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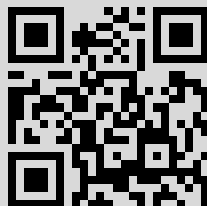
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## On equivalence of some subcategories of modules in Morita contexts

A. I. Kashu

**ABSTRACT.** A Morita context  $(R, {}_R V_S, {}_S W_R, S)$  defines the isomorphism  $\mathcal{L}_0(R) \cong \mathcal{L}_0(S)$  of lattices of torsions  $r \geq r_I$  of  $R\text{-Mod}$  and torsions  $s \geq r_J$  of  $S\text{-Mod}$ , where  $I$  and  $J$  are the trace ideals of the given context. For every pair  $(r, s)$  of corresponding torsions the modifications of functors  $T^W = W \otimes_R -$  and  $T^V = V \otimes_S -$  are considered:

$$R\text{-Mod} \supseteq \mathcal{P}(r) \xrightleftharpoons[\bar{T}^V = (1/r) \cdot T^V]{\bar{T}^W = (1/s) \cdot T^W} \mathcal{P}(s) \subseteq S\text{-Mod},$$

where  $\mathcal{P}(r)$  and  $\mathcal{P}(s)$  are the classes of torsion free modules. It is proved that these functors define the equivalence

$$\mathcal{P}(r) \cap \mathcal{J}_I \approx \mathcal{P}(s) \cap \mathcal{J}_J,$$

where  $\mathcal{P}(r) = \{{}_R M \mid r(M) = 0\}$  and  $\mathcal{J}_I = \{{}_R M \mid IM = M\}$ .

Let  $(R, {}_R V_S, {}_S W_R, S)$  be an arbitrary Morita context with the bimodule morphisms

$$(\cdot) : V \otimes_S W \longrightarrow R, \quad [\cdot] : W \otimes_R V \longrightarrow S,$$

satisfying the conditions of associativity:

$$(v, w)v_1 = v[w, v_1], \quad [w, v]w_1 = w(v, w_1) \quad (1)$$

for  $v, v_1 \in V$  and  $w, w_1 \in W$ . We denote by  $I = (V, W)$  and  $J = [W, V]$  the *trace ideals* of this context, where  $I$  is ideal of  $R$  and  $J$  is ideal of  $S$ .

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They define the torsions  $r_I$  in  $R\text{-Mod}$  and  $r_J$  in  $S\text{-Mod}$  such that the classes of torsion free modules are:

$$\begin{aligned}\mathcal{P}(r_I) &= \{ {}_R M \mid I m = 0, m \in M \implies m = 0 \}, \\ \mathcal{P}(r_J) &= \{ {}_S N \mid J n = 0, n \in N \implies n = 0 \},\end{aligned}$$

i.e.  $r_I$  and  $r_J$  are determined by the smallest Gabriel filters, containing  $I$  and  $J$ , respectively [7].

In the lattices  $\mathcal{L}(R)$  and  $\mathcal{L}(S)$  of all torsions of  $R\text{-Mod}$  and  $S\text{-Mod}$ , respectively, we distinguish the following sublattices:

$$\begin{aligned}\mathcal{L}_0(R) &= \{ r \in \mathcal{L}(R) \mid r \geq r_I \}, \\ \mathcal{L}_0(S) &= \{ s \in \mathcal{L}(S) \mid s \geq r_J \}.\end{aligned}\tag{2}$$

The following result is well known ([1], [4], [5], [7]).

**Theorem 1.** *There exists a preserving order bijection between the torsions of  $R\text{-Mod}$  containing  $r_I$  and torsions of  $S\text{-Mod}$  containing  $r_J$ , i.e.  $\mathcal{L}_0(R) \cong \mathcal{L}_0(S)$ .  $\square$*

This bijection is obtained with the help of the functors:

$$R\text{-Mod} \begin{array}{c} \xrightarrow{H^V = \text{Hom}_R(V, -)} \\ \xleftarrow{H^W = \text{Hom}_S(W, -)} \end{array} S\text{-Mod},\tag{3}$$

acting by  $H^V$  and  $H^W$  to the injective cogenerators of torsions [4]. From the definitions it follows

