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**THE CONFERENCE “DYNAMICS IN SIBERIA”,
NOVOSIBIRSK, FEBRUARY 25 – MARCH 2, 2019**

I.A. DYNNIKOV, A.A. GLUTSYUK, A.E. MIRONOV, I.A. TAIMANOV, A.YU. VESNIN

ABSTRACT. In this article abstracts of talks of the Conference “Dynamics in Siberia” held in Sobolev Institute of Mathematics, February 25 – March 2, 2019 are presented.

The conference “Dynamics in Siberia” was held in the Sobolev Institute of Mathematics SB RAS (Novosibirsk) from February 25 to March 2, 2019. Members of the program committee were as follows: I.A. Dynnikov, A.A. Glutsyuk, A.E. Mironov, I.A. Taimanov and A.Yu. Vesnin.

More than 50 experts on dynamical systems, mathematical physics, geometry and topology participated in the conference. The conference program consisted of plenary talks and short talks. The talks were made by well-known experts from Moscow, Novosibirsk, Troitsk, Dubna, Ufa, Nizhny Novgorod, Vladivostok and also by well-known mathematicians from France, Germany, Poland and UK. About 15 young scientists, graduate and undergraduate students participated in the conference. Most of them gave short talks.

DYNNIKOV, I.A., GLUTSYUK A.A., MIRONOV, A.E., TAIMANOV, I.A., VESNIN, A.YU.,
CONFERENCE "DYNAMICS IN SIBERIA NOVOSIBIRSK, FEBRUARY 25 – MARCH 2, 2019.

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PROGRAM (PLENARY TALKS)

February 25

- 10:00 – 10:50 V. Kozlov (*Moscow*). Tensor invariants and integration of differential equations.
- 10:50 – 11:40 H. Rademacher (*Leipzig, Germany*). On the number of closed geodesics.
- 12:00 – 12:50 Yu. Trakhinin (*Novosibirsk*). On local existence and violent instabilities of a plasma–vacuum interface.

February 26

- 10:00 – 10:30 A. Dobrogowska (*Bialystok, Poland*). Tangent lifts of bi-Hamiltonian structures.
- 10:30 – 11:00 M. Guzev (*Vladivostok*). Stability of rolling particles between elastic plates.
- 11:00 – 11:50 A. Glutsyuk (*Lyon, France*). On polynomially integrable billiards on surfaces of constant curvature.
- 12:10 – 13:00 A. Plakhov (*Aveiro, Portugal*). New results in Newton’s problem of minimal resistance.

February 27

- 9:30 – 10:20 A. Chupahin (*Novosibirsk*). Ovsyannikov vortex: exact solution of classical and relativistic hydrodynamic equations.
- 10:20 – 11:10 V. Nazaikinskii (*Moscow*). Billiards with semi-rigid walls, asymptotic eigenfunctions of the $2D$ operator $\nabla D(x)\nabla$, and trapped coastal waves.
- 11:30 – 12:00 A. Orlov (*Moscow*). Hurwitz numbers and matrix integrals labeled with chord diagrams.

February 28

- 10:00 – 10:50 A. Gaifullin (*Moscow*). On homology of Johnson kernel.
- 10:50 – 11:40 A. Buryak (*Leeds, UK*). Simple Lax description of the ILW hierarchy.
- 12:00 – 12:50 V. Grines (*Nizhny Novgorod*). On the topology of manifolds admitting cascades attractor – repeller of the same dimension.
- 12:50 – 13:40 O. Pochinka (*Nizhny Novgorod*). Topological objects in invariant sets of dynamical systems.

March 1

- 9:30 – 10:20 D. Treschev (*Moscow*). On a quantum heavy particle.
- 10:20 – 11:10 Yu. Kordyukov (*Ufa*). Trace formula for the magnetic Laplacian.
- 11:30 – 12:00 I. Krasil’shchik (*Moscow*). $2D$ –reductions of the Mikhalev–Pavlov equation and their nonlocal symmetries.

March 2

- 10:00 – 10:50 P. Akhmet’ev (*Troitsk*). Invariants of knots in geodesic flows.
- 10:50 – 11:40 V. Timorin (*Moscow*). Invariant spanning trees for quadratic rational maps.

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PLENARY TALKS

Invariants of Knots in Geodesic Flows*P. Akhmet'ev (Troitsk)*

The invariant M_3 of 3-component links is introduced in [1]. A new invariant M_5 of oriented 5-component links with cyclic order of components is introduced. Invariants have asymptotic and ergodic properties. An expression of the invariant M_3 by means of coefficients of the Conway polynomial is in [2].

The invariant M_3 is calculated for knotted trajectories of ergodic flows on the lens space S^3/Z_3 . The flows are defined using the geodesic flow on the Lobachevskii plane with the modular group symmetry. The spaces of the flows have a common volume, metrics depend on a curvature. Analogical calculation for Arnold's asymptotic ergodic linking numbers [3] is in [4].

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Simple Lax description of the ILW hierarchy*A. Buryak (Leeds, UK)*

The Intermediate Long Wave (ILW) equation describes the propagation of waves in a two-layer fluid of finite depth. Recently, it also appeared in the computation of certain intersection numbers on the moduli space of Riemann surfaces. The ILW equation possesses an infinite number of infinitesimal symmetries which form the so-called ILW hierarchy. A Lax representation of the ILW equation already appeared in the literature before, however, an explicit relation between the Lax representation of the ILW equation, its higher flows and the Hamiltonian structure was never clarified. In the talk, using the notion of the logarithm of a pseudo-difference operator, I will present an explicit description of the whole ILW hierarchy together with its Hamiltonian structure in terms of a single Lax difference-differential operator.

The talk is based on a joined work with Paolo Rossi.

