and geometric 2-orbifolds. *Trans. Amer. Math. Soc.* 361 (2009), 5885–5920.

[2] M. A. Pascali, C. Petronio, Branched covers of the sphere and the prime-degree conjecture. *Ann. Mat. Pura Appl.* 191 (2012), 563–594.

Rectangular diagrams and Giroux's convex surfaces

Maxim Prasolov
0x00002a@gmail.com
Department of Mathematics, Moscow State University, Leninskie Gory, Moscow 119991, Russia

Rectangular diagrams can be considered as a special class of plane diagrams of links. Every link can be represented by a rectangular diagram and an analogue of Reidemeister theorem holds that any two rectangular diagrams of the same link are related by a sequence of elementary moves.

There is a natural complexity function on the set of rectangular diagrams for which a trivial knot can be recognized by a monotonic simplification, as shown by I. Dynnikov. Or equivalently, any rectangular diagram of a trivial knot can be transformed into a minimal one by elementary moves which do not increase the complexity.

It is convenient to represent by rectangular diagrams Legendrian links, i.e. which are tangent to the plane distribution $\ker(dz + xdy)$ in $\mathbb{R}^3$. In a recent joint paper with I. Dynnikov it is shown that an extension of monotonic simplification to arbitrary links is closely related to a classification of Legendrian representatives in a fixed topological type.

One of the key instruments of low-dimensional contact topology, and particularly of Legendrian knot theory, is the Giroux's notion of convex surface. In our joint work with I. Dynnikov which is in preparation we show that convex surfaces in $\mathbb{R}^3$ can be nicely described in 'rectangular language'. We give an example of two Legendrian knots which can be distinguished using an analogue of rectangular diagrams for surfaces and which can not be distinguished by known algebraic invariants due to the lack of computational power.

Knot invariants arising from homological operations on Khovanov homology

Alexander Shumakovitch
shurik@gwu.edu
The George Washington University, 801 22nd St. NW, Phillips Hall, Suite 739, Washington, DC 20052, USA

There are several homological operations that can be defined between even and odd Khovanov homology theories using the unified homology theory developed by Putyra. This construction works for both reduced and unreduced versions of the Khovanov homology. We discuss these homological operations, compare different versions of them, and show how they can give rise to new knot invariants with interesting properties.

The talk is based on a joint work with Krzysztof Putyra [1]. The author is partially supported by a Simons Collaboration Grant for Mathematicians #279867.

References:

[1] K. Putyra and A. Shumakovitch, Knot invariants arising from homological operations on Khovanov homology. *J. Knot Th. and Ramif.* 25 (2016), no. 3, 1640012 [18 pages]; arXiv:1601.00798.

Quantum Racah matrices and evolution of knots

Alexey Sleptsov

sleptsov@itep.ru

Insitute for Theoretical and Experimental Physics, Bol'shaya Cheremushkinskaya street 25, Moscow 117259, Russia

Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

We construct a general procedure to extract the exclusive Racah matrices S and $\bar{S}$ from the inclusive 3-strand mixing matrices by the evolution method and apply it to the first simple representations $R = [1]$, $[2]$, $[3]$ and $[2,2]$. The matrices S and $\bar{S}$ relate respectively the maps $(R\otimes R)\otimes\bar{R} \longrightarrow R$ with $R\otimes(R\otimes\bar{R}) \longrightarrow R$ and $(R \otimes \bar{R}) \otimes R \longrightarrow R$ with $R \otimes (\bar{R} \otimes R) \longrightarrow R$. They are building blocks for the colored HOMFLY polynomials of arbitrary arborescent knots.

The talk is based on the joint work with A.Mironov, A.Morozov and An.Morozov [1]. The author is partially supported by the Laboratory of Quantum Topology of Chelyabinsk State University (Russian Federation government grant 14.Z50.31.0020), RFBR grant mol-a-dk 16-31-60082 and MK-8769.2016.1

References:

[1] A. Mironov, A. Morozov, An. Morozov, A. Sleptsov, Racah matrices and hidden integrability in evolution of knots. arXiv:1605.04881

The spaces of non-contractible closed curves in compact space forms

Iskander Taimanov

taimanov@math.nsc.ru

Sobolev Institute of Mathematics, Koptyuga 4, Novosibirsk 630090, Russia

We calculate the rational equivariant cohomology of the spaces of non-contractible loops in compact space forms and show how to apply these calculations for proving the existence of closed geodesics.