**Lemma 2.** ([4], Lemma 4). *If  $(r, s)$  is a pair of corresponding torsions in the sense of Theorem 1 (i.e.  $H^V(r) = s$  and  $H^W(s) = r$ ), then  $H^V(\mathcal{P}(r)) \subseteq \mathcal{P}(s)$  and  $H^W(\mathcal{P}(s)) \subseteq \mathcal{P}(r)$ , where  $\mathcal{P}(r)$  and  $\mathcal{P}(s)$  are the classes of torsion free modules.  $\square$*

Now we consider the following functors accompanying the given Morita context:

$$R\text{-Mod} \begin{array}{c} \xrightarrow{T^W = W \otimes_R -} \\ \xleftarrow{T^V = V \otimes_S -} \end{array} S\text{-Mod}\tag{4}$$

with the natural transformations

$$\eta : T^V T^W \longrightarrow 1_{R\text{-Mod}}, \quad \rho : T^W T^V \longrightarrow 1_{S\text{-Mod}},$$

defined by the rules:

$$\eta_M(v \otimes w \otimes m) = (v, w)m, \quad \rho_N(w \otimes v \otimes n) = [w, v]n,\tag{5}$$

for  $v \otimes w \otimes m \in T^V T^W(M)$ ,  $M \in R\text{-Mod}$  and  $w \otimes v \otimes n \in T^W T^V(N)$ ,  $N \in S\text{-Mod}$ . By definitions it follows:

$$\text{Im } \eta_M = IM, \quad \text{Im } \rho_N = JN.$$

It is easy to verify the following relations:

$$T^W(\eta_M) = \rho_{T^W(M)}, \quad (6)$$

$$T^V(\rho_N) = \eta_{T^V(N)}, \quad (7)$$

for every  $M \in R\text{-Mod}$  and  $N \in S\text{-Mod}$  (i.e.  $(T^W, T^V)$  and  $(\eta, \rho)$  define a wide Morita context in the sense of [3]).

For an arbitrary class of modules  $\mathcal{K} \subseteq R\text{-Mod}$  we denote:

$$\begin{aligned} \mathcal{K}^\uparrow &= \{X \in R\text{-Mod} \mid \text{Hom}_R(X, Y) = 0 \ \forall Y \in \mathcal{K}\}, \\ \mathcal{K}^\downarrow &= \{Y \in R\text{-Mod} \mid \text{Hom}_R(X, Y) = 0 \ \forall X \in \mathcal{K}\}. \end{aligned}$$

If  $r$  is a torsion of  $R\text{-Mod}$ ,  $\mathcal{R}(r) = \{M \in R\text{-Mod} \mid r(M) = M\}$  and  $\mathcal{P}(r) = \{M \in R\text{-Mod} \mid r(M) = 0\}$ , then  $\mathcal{R}(r) = \mathcal{P}(r)^\uparrow$  and  $\mathcal{P}(r) = \mathcal{R}(r)^\downarrow$  ([5], [7], [8]).

The following statement is known ([6], lemma 3), but for convenience we give the proof.

**Lemma 3.** *If  $(r, s)$  is a pair of corresponding torsions in the sense of Theorem 1, then  $T^W(\mathcal{R}(r)) \subseteq \mathcal{R}(s)$  and  $T^V(\mathcal{R}(s)) \subseteq \mathcal{R}(r)$ .*

*Proof.* Let  ${}_S N \in \mathcal{R}(s) = \mathcal{P}(s)^\uparrow$ , i.e.  $\text{Hom}_S(N, Y) = 0$  for every  $Y \in \mathcal{P}(s)$ . If  $M \in \mathcal{P}(r)$ , then by Lemma 2  ${}_S H^V(M) = \text{Hom}_R(V, M) \in \mathcal{P}(s)$ . Now from  $N \in \mathcal{R}(s)$  it follows that  $\text{Hom}_S(N, \text{Hom}_R(V, M)) = 0$ . By adjunction

$$\text{Hom}_R(V \otimes_S N, M) \cong \text{Hom}_S(N, \text{Hom}_R(V, M)) = 0$$

for every  $M \in \mathcal{P}(r)$ , therefore  $V \otimes_S N \in \mathcal{P}(r)^\uparrow = \mathcal{R}(r)$ , i.e.  $T^V(\mathcal{R}(s)) \subseteq \mathcal{R}(r)$ . By symmetry the relation  $T^W(\mathcal{R}(r)) \subseteq \mathcal{R}(s)$  is true.  $\square$