Ovsyannikov vortex: exact solution of classical and relativistic hydrodynamic equations*A. Cherevko, A. Chupakhin, A. Yanchenko (Novosibirsk)*

Ovsyannikov vortex is a exact solution of continuum mechanics equations which is partially symmetric with respect to the group of rotations $SO(3)$ in the space $R^3(\vec{x}) \times R^3(\vec{u})$. Physical interpretation of this solution is vortex gas or fluid flowing from spherical surface with transverse velocity component. This solution was discovered and first studied by Ovsyannikov [1]. Later, it was studied in a series of papers with additional symmetry for the equations of gas dynamics [2–5], magnetic

hydrodynamics [6], relativistic hydrodynamics [7]. The report provides an overview of previously obtained results and new concerning the mathematical properties of this solution and its physical interpretation.

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Tangent lifts of bi-Hamiltonian structures

A. Dobrogowska (Białystok, Poland)

We construct several Poisson structures on the tangent bundle TM to a Poisson manifold M using the Lie algebroid structure on the cotangent bundle T^*M . We also show that bi-Hamiltonian structure from M can be transferred to its tangent bundle TM . Moreover, we present how to find Casimir functions for those Poisson structures and we discuss some particular examples.

On homology of Johnson kernel

A. Gaifullin (Moscow)

Let S_g be an orientable closed surface. The *mapping class group* Mod_g is the quotient of the group of orientation preserving homeomorphisms of S_g onto itself by the group of homeomorphisms isotopic to the identity. The mapping class group contains a lot of interesting subgroups. The most important of them are the *Torelli group* $\mathcal{I}_g$ and the *Johnson kernel* $\mathcal{K}_g$, $\mathcal{K}_g < \mathcal{I}_g < \text{Mod}_g$. Recall that the Torelli group consists of all mapping classes in Mod_g that act trivially on homology of S_g . In other words, $\mathcal{I}_g$ is the kernel of the natural surjective homomorphism $\text{Mod}_g \rightarrow \text{Sp}_{2g}(\mathbb{Z})$. The Johnson kernel $\mathcal{K}_g$ admits two different definitions. First, it is the kernel of the so-called *Johnson homomorphism*

$$\tau: \mathcal{I}_g \rightarrow \frac{\bigwedge^3 H_1(S_g)}{\ker \left[H_1(S_g) \xrightarrow{-\wedge \Omega} \bigwedge^3 H_1(S_g) \right]}$$

induced by the action of $\mathcal{I}_g$ on $\pi/[[\pi, \pi], \pi]$, where $\pi = \pi_1(S_g)$. (Here $\Omega \in \bigwedge^2 H_1(S_g)$ is the inverse of the intersection form.) Second, $\mathcal{K}_g$ is the subgroup of Mod_g generated by Dehn twists about those curves that separate S_g . So the group $\mathcal{K}_g$ is often called the *bounding twist group*. The equivalence of the above two definitions of $\mathcal{K}_g$ is a deep result due to Johnson (1985).

In 2007, Bestvina, Bux, and Margalit constructed a special cell complex called the *complex of cycles* on which the groups $\mathcal{I}_g$ and $\mathcal{K}_g$ act, and used it to study the homology of $\mathcal{I}_g$ and $\mathcal{K}_g$. They proved that the cohomological dimensions of the groups $\mathcal{I}_g$ and $\mathcal{K}_g$ are equal to $3g - 5$ and to $2g - 3$, respectively. Further, they proved that the top homology group $H_{3g-5}(\mathcal{I}_g)$ is not finitely generated. However, they did not manage to obtain the same result for the top homology group of $\mathcal{K}_g$. This will be the main result of the talk. Namely, we shall show that the group $H_{2g-3}(\mathcal{K}_g)$ contains a free abelian subgroup of infinite rank.

On polynomially integrable billiards on surfaces of constant curvature

A. Glutsyuk (Lyon, France)

The famous Birkhoff Conjecture deals with convex bounded planar billiard Ω with smooth boundary. A particle moves in Ω with constant speed, and as it hits the boundary, it reflects and moves in the reflected direction with velocity of the same module etc. A billiard is *Birkhoff integrable*, if the above dynamical system has a first integral independent with the module of the speed on a neighborhood of the unit tangent bundle to the boundary. If $\partial\Omega$ is an ellipse, then there exists a non-trivial integral quadratic in the velocity. The Birkhoff Conjecture states that every Birkhoff integrable planar billiard is an ellipse. Recently V. Kaloshin and A. Sorrentino proved its local version: every Birkhoff integrable deformation of an ellipse is an ellipse [1].

The polynomial version of the Birkhoff Conjecture, which was first stated and studied by Sergey Bolotin in 1990, concerns *polynomially integrable* billiards, where there exists a first integral polynomial in the velocity that is non-constant on the unit level hypersurface of the module of the velocity.

In this talk we present a brief survey of Birkhoff Conjecture and a complete solution of its polynomial version. We prove that *each bounded polynomially integrable planar billiard with C^2 -smooth non-linear connected boundary is an ellipse*. We prove analogous statement on surfaces of constant curvature (plane, sphere, hyperbolic plane) and classify polynomially integrable billiards with piecewise smooth boundaries on all these surfaces. These are joint results with Mikhail Bialy and Andrey Mironov [2–5].

Thanks. Research supported by RFBR grants 13-01-00969-a, 16-01-00748, 16-01-00766.

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**О топологии многообразий, допускающих каскады
аттрактор – репеллер одинаковой размерности**

В.З. Гринес (Нижний Новгород)

Пусть $f : M^n \rightarrow M^n$ — сохраняющий ориентацию диффеоморфизм гладкого замкнутого ориентируемого многообразия M^n , удовлетворяющий аксиоме А С. Смейла (то есть такой, что его неблуждающее множество $NW(f)$ является гиперболическим и множество его периодических точек плотно в $NW(f)$).

Согласно спектральной теореме С. Смейла множество $NW(f)$ представляется в виде конечного объединения непересекающихся замкнутых инвариантных множеств (называемых базисными), каждое из которых содержит всюду плотную орбиту.

