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THE INFRARED BOUNDS METHOD IN THE STUDY OF BOSON SYSTEMS

This is a study of the equilibrium thermodynamics of a lattice boson gas with on-site repulsion and nearest neighbor site attraction. For this system, the existence of Bose condensate is proven and an equation for the lower estimate on the critical temperature is obtained. Moreover, the upper and lower bounds for the structure factor are derived. Finally, in the framework of the infrared bounds method, a Huang–Davies type Bose system is studied.

1. INTRODUCTION

The aim of this paper is to study a ν -dimensional lattice system of bosons with on-site repulsion and nearest neighbour attraction. In this work, we apply the so-called method of infrared bounds [1–7] to prove Bose condensate.

The proof of phase transitions by this method involves three steps. Verify the reflection positivity for a suitable defined reflection. Choose the quantity, typically the Fourier transform of a two point function, for which an infrared bound is to be proven. Finally, complete the argument of the proof of a sum rule on the Fourier transform.

This paper is organized as follows. In Section 2, the reflection positivity and the Gaussian dominance property is proven for a quantum lattice system of anharmonic oscillators. Following the standard procedure, an upper bound on Duhamel’s two point function $(a_p^\dagger, a_p)$ is derived, $a_p^\dagger, a_p$ being the well-known Bose operators of the creation and annihilation of particles.

In Section 3, we prove an upper bound for the two point function $\langle a_p^\dagger a_p \rangle$, where $\langle \rangle$ represents the state of thermodynamic equilibrium.

The above bound leads to an equation for the inverse temperature, whose solution gives a lower estimation for the critical temperature of the phase transition.

In Section 4, we derive the upper [2] and lower bounds for the static structure factor of the system by using the infrared bounds method and Bogoliubov’s well-known formula.

Finally, in Section 5, we study the critical behaviour of a quantum Huang–Davies type Bose system.

2. AN UPPER BOUND ON DUHAMEL'S TWO POINT FUNCTION

We consider the following Hamiltonian:

$$H = -t \sum_{\langle \alpha \gamma \rangle} a_\alpha^\dagger a_\gamma - (\mu - \nu t) \sum_\alpha a_\alpha^\dagger a_\alpha + (\lambda + \nu u) \sum_\alpha (a_\alpha^\dagger a_\alpha)^2 - u \sum_{\langle \alpha \gamma \rangle} n_\alpha n_\gamma, \quad (1)$$

defined on a hypercube Λ with periodic boundary conditions in the lattice Z^ν , where α is the site index, $\langle \alpha \gamma \rangle$ denotes the summation over pairs of the nearest neighbor sites, $a_\alpha^\dagger, a_\alpha$ are the creation-annihilation Bose operators, $n_\alpha = a_\alpha^\dagger a_\alpha$ is the number operator, μ is the chemical potential, and t, u, λ are positive constants.

Rewriting (1) as

$$H = \frac{t}{2} \sum_{\langle \alpha \gamma \rangle} (a_\alpha^\dagger - a_\gamma^\dagger)(a_\alpha - a_\gamma) - \mu \sum_\alpha a_\alpha^\dagger a_\alpha + \lambda \sum_\alpha (a_\alpha^\dagger a_\alpha)^2 + \frac{u}{2} \sum_{\langle \alpha \gamma \rangle} (n_\alpha - n_\gamma)^2, \quad (2)$$

it is easy to see that

$$H \geq -\mu \hat{N}, \quad (3)$$

where $\hat{N}$ is the total number operator. Moreover, introducing the operators

$$a_\alpha^\dagger = \frac{1}{\sqrt{2}}(\hat{x}_\alpha - i\hat{p}_\alpha), \quad a_\alpha = \frac{1}{\sqrt{2}}(\hat{x}_\alpha + i\hat{p}_\alpha)$$

into (2), where $\hat{x}_\alpha, \hat{p}_\alpha$ are the position and momentum operators, we can see that (1) is mathematically equivalent to a thermodynamically stable coupled anharmonic oscillator system. In this case, the Hamiltonian takes the form

$$\begin{aligned} H = & \frac{t}{4} \sum_{\langle \alpha \gamma \rangle} (\hat{x}_\alpha - \hat{x}_\gamma)^2 + (\hat{p}_\alpha - \hat{p}_\gamma)^2 - \\ & - \frac{1}{2} \mu \sum_\alpha \hat{w}_\alpha + \frac{1}{4} \lambda \sum_\alpha \hat{w}_\alpha^2 + \frac{u}{8} \sum_{\langle \alpha \gamma \rangle} (\hat{w}_\alpha - \hat{w}_\gamma)^2, \end{aligned} \quad (4)$$

where

$$\hat{w}_\alpha = \hat{x}_\alpha^2 + \hat{p}_\alpha^2 - 1 = 2a_\alpha^\dagger a_\alpha.$$

Let U_α be the algebra of observables at site α (a family of operators formed with the operators $\hat{x}_\alpha, \hat{p}_\alpha$). We define $U_{\Lambda_\pm} = \bigotimes_{\alpha \in \Lambda_\pm} U_\alpha$, where

$$\Lambda_+ = \{\alpha \in \Lambda / \alpha_1 \geq 1\}, \quad \Lambda_- = \{\alpha \in \Lambda / \alpha_1 \leq 0\},$$

and α_1 denotes the first component of the vector α .