In continuation we mention some facts about the classes of modules determined by trace ideals  $I \triangleleft R$  and  $J \triangleleft S$  in the categories  $R\text{-Mod}$  and  $S\text{-Mod}$ , respectively. The ideal  $I \triangleleft R$  defines in  $R\text{-Mod}$  the following classes of modules:

$$\begin{aligned} \mathcal{A}(I) &= \{M \in R\text{-Mod} \mid IM = 0\}, \\ \mathcal{J}_I &= \{M \in R\text{-Mod} \mid IM = M\}, \\ \mathcal{F}_I &= \{M \in R\text{-Mod} \mid Im = 0, m \in M \implies m = 0\} = \mathcal{P}(r_I). \end{aligned}$$

The modules of  $\mathcal{J}_I$  are called *I-accessible* and

$$\mathcal{J}_I = \{M \in R\text{-Mod} \mid \text{Im } \eta_M = M\}.$$

The following relations are known ([7], [8]):

$$\mathcal{J}_I = \mathcal{A}(I)^\uparrow, \quad \mathcal{F}_I = \mathcal{A}(I)^\downarrow. \quad (8)$$

Similarly we define the classes  $\mathcal{A}(J)$ ,  $\mathcal{J}_J$  and  $\mathcal{F}_J$  in  $S\text{-Mod}$  with the relations  $\mathcal{J}_J = \mathcal{A}(J)^\uparrow$  and  $\mathcal{F}_J = \mathcal{A}(J)^\downarrow$ , where  $\mathcal{F}_J = \mathcal{P}(r_J)$ .

**Lemma 4.** *Let  $(r, s)$  be a pair of corresponding torsions (Theorem 1). Then  $\mathcal{A}(I) \subseteq \mathcal{R}(r)$  and  $\mathcal{A}(J) \subseteq \mathcal{R}(s)$ .*

*Proof.* From  $r \geq r_I$  it follows  $\mathcal{P}(r) \subseteq \mathcal{P}(r_I) = \mathcal{F}_I$  and by (8) we obtain

$$\mathcal{R}(r) = \mathcal{P}(r)^\uparrow \supseteq \mathcal{P}(r_I)^\uparrow = \mathcal{F}_I^\uparrow = \mathcal{A}(I)^\downarrow \supseteq \mathcal{A}(I).$$

Similarly,  $\mathcal{R}(s) \supseteq \mathcal{A}(J)$ . □

From now on we fix an arbitrary pair  $(r, s)$  of corresponding torsions, i.e.  $r \geq r_I$ ,  $s \geq r_J$ ,  $s = H^V(r)$  and  $r = H^W(s)$  (Theorem 1). We consider the following modifications of the functors  $T^W$  and  $T^V$ :

$$\begin{array}{ccc} R\text{-Mod} & \begin{array}{c} \xleftarrow{T^W} \\ \xrightarrow{T^V} \end{array} & S\text{-Mod} \\ \downarrow 1/r & & \downarrow 1/s \\ R\text{-Mod} & \begin{array}{c} \xleftarrow{\bar{T}^W} \\ \xrightarrow{\bar{T}^V} \end{array} & S\text{-Mod}, \end{array}$$

where  $(1/r)(M) = M/r(M)$ ,  $(1/s)(N) = N/s(N)$ ,  $\bar{T}^W = (1/s) \cdot T^W$  and  $\bar{T}^V = (1/r) \cdot T^V$ . So, by definition:

$$\bar{T}^W({}_R M) = (W \otimes_R M)/s(W \otimes_R M), \quad \bar{T}^V({}_S N) = (V \otimes_S N)/r(V \otimes_S N) \quad (9)$$

for  $M \in R\text{-Mod}$  and  $N \in S\text{-Mod}$ . Denote by  $\alpha$  and  $\beta$  the natural transformations:

$$\alpha : T^W \longrightarrow \bar{T}^W, \quad \beta : T^V \longrightarrow \bar{T}^V,$$