Хорошо известно, что если неблуждающее множество диффеоморфизма f состоит из источниковой и стоковой неподвижных точек, то многообразие M^n диффеоморфно n -мерной сфере S^n . Если же размерность какого-либо базисного множества совпадает с размерностью исходного многообразия, то диффеоморфизм f является диффеоморфизмом Аносова, а само базисное множество является одновременно аттрактором и репеллером и совпадает с многообразием M^n . Как было показано Дж. Френксом и Ш. Ньюхаусом в случае, когда размерность устойчивого или неустойчивого многообразия какой-либо периодической точки диффеоморфизма Аносова равна 1, многообразие M^n гомеоморфно тору размерности n (см. [1], [2]).

В докладе приводятся результаты, полученные в работах В.З. Гринеса, Е.В. Жужомы, Ю.А. Левченко, В.С. Медведева, О.В. Починки (см. [3]–[5]), из которых следует топологическая классификация многообразий M^n , допускающих диффеоморфизмы f , неблуждающее множество которых состоит из аттрактора и репеллера одинаковой размерности. Кроме того, приводятся достаточные условия на диффеоморфизм f , при которых его неблуждающее множество не может состоять из двух базисных множеств одинаковой размерности.

Благодарности. Доклад подготовлен при финансовой поддержке Российского Научного Фонда (проект 17–11–01041).

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Stability of rolling particles between elastic plates

M. Guzev (Vladivostok)

In many applications stability of systems with rotating non-spherical particles is of essence. We show that in the simplest case such systems can be modelled as

a system of two linked oscillators (masses on rods) and a modified system of these oscillators whose rods intersect and slide without friction relative to each other. The oscillators are linked by a linear elastic spring and posed vertically in a uniform gravity field. We demonstrate that both models have symmetrical and asymmetrical equilibrium solutions depending on the spring stiffness and distance between the oscillators' suspension centres (points of contacts of the particles). Relations of these parameters are obtained identifying the stability region at the upper and bottom oscillator positions.

Trace formula for the magnetic Laplacian

Yu. Kordyukov (Ufa)

Given a Riemannian manifold equipped with a magnetic field 2-form, the associated magnetic Laplacian is the Schroedinger operator with magnetic field and vanishing electric potential. In this talk, we will discuss the Guillemin–Uribe trace formula, which relates some asymptotic spectral invariants of the magnetic Laplacian with geometric and dynamical invariants of the associated magnetic geodesic flow. First, we will explain the formula. Then we will describe concrete examples of its computation for two-dimensional constant curvature surfaces with constant magnetic fields and for the Katok example.

This is joint work with Iskander A. Taimanov.

Тензорные инварианты и интегрирование дифференциальных уравнений

В.В. Козлов (Москва)

Обсуждается связь тензорных инвариантов систем дифференциальных уравнений с проблемой их точного интегрирования. Доказана общая теорема об интегрируемости динамических систем, допускающих полный набор интегральных инвариантов по Картану. Наличие инвариантной 1-формы связано с возможностью представления динамической системы в гамильтоновой форме (возможно, с вырожденной симплектической структурой). Эта общая идея продемонстрирована на примере линейных систем дифференциальных уравнений. Введено общее понятие флагов тензорных инвариантов. Установлены общие соотношения между показателями Ковалевской квазиоднородных систем дифференциальных уравнений и флагами квазиоднородных тензорных инвариантов известной структуры. Результаты общего характера применены, в частности, для доказательства ветвления общего решения уравнений вращения твердого тела в случае Горячева–Чаплыгина.

2D-reductions of the Mikhalev–Pavlov equation and their nonlocal symmetries

I. Krasil'shchik (Moscow)

The Mikhalev–Pavlov equation (MPE) reads $u_{yy} = u_{tx} + u_y u_{xx} - u_x u_{xy}$ and belongs to the class of integrable linearly degenerate equations. It admits an infinite-dimensional Lie algebra of symmetries and all its 2D-symmetry reductions were described in [1]. Among these reductions, the following ones are of a special interest: (1) $u_y u_{xy} - u_x u_{yy} = e^y u_{xx}$, (2) $u_{xx} = (x - u_y) u_{xy} + (2y + u_x) u_{yy} - u_y$, (3) $u_{yy} = (u_y + y) u_{xx} - u_x u_{xy} - 2$ (the last one is equivalent to the Gibbons–Tsarev equation). Under

the reductions, the isospectral Lax pair of MPE transforms to rational differential coverings of the form

$$(1) \quad w_x = \frac{a_2 w^2 + a_1 w + a_0}{w^2 + c_1 w + c_0}, \quad w_y = \frac{b_2 w^2 + b_1 w + b_0}{w^2 + c_1 w + c_0}$$

for the reduced equations, where a_i , b_i , and c_i are real constants, [2]. The standard “reversion procedure”, makes it possible to introduce a fake spectral parameter to (1) and construct infinite series of conservation laws together with the corresponding infinite-dimensional coverings. Using the known description of nonlocal symmetries for MPE, [3], it is proved that the algebras of nonlocal symmetries for reductions are isomorphic to the Witt algebra, cf. [4].

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Billiards with semi-rigid walls, asymptotic eigenfunctions of the 2D operator $\nabla D(x)\nabla$, and trapped coastal waves

V. Nazaikinskii (Moscow)

We construct asymptotic eigenfunctions of the two-dimensional operator $\hat{L} = \nabla D(x)\nabla$ in a domain Ω with coefficient $D(x)$ degenerating on the boundary $\partial\Omega$. These eigenfunctions are associated with Liouville tori of integrable geodesic flows with a metric degenerating on $\partial\Omega$. Such geodesic flows can be called “billiards with semi-rigid walls”.

The talk is based on joint work with A.Yu. Anikin, S.Yu. Dobrokhotov, and A.V. Tsvetkova.