References:
[1] I.A. Taimanov, The spaces of non-contractible closed curves in compact space forms. arXiv:1604.05237.

Additive posets, CW-complexes, and graphs

Vladimir Turaev
vturaev@yahoo.com
Department of Mathematics, Indiana University, Bloomington IN47405, USA
Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

We introduce additive posets and study their properties and invariants. We show that the top homology group of a finite dimensional CW-complex carries a natural structure of an additive poset invariant under subdivisions of the CW-complex. Applications to graphs are discussed.

The talk is based on my preprint [1]. The author is partially supported by the Laboratory of Quantum Topology of Chelyabinsk State University (Russian Federation government grant 14.Z50.31.0020).

References:
[1] V. Turaev, Additive posets, CW-complexes, and graphs. arXiv:1605.07798.

Linking numbers in non-orientable 3-manifolds

Victor Vassiliev
vva@mi.ras.ru
Steklov Mathematical Institute, Gubkin st 8, Moscow 119991, Russia

The construction of integer linking numbers of closed curves in three-dimensional manifolds usually appeals to the orientability of these manifolds. I will discuss how (and when) it is possible to avoid this restriction in constructing similar invariants of links.

On Virtual Braids

Vladimir Vershinin
versh@math.nsc.ru
Université de Montpellier, Place Eugène Bataillon, 34095 Montpellier cedex 5, France
Sobolev Institute of Mathematics, Koptyuga 4, Novosibirsk 630090, Russia
Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

We give a survey of some recent results on virtual braids, in particular of the joint work with V. G. Bardakov, R. Mikhailov, and J. Wu [1]. The author is partially supported by the Laboratory of Quantum Topology of Chelyabinsk State University (Russian Federation government grant 14.Z50.31.0020)

References:

[1] V.G. Bardakov, R. Mikhailov, V.V. Vershinin, J. Wu, On the pure virtual braid group PV_3. *Comm. Algebra* 44 (2016), 1350–1378.

On volumes of compact and non-compact right-angled hyperbolic polyhedra

Andrei Vesnin
vesnin@math.nsc.ru
Sobolev Institute of Mathematics, Koptyuga 4, Novosibirsk 630090, Russia
Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia

There is a recent progress in study of Platonic tessellations of a hyperbolic 3-space and related hyperbolic 3-manifolds by algorithmic topology methods [1, 2]. It is known that some of hyperbolic Platonic solids are right-angled. There we are interested in a class of hyperbolic 3-manifolds which can be decomposed into right-angled hyperbolic polyhedra. Necessary and sufficient conditions for a polyhedron of a given combinatorial type to be realized as a compact right-angled polyhedron in a hyperbolic 3-space were described by Pogorelov in 1967 in the very first issue of "Matematicheskie Zametki" (Mathematical Notes) [3]. The simplest compact right-angled hyperbolic polyhedron is a dodecahedron.

The universal method to construct a hyperbolic 3-manifold from few copies of an arbitrary right-angled hyperbolic polyhedron was given in [4]. This motivates the study of the census of right-angled hyperbolic polyhedra.

Recently, Inoue [5] presented 825 smallest compact right-angled hyperbolic polyhedra. We will discuss a census of non-compact right-angled hyperbolic polyhedra.

For compact and non-compact cases both we will present results of numerical computations.

References:

[1] B. Everitt, 3-manifolds from Platonic solids, *Topology Appl.* 138 (2004), no. 1-3, 253–263.

[2] M. Goerner, A census of hyperbolic Platonic manifolds and augmented knotted trivalent graphs, arxiv1602.02208.

[3] A.V. Pogorelov, Regular decomposition of Lobachevskii space, *Mat. Zametki* 1, No. 1, 3–8 (1967).

[4] A.Yu. Vesnin, Three-dimensional hyperbolic manifolds of Loebell type, *Siberian Math. J.* 28, No. 5, 731–734 (1987).

[5] T. Inoue, The 825 smallest right-angled hyperbolic polyhedra, arxiv:1512.01761.