where

$$\alpha_M : T^W(M) \longrightarrow T^W(M)/s(T^W(M))$$

and

$$\beta_N : T^V(N) \longrightarrow T^V(N)/r(T^V(N))$$

are the natural epimorphisms. Since the functors  $T^W$  and  $T^V$  are right exact, it is clear that the functors  $\bar{T}^W$  and  $\bar{T}^V$  preserve epimorphisms. By definitions of  $\bar{T}^W$  and  $\bar{T}^V$  it follows that  $\bar{T}^W(M) \in \mathcal{P}(s)$  and  $\bar{T}^V(N) \in \mathcal{P}(r)$  for every  $M \in R\text{-Mod}$  and  $N \in S\text{-Mod}$ , therefore we can consider the restrictions of these functors on the subcategories  $\mathcal{P}(r)$  and  $\mathcal{P}(s)$ :

$$\mathcal{P}(r) \begin{array}{c} \xrightarrow{\bar{T}^W} \\ \xleftarrow{\bar{T}^V} \end{array} \mathcal{P}(s). \quad (10)$$

In the situation (10) there exist the modifications of natural transformations  $\eta$  and  $\rho$ :

$$\bar{\eta}: \bar{T}^V \bar{T}^W \longrightarrow 1_{\mathcal{P}(r)}, \quad \bar{\rho}: \bar{T}^W \bar{T}^V \longrightarrow 1_{\mathcal{P}(s)},$$

which are defined (see [3]) as follows. For every  $M \in \mathcal{P}(r)$  applying  $T^V$  to the exacte sequence

$$0 \rightarrow s(T^W(M)) \xrightarrow{i_M} T^W(M) \xrightarrow[\text{nat}]{\alpha_M} T^W(M)/s(T^W(M)) \rightarrow 0, \quad (11)$$

we obtain the diagram:

$$\begin{array}{ccccccc} & & & & r(T^V \bar{T}^W(M)) & & \\ & & & & \downarrow i \cap | & & \\ & & & & \downarrow & & \\ T^V(s(T^W(M))) & \xrightarrow{T^V(i_M)} & T^V T^W(M) & \xrightarrow{T^V(\alpha_M)} & T^V \bar{T}^W(M) & \xrightarrow{\beta_{\bar{T}^W(M)}} & \bar{T}^V \bar{T}^W(M) \rightarrow 0 \\ & \searrow \eta_M & & \downarrow \eta'_M & & \nearrow \bar{\eta}_M & \\ & & & M & & & \end{array} \quad (12)$$

Since  $s(T^W(M)) \in \mathcal{R}(s)$ , by Lemma 3  $T^V(s(T^W(M))) \in \mathcal{R}(r)$ , so from  $M \in \mathcal{P}(r)$  it follows  $\text{Hom}_R(T^V(s(T^W(M))), M) = 0$ , therefore  $\eta_M \cdot T^V(i_M) = 0$ . Since  $\text{Im } T^V(i_M) = \text{Ker } T^V(\alpha_M) \subseteq \text{Ker } \eta_M$  and  $T^V(\alpha_M)$  is an epimorphism, there exists a unique morphism  $\eta'_M$  such that  $\eta'_M \cdot T^V(\alpha_M) = \eta_M$ . The following step: from  $M \in \mathcal{P}(r)$  and  $r(T^V \bar{T}^W(M)) \in \mathcal{R}(r)$  it follows  $\eta'_M \cdot i = 0$  and there exists a unique morphism  $\bar{\eta}_M$  such that  $\bar{\eta}_M \cdot \beta_{\bar{T}^W(M)} = \eta'_M$ . So, by definitions we have:

$$\eta_M = \bar{\eta}_M \cdot \beta_{\bar{T}^W(M)} \cdot T^V(\alpha_M). \quad (13)$$

In such a way it is obtained a natural transformations  $\bar{\eta}$  ([3]) and symmetrically  $\bar{\rho}$  is defined. From these definitions follows immediately

**Lemma 5.** a) If the module  $M \in \mathcal{P}(r)$  is  $I$ -accessible (i.e.  $\eta_M$  is epi), then  $\bar{\eta}_M$  is an epimorphism.

b) If the module  $N \in \mathcal{P}(s)$  is  $J$ -accessible, then  $\bar{\rho}_N$  is an epimorphism.  $\square$

Now we consider in  $\mathcal{P}(r)$  and  $\mathcal{P}(s)$  the following subcategories of torsion free and accessible modules:

$$\mathcal{A} = \mathcal{P}(r) \cap \mathcal{J}_I \subseteq R\text{-Mod}, \quad \mathcal{B} = \mathcal{P}(s) \cap \mathcal{J}_J \subseteq S\text{-Mod}.$$

**Lemma 6.** The functors  $\bar{T}^W$  and  $\bar{T}^V$  transfer subcategories  $\mathcal{A}$  and  $\mathcal{B}$  each one in another, i.e.  $\bar{T}^W(\mathcal{A}) \subseteq \mathcal{B}$  and  $\bar{T}^V(\mathcal{B}) \subseteq \mathcal{A}$ .