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Hurwitz numbers and matrix integrals labeled with chord diagrams

A. Orlov (Moscow)

We shall consider the product of complex random matrices from the independent complex Ginibre ensembles. The product includes complex matrices $Z_i, Z_i^\dagger, i = 1, \dots, n$ and $2n$ sources (complex matrices C_i and C_i^*). Any such product can be represented by a chord diagram that encodes the order of the matrices in the product. We introduce the Euler characteristic E^* of the chord diagram and show that the spectral correlation functions of the product generate Hurwitz numbers that enumerate nonequivalent branched coverings of Riemann surfaces of genus g^* .

The role of sources is the generation of branching profiles in critical points which are assigned to the vertices of the graph drawn on the base surface obtained as a result of gluing of the $2n$ -gon related to the chord diagram in a standard way. Hurwitz numbers for Klein surfaces may also be obtained by a slight modification of the model. Namely, we consider $2n+1$ polygon and consider pairing of the extra matrix Z_{2n+1} with a “Möbius” tau function. Thus, the presented matrix models labelled by chord diagrams generate Hurwitz numbers for any given Euler characteristic of the base surface and for any given set of ramification profiles.

New results in Newton’s problem of minimal resistance

A. Plakhov (Moscow)

Isaac Newton posed this problem nearly 330 years ago in his *Mathematical Principles of Natural Philosophy*. It is as follows. A body moves in a highly rarefied medium of point particles at rest, and the particles reflect elastically when colliding with the body’s surface. It is required to find the shape of the body that minimizes the force of aerodynamic resistance of the medium. Starting from 1993, new interest in mathematical community to Newton’s problem has been raised. The problem proved to be highly interdisciplinary, and various aspects of it were studied using methods borrowed from multidimensional variational analysis, theory of billiards, optimal mass transport, Kakeya needle problem, and theory of convex bodies. Interesting connections of the problem with geometric optics were found, including invisibility, retroreflectors, and the problem of camouflaging. A review of the state of art in this area will be given.

Topological objects in invariant sets of dynamical systems

O. Pochinka (Nizhny Novgorod)

Various topological constructions naturally emerge in the modern theory of dynamical systems. For instance, the Cantor set, discovered as an example of a set with cardinality of the continuum and zero Lebesgue measure, clarified the structure of expanding attractors and contracting repellers. Fractals, being self-similar objects with fractional dimension, are naturally found in complex dynamics. For example, the basin boundary of an attracting point can be the Julia set. The lakes of Wada, showing the phenomenon of a curve dividing the plane into more than two domains, were used in the construction of the Plykin attractor on the 2-sphere. A curve contained in the 2-torus and having an irrational winding number, being an injectively immersed subset but not a topological submanifold, was realized as an invariant manifold of a fixed point of the Anosov diffeomorphism of the 2-torus. The Artin–Fox arc [1] and the mildly wild frame of Debrune–Fox arcs [2], symbolizing a wild set of hand arcs in $\mathbb{R}^3$, are realized by a frame of one-dimensional separatrix of Morse–Smale diffeomorphism on the three-dimensional sphere [3–5]. These parallels can be continued for quite a long time, and this report is devoted to the construction of dynamic elements based on known topological objects.

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On the number of closed geodesics

H. Rademacher (Leipzig, Germany)

It is an open question whether any Riemannian metric on a compact manifold has infinitely many geometrically distinct closed geodesics. We first revisit results for generic metrics in the case of simply-connected manifolds. The genericity assumption is defined in terms of the linearization of the local return map of the periodic orbits of the geodesic flow. Then we discuss recent results obtained jointly with Iskander Taimanov on the number of closed geodesics on connected sums of non-simply connected and compact manifolds. In particular we conclude that any Riemannian metric on a three-dimensional compact manifold with infinite fundamental group has infinitely many closed geodesics.

Invariant spanning trees for quadratic rational maps

V. Timorin (Moscow)

This is a joint project with A. Shepelevtseva.

A theorem of W. Thurston (sometimes called the fundamental theorem of complex dynamics) opens a door for algebraic, topological and combinatorial methods into dynamics of rational maps on the Riemann sphere. We study Thurston equivalence classes of quadratic post-critically finite rational maps by means of “visual” invariants, the so called invariant spanning trees. A computational procedure for searching for invariant spanning trees will be described. This procedure uses bisets over the fundamental group of a punctured sphere.

On local existence and violent instabilities of a plasma–vacuum interface

Yu. Trakhinin (Novosibirsk)

We discuss the influence of electric and magnetic fields on the well-posedness in Sobolev spaces of the plasma–vacuum interface problem. For the classical statement of the problem [1], the plasma flow is governed by the equations of ideal compressible or incompressible magnetohydrodynamics while the vacuum magnetic field obeys the div–curl system of pre-Maxwell dynamics. We show that the Rayleigh–Taylor sign condition is not necessary for well-posedness because the plasma and vacuum magnetic fields play a stabilizing role if they are not collinear at each point of the interface [2, 3]. At the same time, the simultaneous failure of the non-collinearity condition and the Rayleigh–Taylor sign condition leads to Rayleigh–Taylor instability [4]. For the non-classical statement of the problem [5], when we do not neglect the displacement current in the vacuum region and consider

the Maxwell equations for electric and magnetic fields, we show that a sufficiently large vacuum electric field makes the interface violently unstable. Moreover, for the technically simpler case of incompressible plasma [6] one can show that as soon as the plasma and vacuum magnetic fields are collinear on the interface, the interface is always violently unstable for any nonzero and even very small vacuum electric field.

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On a quantum heavy particle

D. Treschev (Moscow)

We consider Schrödinger equation for a particle on a flat n -torus in a bounded potential, depending on time. Mass of the particle equals $1/\mu^2$, where μ is a small parameter. We show that the Sobolev H^ν -norms, $\nu \geq 1$ of the wave function grow approximately as t^ν on the time interval $t \in [0, t_*]$, where t_* is slightly less than $O(1/\mu)$.

SHORT TALKS

Существование энергетической функции для 3–диффеоморфизмов с одномерными поверхностными аттрактором и репеллером*М. Баринаева (Нижний Новгород)*

Функция Ляпунова была введена А.М. Ляпуновым для изучения устойчивости состояний равновесия дифференциальных уравнений. Гладкая функция Ляпунова, множество критических точек которой совпадает с цепно–рекуррентным множеством системы, называется энергетической функцией. Такие функции всегда существуют для потоков. В то же время построены примеры каскадов, которые не имеют энергетической функции. Первый пример такого диффеоморфизма с регулярной динамикой был построен на 3–сфере Д. Пикстоном [1] в 1997 году с использованием дикой дуги Артина–Фокса [2]. Тем удивительнее факт существования энергетической функции для систем с хаотической динамикой, обнаруженный в 2015 году В.З. Гринесом, М.К. Баринаевой и О.В. Починкой [3]–[5]. Ими было доказано существование энергетической функции для поверхностных Ω –устойчивых диффеоморфизмов с одномерными нетривиальными базисными множествами и для грубых трехмерных диффеоморфизмов с поверхностными базисными множествами. В докладе рассматриваются Ω –устойчивые 3–диффеоморфизмы, неблуждающее множество которых состоит в точности из одного поверхностного аттрактора и одного поверхностного репеллера. Доказывается существование энергетической функции для любого диффеоморфизма из этого класса.

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**Separation of variables for type D_n
Hitchin systems on hyperelliptic curves**

P. Borisova (Moscow)

Darboux coordinates for the Hitchin systems were known only in case of Lie algebra $sl(2)$ and a curve of genus 2, until recently. A description of the class of

spectral curves for the Hitchin systems on hyperelliptic curves of all genera was given by O.K. Sheinman (2018).

For Lie algebras of types A_n, B_n, C_n the Darboux coordinates were found there in an explicit form. The main goal of my talk is to exhibit the difficulties arising in the case D_n , and find out Darboux coordinates for $n = 2$ (Lie algebra $so(4)$) and genus two.

The talk is a natural continuation of the talk by O.K. Sheinman, where the cases of Lie algebras A_n, B_n, C_n are discussed.

Solving triangular Schlesinger systems via periods of meromorphic differentials

R. Gontsov (Moscow)

We study the Schlesinger system of PDEs for N matrices of size $(p \times p)$ in the case when they are triangular and the eigenvalues of each matrix form an arithmetic progression with a rational difference q , the same for all matrices. We show that such a system possesses a family of solutions expressed via periods of meromorphic differentials on the Riemann surfaces of superelliptic curves. As an application to the (2×2) -case, explicit solutions of Painlevé VI equations and Garnier systems are obtained.

On the convergence of various formal series solutions of algebraic ODEs

I. Goryuchkina (Moscow)

We extend studies of the convergence of formal power series solutions of algebraic ODEs investigating in that context formal solutions of a more general form: generalized power series, Dulac series and exotic series solutions. Sufficient conditions of convergence are proposed for such series.

О топологической классификации каскадов Морса–Смейла посредством комбинаторных инвариантов

Е.Я. Гуревич (Нижний Новгород)

Диффеоморфизм $f : M^n \rightarrow M^n$ гладкого замкнутого многообразия M^n называется *диффеоморфизмом Морса–Смейла*, если его неблуждающее множество Ω_f конечно, состоит из гиперболических периодических точек, и для любых двух точек $p, q \in \Omega_f$ пересечение устойчивого многообразия точки p и неустойчивого многообразия точки q трансверсально (см., например, [1] для знакомства с основными понятиями).

Условие конечности множества неблуждающих орбит дает возможность выделять содержательные классы систем Морса–Смейла, для которых проблема топологической классификации сводится к комбинаторной задаче описания взаимного расположения инвариантных многообразий неблуждающих орбит в несущем многообразии. Этот подход был впервые применен Е.А. Леонтович и А.Г. Майером для классификации потоков на двумерной сфере с конечным числом особых траекторий и позднее был развит М. Пейшото, А.А. Ошемковым, В.В. Шарко, Я.Л. Уманским и С.Ю. Пилюгиным, которые решали аналогичную задачу для потоков Морса–Смейла на многообразиях размерности

2, 3 и выше, а также Х. Бонатти, А.Н. Безденежных, В.З. Гринесом, Е.Я. Гуревич, В.С. Медведевым, Р. Ланжевром, О.В. Починкой и др. для каскадов Морса–Смейла на многообразиях (см. для ссылок обзор [2]).

В докладе устанавливается, что диффеоморфизмы Морса–Смейла, заданные на сфере S^n , $n \geq 4$, и не имеющие гетероклинических пересечений, допускают полную топологическую классификацию на комбинаторном языке. Этот результат контрастирует со случаем каскадов Морса–Смейла на многообразиях размерности $n = 3$ (см. [2], [3]).

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Топологическая сопряжённость градиентно-подобных потоков на n -мерной сфере

В. Круглов (Нижегород)

Градиентно-подобные потоки — это непрерывные динамические системы, у которых неблуждающее множество состоит из конечного числа гиперболических неподвижных точек, чьи инвариантные многообразия пересекаются трансверсально.

В зависимости от целей исследования важно знать как особенности качественного поведения системы (то есть разбиения многообразия на траектории), так и повременного движения по траекториям. В теории динамических систем *топологическая эквивалентность* потоков есть наличие гомеоморфизма, переводящего траектории одного потока в траектории другого с сохранением направления движения; если такой гомеоморфизм ещё сохраняет время движения по траекториям, говорят о *топологической сопряжённости* потоков. Нахождение инварианта, определяющего класс топологической эквивалентности для системы, называется *топологической классификацией*.