*Proof.* Let  $M \in \mathcal{A}$ . Since  $\bar{T}^W(M) \in \mathcal{P}(s)$ , it is sufficient to check that  $\bar{T}^W(M) \in \mathcal{J}_J$ . For that we consider the following commutative diagram:

$$\begin{array}{ccc} T^W T^V T^W(M) & \xrightarrow{\rho_{T^W(M)}} & T^W(M) \\ \downarrow T^W T^V(\alpha_M) & & \downarrow \alpha_M \\ T^W T^V \bar{T}^W(M) & \xrightarrow{\rho_{\bar{T}^W(M)}} & \bar{T}^W(M) \end{array} \quad (14)$$

Since  $M \in \mathcal{J}_I$ ,  $\eta_M$  is epi, therefore  $T^W(\eta_M)$  is epi. From (6)  $\rho_{T^W(M)} = T^W(\eta_M)$ , so  $\rho_{T^W(M)}$  is epi, therefore  $\alpha_M \cdot \rho_{T^W(M)}$  also is epi. Now diagram (14) shows that  $\rho_{\bar{T}^W(M)}$  is epimorphism, i.e.  $\bar{T}^W(M) \in \mathcal{J}_J$ . This proves that  $\bar{T}^W(\mathcal{A}) \subseteq \mathcal{B}$ . By symmetry  $\bar{T}^V(\mathcal{B}) \subseteq \mathcal{A}$ .  $\square$

Another proof of Lemma 6 follows from the remark that

$$T^W(\mathcal{J}_I) \subseteq \mathcal{J}_J, \quad T^V(\mathcal{J}_J) \subseteq \mathcal{J}_I. \quad (15)$$

Indeed, if  $M \in \mathcal{J}_I$  then:

$$\begin{aligned} J(W \otimes_R M) &= [W, V]W \otimes_R M = W(V, W) \otimes_R M = \\ &= W \otimes_R (V, W)M = W \otimes_R IM = W \otimes_R M, \end{aligned}$$

i.e.  $T^W(M) \in \mathcal{J}_J$ , and similarly for the second relation.

Now from (15) for every  $M \in \mathcal{J}_I$  we obtain:

$$\begin{aligned} J \cdot \bar{T}^W(M) &= J \cdot [(W \otimes_R M)/s(W \otimes_R M)] = \\ &= [J(W \otimes_R M) + s(W \otimes_R M)]/s(W \otimes_R M) \stackrel{(15)}{=} \\ &= [W \otimes_R M + s(W \otimes_R M)]/s(W \otimes_R M) = \\ &= (W \otimes_R M)/s(W \otimes_R M) = \bar{T}^W(M), \end{aligned}$$

therefore  $\bar{T}^W(M) \in \mathcal{J}_J$ .

Lemma 6 permits to obtain by restriction the functors:

$$\mathcal{A} \begin{array}{c} \xrightarrow{\bar{T}^W} \\ \xleftarrow{\bar{T}^V} \end{array} \mathcal{B} \quad (16)$$

with the natural transformations  $\bar{\eta}$  and  $\bar{\rho}$ .

**Lemma 7.** a) For every  $M \in \mathcal{P}(r)$ ,  $I \cdot \text{Ker } \bar{\eta}_M = 0$ , i.e.  $\text{Ker } \bar{\eta}_M \in \mathcal{A}(I) \subseteq \mathcal{R}(r)$ .

b) For every  $N \in \mathcal{P}(s)$ ,  $J \cdot \text{Ker } \bar{\rho}_N = 0$ , i.e.  $\text{Ker } \bar{\rho}_N \in \mathcal{A}(J) \subseteq \mathcal{R}(s)$ .