Конечность множества неблуждающих траекторий градиентно-подобного потока позволяет свести задачу топологической классификации к комбинаторной проблеме. Впервые это было сделано Е.А. Леонтович и А.Г. Майером в [1], [2] для классификации потоков с конечным числом особых траекторий на двумерной сфере. Эти результаты позже получили развитие в исследованиях М. Пейшото [3], А.А. Ошемкова, В.В. Шарко [4], С.Ю. Пилюгина [5], А.О. Пришляка [6], где аналогичная проблема была решена для потоков Морса–Смейла на замкнутых многообразиях размерностей 2,3 и выше. Перечисленные работы были посвящены топологической эквивалентности потоков. В работе [7] показано, что для градиентно-подобных потоков на поверхностях эквивалентность

и сопряжённость совпадают, следовательно, все результаты относительно эквивалентности можно использовать и для проверки сопряжённости. В настоящей работе мы получили аналогичный результат для класса G градиентно-подобных потоков без гетероклинических траекторий на n -сфере, $n \geq 3$. Кроме того, мы вводим топологический комбинаторный инвариант для таких потоков — *двухцветный граф*, и доказываем, что два потока рассматриваемого класса топологически сопряжены тогда и только тогда, когда их двухцветные графы изоморфны.

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Об одномерных базисных множествах эндоморфизмов двумерного тора *Е.Д. Куренков (Нижний Новгород)*

В 1967 году Смейл [1] предложил способ построения нетривиальных базисных множеств, отталкиваясь от диффеоморфизма Аносова. Фактически, Смейл схематично описал хирургическую операцию над диффеоморфизмом Аносова, в результате которой получается ДА-диффеоморфизм с базисными множествами, имеющими топологическую размерность на единицу меньшую, чем размерность несущего многообразия.

Идея хирургической операции состоит в том, что вместо неподвижной седловой точки в ее малой окрестности образуется определенным образом одна узловая (стоковая или источниковая) и две седловые неподвижные точки. При этом одинаково возможны следующие два сценария 1) узловая неподвижная точка является источником; 2) узловая неподвижная точка является стоком.

В первом случае неблуждающее множество полученного ДА-диффеоморфизма состоит из источниковой неподвижной точки одномерного растягивающегося аттрактора. Во втором случае неблуждающее множество

полученного ДА-диффеоморфизма состоит из стоковой неподвижной точки и одномерного сжимающегося репеллера.

В [2] был введен класс не взаимно однозначных гладких отображений, обобщающих диффеоморфизмы Аносова. Представляется вполне естественным рассмотреть хирургическую операцию для алгебраических эндоморфизмов, например, для эндоморфизма Аносова $f_0: \mathbb{T}^2 \rightarrow \mathbb{T}^2$, заданного формулой

$$\begin{cases} \bar{x} = 3x + y \\ \bar{y} = x + y \end{cases} \pmod{1}. \quad (1)$$

Поскольку f_0 является локальным диффеоморфизмом, то оба сценария хирургической операции формально возможны.

В работе [3] был численно реализован второй сценарий хирургической операции Смейла для алгебраического эндоморфизма Аносова вида (1) в результате которого был построен A -эндоморфизм $f: \mathbb{T}^2 \rightarrow \mathbb{T}^2$ двумерного тора, неблуждающее множество которого состоит из гиперболического сжимающегося репеллера Λ и одной стоковой неподвижной точки O .

Основной результат настоящего доклада состоит в том, что первый сценарий для эндоморфизма Аносова (1) не приводит к A -эндоморфизму с нетривиальным одномерным растягивающимся аттрактором, обладающим аналогичными свойствами [4].

Теорема. Пусть $f: \mathbb{T}^2 \rightarrow \mathbb{T}^2$ — A -эндоморфизм, являющийся k -накрытием, $k \geq 2$. Тогда f не имеет одномерных аттракторов Λ , удовлетворяющих следующим условиям:

- Λ является строго инвариантным;
- неустойчивое многообразие $W^u(x)$ каждой точки $x \in \Lambda$ не зависит от ее отрицательной полуорбиты и является одномерной незамкнутой кривой;
- имеет место равенство $\bigcup_{x \in \Lambda} W^u(x) = \Lambda$ и Λ образует ламинацию локально гомеоморфную произведению отрезка на канторовское множество;
- любая достижимая изнутри граница компоненты связности множества $\mathbb{T}^2 \setminus \Lambda$ состоит из конечного числа слоев ламинации Λ .

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**Numerical algorithms for the direct spectral
Zakharov–Shabat problem with application
to the solution of nonlinear equations**

S. Medvedev (Novosibirsk), I. Vaseva, I. Chekhovskoy, M. Fedoruk

The numerical implementation of the nonlinear Fourier transformation (NFT) for the nonlinear Schrödinger equation (NLSE) requires effective numerical algorithms for each stage of the method. The very first step in this scheme is the solution of the direct scattering problem for the Zakharov–Shabat system. One of the most efficient methods for the solution of this problem is the second-order Boffetta–Osborne algorithm [1].

In this report we propose a generalization of the Boffetta–Osborne method. A two-parametric family of one-step fourth-order difference schemes is constructed. It requires the potential values at three neighboring points and reduces to the Boffetta–Osborne scheme in case of constant potentials. The family contains a scheme (super-scheme) that preserves the integral of the system for the continuous spectrum.

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**О простом изотопическом классе градиентно-подобных
диффеоморфизмов на двумерной сфере**

Е. Ноздринова (Нижегород)

В 1976 году Ш. Ньюхаусом, Дж. Палисом, Ф. Такенсом [1] было введено понятие устойчивой дуги, соединяющей две структурно устойчивые системы на многообразии. Такая дуга не меняет своих качественных свойств при малом шевелении. В том же году Ш. Ньюхаус и М. Пейшото [2] доказали существование устойчивой дуги между любыми двумя потоками Морса–Смейла. Для диффеоморфизмов Морса–Смейла на многообразиях любой размерности известны примеры систем, которые не могут быть соединены устойчивой дугой. В связи с этим возникает важный вопрос о нахождении инварианта, однозначно определяющего класс эквивалентности диффеоморфизма Морса–Смейла относительно отношения связности устойчивой дугой (компонента устойчивой связности). Так для сохраняющих ориентацию грубых преобразований окружности таким инвариантом является число вращения Пуанкаре [3]. Для диффеоморфизмов Морса–Смейла на поверхностях П. Бланшаром [4] установлены некоторые необходимые условия существования соединяющей их устойчивой дуги. Из этих условий в частности следует, что даже на двумерной сфере компонент устойчивой связности бесконечно много. Настоящий доклад посвящен определению необходимых и достаточных условий существования устойчивой дуги, соединяющей градиентно-подобные диффеоморфизмы на 2-мерной сфере.

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A new class of exact solutions for three–dimensional quasilinear systems of first order

M. Pavlov (Moscow)

Well-known Lin–Reissner–Tsien equation in aerodynamics (1948) will be considered. This equation also is known as the Khokhlov–Zabolotskaya equation in nonlinear acoustics, and is known as a dispersionless limit of the Kadomtsev–Petviashvili equation in hydrodynamics.

New ansatz for construction of infinitely many two-dimensional reductions is found for this three-dimensional equation. They are generalisations of two-dimensional hydrodynamic reductions. In one-component case, a corresponding particular solution is found with five arbitrary functions of a single variable. Also some other three-dimensional integrable quasilinear systems of first order will be considered.

Deformation quantization of commutative families

G. Sharygin (Moscow)

In my talk I will describe a series of cohomological obstructions, that regulate the existence of elements in deformation quantization of a Poisson manifold, that would extend the given commutative (w.r. to the Poisson brackets) family of smooth functions to a family of commuting (in the usual sense) elements in the deformed algebra of functions on this manifold. In the end I will describe a conjecture, that gives a more geometric way to construct such obstructions in the particular case, when the manifold in question is symplectic.

Thanks. The talk is based on work, supported by the grant NSh–6399.2018.1.

A viscous approximation of crack propagation in elastic bodies

V. Shcherbakov (Novosibirsk)

We discuss a rate-independent model for crack propagation in 2D elastic bodies without prescribing a priori the crack path. Due to the dependence on the crack path the energy is nonconvex; therefore, solutions may have jumps as a function of time. We employ a viscous approximation of the model and consider it as a limit of systems driven by viscous, rate-dependent dissipation in order to prove the existence of solutions that satisfy the Griffith fracture criterion and to describe accurately the behavior of the solutions at jumps.

**Spectral curves and separation of variables for
hyperelliptic Hitchin systems of types A_n, B_n, C_n**
O. Sheinman (Moscow)

We give a description of families of spectral curves for Hitchin systems of types A_n, B_n, C_n on hyperelliptic curves of all genera. On this basis, we develop the separation of variables scheme for those systems, and explicitly find out Darboux coordinates for them. The case D_n will be considered in the related talk by P. Borisova.

Automorphism groups of Hopf and Kodaira surfaces
C. Shramov (Moscow)

Hopf and Kodaira surfaces are certain compact complex (analytic) surfaces that cannot be deformed to projective surfaces. I will survey various results concerning their automorphism groups, in particular, the structure of finite subgroups acting on such surfaces and their relation to elliptic fibrations on them.

**On a complexification of the moduli space
of Bohr–Sommerfeld lagrangian cycles.**
N. Tyurin (BLTP Ph JINR (Dubna) and NRU HSE (Moscow))

Let (M, ω) — be a compact simply connected symplectic manifold of real dimension $2n$ with integer symplectic form such that $[\omega] \in H^2(M, \mathbb{Z}) \subset H^2(M, \mathbb{R})$. Consider prequantization data (L, a) , where $L \rightarrow M$ — is a complex line bundle with a fixed hermitian structure and $a \in \mathcal{A}_h(L)$ — a hermitian connection whose curvature form satisfies $F_a = 2\pi i \omega$. Choose and fix an appropriate topological type S of smooth oriented n -dimensional manifold and a homology class $[S] \in H_n(M, \mathbb{Z})$, then it leads to the derivation of the corresponding moduli space $\mathcal{B}_S$ of Bohr–Sommerfeld lagrangian cycles of fixed topological type, which has been constructed in [1]. This moduli space is an infinite dimensional Frechet smooth real manifold, locally isomorphic to space $C^\infty(S, \mathbb{R})$ modulo constants. The points of $\mathcal{B}_S$ can be presented by lagrangian submanifolds of fixed topological type satisfying the Bohr–Sommerfeld condition: the restriction of the prequantization data $(L, a)|_S$ admits covariantly constant sections. The details of the construction can be found in [1].

The moduli space $\mathcal{B}_S$ can be exploited in the lagrangian approach to Geometric Quantization: the corresponding “quantum” mechanics can be directly described while the measurement process, which any quantum mechanical system must admit, can be defined only after an appropriate “complexification” of the moduli space $\mathcal{B}_S$ is given. This problem is in the focus of our studies.

Consider the space $\Gamma(M, L)$ of all smooth sections of the prequantization bundle and its projectivization $\mathbb{P}\Gamma(M, L)$. One can attach to every element of the last space certain complex 1-form, defined on the open complement M_α where D_α is the zero set of the corresponding section. Namely for every $[\alpha] \in \mathbb{P}\Gamma(M, L)$ take a smooth section α , defined up to scaling, and consider the following expression

$$\rho(\alpha) = \frac{\nabla_a \alpha}{\alpha} = \frac{\langle \nabla_a \alpha, \alpha \rangle_h}{\langle \alpha, \alpha \rangle_h} \in \Omega^1_{M_\alpha} \otimes \mathbb{C},$$

where D_α corresponds to α . Evidently this 1-form $\rho(\alpha)$ does not depend on the scaling $c\alpha$, therefore this form corresponds not to a section but to the class $[\alpha] \in \mathbb{P}\Gamma(M, L)$.