*Proof.* From definition of  $\bar{\eta}_M$  (see (12), (13)) it is clear that  $\bar{\eta}_M$  acts as follows:

$$\bar{\eta}_M(\overline{v \otimes (w \otimes m + s(W \otimes_R M))}) = \eta_M(v \otimes w \otimes m) = (v, w)m,$$

where  $\overline{v \otimes (w \otimes m + s(W \otimes_R M))} = \beta_{\bar{T}^W(M)} T^V(\alpha_M)(v \otimes w \otimes m)$ .

If  $\overline{v \otimes (w \otimes m + s(W \otimes_R M))} \in \text{Ker } \bar{\eta}_M$ ,

then  $\eta_M(v \otimes w \otimes m) = (v, m)m = 0$  and for every  $(v', w') \in I$  we obtain:

$$\begin{aligned} (v', w') \overline{v \otimes (w \otimes m + s(W \otimes_R M))} &= \\ &= \overline{(v', w')v \otimes (w \otimes m + s(W \otimes_R M))} = \\ &= \overline{v'[w', v] \otimes (w \otimes m + s(W \otimes_R M))} = \\ &= \overline{v' \otimes ([w', v]w \otimes m + s(W \otimes_R M))} = \\ &= \overline{v' \otimes (w'(v, w) \otimes m + s(W \otimes_R M))} = \\ &= \overline{v' \otimes (w' \otimes (v, w)m + s(W \otimes_R M))} = 0, \end{aligned}$$

because  $(v, w)m = 0$ . From this we can conclude that  $I \cdot \text{Ker } \bar{\eta}_M = 0$  and by Lemma 4  $\text{Ker } \bar{\eta}_M \in \mathcal{A}(I) \subseteq \mathcal{R}(r)$ . The statement (b) follows from symmetry.  $\square$

**Lemma 8.** a)  $\text{Ker } \bar{\eta}_M = 0$  for every  $M \in \mathcal{P}(r)$ . b)  $\text{Ker } \bar{\rho}_N = 0$  for every  $N \in \mathcal{P}(s)$ .

*Proof.* Since  $\text{Ker } \bar{\eta}_M \subseteq \bar{T}^V \bar{T}^W(M) \in \mathcal{P}(s)$ , we have  $\text{Ker } \bar{\eta}_M \in \mathcal{P}(r)$ . By Lemma 7  $\text{Ker } \bar{\eta}_M \in \mathcal{R}(r)$ , therefore  $\text{Ker } \bar{\eta}_M \in \mathcal{R}(r) \cap \mathcal{P}(r) = \{0\}$ . Similarly  $\text{Ker } \bar{\rho}_N = 0$  for  $N \in \mathcal{P}(s)$ .  $\square$

**Theorem 9.** For every pair  $(r, s)$  of corresponding torsions (in the sense of Theorem 1) the functors  $\bar{T}^W$  and  $\bar{T}^V$  (see (10)) with natural transformations  $\bar{\eta}$  and  $\bar{\rho}$  define an equivalence between the subcategories of torsion free and accessible modules  $\mathcal{A} = \mathcal{P}(r) \cap \mathcal{J}_I \subseteq R\text{-Mod}$  and  $\mathcal{B} = \mathcal{P}(s) \cap \mathcal{J}_J \subseteq S\text{-Mod}$ .



*Proof.* If  $M \in \mathcal{A}$ , then by Lemma 5 a)  $\bar{\eta}_M$  is epi. Moreover, from  $M \in \mathcal{P}(r)$  by Lemma 8 a) we conclude that  $\bar{\eta}_M$  is mono, so  $\bar{\eta}_M$  is an isomorphism. Symmetrically, for every  $N \in \mathcal{B}$  we obtain that  $\bar{\rho}_N$  is an isomorphism. Therefore the functors  $\bar{T}^W$  and  $\bar{T}^V$  with the natural transformations  $\bar{\eta}$  and  $\bar{\rho}$  establish the equivalence  $\mathcal{A} \approx \mathcal{B}$ .  $\square$

The more general situation of wide Morita contexts is studied in [3]. The equivalence of Theorem 9 can be proved by [3, Theorem 2.6], using the preceding lemmas. We exposed the direct proof of this result.

For the particular case of the smallest pair  $(r_I, r_J)$  of corresponding torsions we have

**Corollary 10.** ([2], [3]). *The subcategories of torsion free and accessible modules  $\mathcal{P}(r_I) \cap \mathcal{J}_I$  and  $\mathcal{P}(r_J) \cap \mathcal{J}_J$  are equivalent.*  $\square$

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