The main properties of this 1-form $\rho(\alpha)$ are the following: its real part $\text{Re}\rho(\alpha)$ is exact, and the differential of the imaginary part satisfies $d\rho(\alpha) = 2\pi\omega$ (see [2]).

Then we say that

Definition 1. *Lagrangian submanifold $S \subset M$ is special Bohr–Sommerfeld with respect to α (or α – SBS for short) iff the restriction $\text{Im}\rho(\alpha)|_S$ identically vanishes.*

In particular it implies that S does not intersect zeroset D_α .

Remark 1. *In paper [2] we originally used another definition of SBS and then deduced the vanishing condition for the imaginary part as a proposition. However in the present situation we skip the story and use the vanishing condition as the definition.*

SBS — condition, introduced above, leads to the following

Definition 2. *Subset $\mathcal{U}_{SBS} \subset \mathcal{B}_S \times \mathbb{P}\Gamma(M, L)$ is formed by the pairs $(S, [\alpha])$, such that S satisfies α – SBS condition.*

By the very definition the subset $\mathcal{U}_{SBS}$ admits two canonical projections $p : \mathcal{U}_{SBS} \rightarrow \mathbb{P}\Gamma(M, L)$ and $q : \mathcal{U}_{SBS} \rightarrow \mathcal{B}_S$. The first projection has been studied in [2], there one establishes that the fibers of the projections are discrete; the image of the projection is an open subset in the projective space; the differential dp has trivial kernel at generic point (Theorem 1, [2]). It follows that $\mathcal{U}_{SBS}$ is weakly Kahler: the standard Kahler form from the projective space $\mathbb{P}\Gamma(M, L)$ can be lifted using the differential dp .

In preprint [3] we use the correspondence $[\alpha] \leftrightarrow \rho(\alpha)$ to prove that the differential dp is an isomorphism: any tangent to $\mathbb{P}\Gamma(M, L)$ vector can be canonically lifted to a tangent one to the moduli space $\mathcal{U}_{SBS}$; in particular it implies the existence of the complex structure on $\mathcal{U}_{SBS}$, lifted by the differential dp .

However the space $\mathcal{U}_{SBS}$ is not a complexification of the moduli space $\mathcal{B}_S$, since it is too big. Indeed, the “dimension” of the moduli space $\mathcal{B}_S$ equals to the dimension of $C^\infty(S, \mathbb{R})$ modulo constants while the “dimension” of $\mathcal{U}_{SBS}$ is the same as for $\mathbb{P}\Gamma(M, L)$, so it is of order $C^\infty(M, \mathbb{C})$ modulo constants. On the other hand it exists a natural map $\tau : \mathcal{U}_{SBS} \rightarrow T\mathcal{B}_S$, which splits the second projection q , namely one has the representation $q = \tau \circ \pi$, where $\pi : T\mathcal{B}_S \rightarrow \mathcal{B}_S$ is the canonical projection to the base. The map τ is given by a simple and exact expression: since Definition 1 implies that for any point $(S, [\alpha]) \in \mathcal{U}_{SBS}$ the restriction $\rho(\alpha)|_S$ is an exact real 1-form, but from [1] we know that any exact real 1-form is a tangent vector to $\mathcal{B}_S$ at point S , consequently we get a simple formula

$$\tau(S, [\alpha]) = (\rho(\alpha))|_S \in T_S\mathcal{B}_S,$$

and it is easy to see that applying π we get the image $q(S, [\alpha]) = S \in \mathcal{B}_S$.

In [3] we show that a generic fiber $\tau^{-1}(S, df) \subset \mathcal{U}_{SBS}$ is a complex subset in $\mathcal{U}_{SBS}$ with respect to the complex structure, given by the previous construction. It should lead to a construction of derived complex structures on the tangent bundle $T\mathcal{B}_S$ itself: suppose that one finds an appropriate section of the fibered map $\tau : \mathcal{U}_{SBS} \rightarrow T\mathcal{B}_S$, which is Kahler orthogonal to the fibers or at least which is a complex subset

in $\mathcal{U}_{SBS}$. Right now we can not present a good candidate, hoping that it will be found in the nearest future.

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Vector bundles and difference equations in a complex domain

I. Vyugin (Moscow)

Applications of the theory of holomorphic vector bundles with meromorphic connections to the classical Riemann–Hilbert problem are well known. We are going to apply holomorphic vector bundles with meromorphic additive shift or q -shift to studying of generalized Riemann–Hilbert–Birkhoff problem for difference and q -difference systems.

As the application of this approach we obtain a generalization of Birkhoff’s existence theorem and reprove the local existence theorem. We prove that for any admissible set of characteristic constants and monodromy there exists a system

$$Y(z+1) = A(z)Y(z) \quad \text{or} \quad Y(qz) = Q(z)Y(z)$$

which has the given monodromy and characteristic constants.

Rate of equidistribution for the unstable manifolds of Anosov diffeomorphisms

D. Zubov (Moscow)

Let M be a compact Riemannian manifold. For a C^3 smooth topologically mixing Anosov diffeomorphism $F : M \rightarrow M$, we study the equidistribution properties of the unstable manifolds with respect to the Margulis measure of maximal entropy $\mathbf{m}$. Extending the results of Bufetov and Bufetov–Forni on geodesic/horocycle flows on compact Riemann surfaces of constant negative curvature to a non-linear setting, we prove that, under certain bounded distortion assumptions on the diffeomorphism, the leafwise averages on the unstable leaves of a C^2 smooth function $\psi : M \rightarrow \mathbb{R}$ with $\mathbf{m}(\psi) = 0$ are controlled by a finitely additive measure on the unstable foliation, invariant under the holonomy along stable leaves.

Using the method Gouzel and Liverani, we construct a Banach space of currents which admits an F -invariant finite dimensional subspace whose elements induce holonomy invariant finitely additive measures.

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