We introduce the *automorphism* [1] $\theta: U_{\Lambda_+} \rightarrow U_{\Lambda_-}$ in the following way:

$$\theta(z\hat{x}_\alpha) = z^* \hat{x}_{r(\alpha)}, \quad \theta(z\hat{p}_\alpha) = z^* \hat{p}_{r(\alpha)}, \quad z \in C,$$

where r is the reflection from Λ_+ into Λ_- , defined as

$$r(\alpha) = (-\alpha_1 + 1, \alpha_2, \dots, \alpha_\nu).$$

Now we define the functional $\langle \cdot \rangle_0$ on U ($U = U_{\Lambda_+} \otimes U_{\Lambda_-}$) as

$$\langle - \rangle_0 = \text{Tr}(-). \quad (5)$$

It is possible to prove that

$$\langle A\theta(A) \rangle_0 = \text{Tr } A\theta(A) = (\text{Tr } A)^2 \geq 0 \quad (6)$$

for any self-adjoint Tr-class operator $A \in U_{\Lambda_+}$. In this case, the functional $\langle - \rangle_0$ is the positive reflection (PR) one [1–10].

Let us define a family of Hamiltonians by

$$H(\{\lambda h_\alpha\}) = \frac{t}{4} \sum_{\langle \alpha \gamma \rangle} (\hat{x}_\alpha^\dagger - \hat{x}_\gamma^\dagger - \lambda^* h_\alpha^* + \lambda^* h_\gamma^*) (\hat{x}_\alpha - \hat{x}_\gamma - \lambda h_\alpha + \lambda h_\gamma) + H_1,$$

where

$$H_1 = \frac{t}{4} \sum_{\langle \alpha \gamma \rangle} (\hat{p}_\alpha - \hat{p}_\gamma)^2 - \frac{1}{2} \mu \sum_{\alpha} \hat{w}_\alpha + \lambda \sum_{\langle \alpha \gamma \rangle} \hat{w}_\alpha^2 - \frac{u}{4} \sum_{\langle \alpha \gamma \rangle} \hat{w}_\alpha \hat{w}_\gamma.$$

In other words,

$$H(\{\lambda h_\alpha\}) = H - \lambda^* A - \lambda A^\dagger + |\lambda|^2 P,$$

where

$$A = \frac{t}{4} \sum_{\langle \alpha \gamma \rangle} (\hat{x}_\alpha^\dagger - \hat{x}_\gamma^\dagger) h_{\alpha\gamma}, \quad P = \frac{t}{4} \sum_{\langle \alpha \gamma \rangle} |h_{\alpha\gamma}|^2,$$

and $h_{\alpha\gamma} = h_\alpha - h_\gamma$. In this case, we shall have

$$-H = A + \theta(B) + \sum_i C_i \theta(D_i),$$

where $A, B, C_i, D_i \in U_{\Lambda_+}$.

From (6), using the fact that the algebras U_{Λ_+} and U_{Λ_-} commute, it follows that the functional

$$\langle - \rangle_H = Z^{-1} \text{Tr}(-) e^{-\beta H},$$

is an PR, too. Here, $Z = \text{Tr } e^{-\beta H}$ and β represents the inverse temperature.

In this case, the thermodynamic stability is assured by (3).

Now, using the inequality

$$\text{Tr}^2 e^{A+\theta(B)+\sum C_i \theta(D_i)} \leq \text{Tr } e^{A+\theta(A)+\sum C_i \theta(C_i)} \text{Tr } e^{B+\theta(B)+\sum D_i \theta(D_i)},$$

we can see that

$$Z^2(\{\lambda h_\gamma\}) \leq Z(\{\lambda h_\alpha\}) Z(\{\lambda h_{r(\alpha)}\}), \quad \alpha \in \Lambda_+, \quad (7)$$

where

$$Z(\{\lambda h_\alpha\}) = \text{Tr } e^{-\beta H(\{\lambda h_\alpha\})}.$$

Finally, (7) leads to the trace inequality

$$Z(\lambda h_\alpha) \leq Z(\lambda h_\alpha = \lambda h) = Z(0). \quad (8)$$

Thus, (8) is a G.D. (Gaussian domination) estimate [4, 5].

On the other hand, the Hamiltonian H satisfies:

$$[H, \hat{N}] = 0.$$

This fact leads to the following selection rules for the system:

$$\langle a_\alpha^\dagger \rangle = \langle a_\alpha \rangle = 0,$$

for all $\alpha \in \Lambda$. This implies

$$\left. \frac{\partial \ln Z(\lambda h_\alpha)}{\partial \lambda^*} \right|_{\lambda^*=0} = 0. \quad (9)$$

From (8) and (9), by straightforward calculations, we obtain

$$\left. \frac{\partial^2 \ln Z(\lambda h_{\alpha\gamma})}{\partial \lambda \partial \lambda^*} \right|_{\lambda^*=0} = -\beta P + \frac{1}{Z} \left. \frac{\partial^2 \text{Tr } e^{-\beta H(\lambda h_{\alpha\gamma})}}{\partial \lambda \partial \lambda^*} \right|_{\lambda^*=0} \leq 0. \quad (10)$$

Equation (10) can be rewritten in the following way:

$$-\beta P + \beta^2 (A^\dagger, A) \leq 0, \quad (11)$$

where the function $(\cdot)$ is Duhamel's two-point function, defined as

$$(A, B) = Z^{-1} \int_0^1 \text{Tr } e^{-x\beta H} A e^{-(1-x)\beta H} B dx.$$

Let us define the complex parameters h_α as

$$h_\alpha = \frac{e^{-ip\alpha}}{|\Lambda|^{1/2}},$$

with p belonging to the dual lattice Λ^* .

With the above defined h_α 's, the following identity holds:

$$\sum_{\langle \alpha\gamma \rangle} (\hat{x}_\alpha^\dagger - \hat{x}_\gamma^\dagger) h_{\alpha\gamma} = 2 \sum_i (1 - e^{-ip\alpha}) \hat{x}_p^\dagger,$$

where $\hat{x}_{-p} = \hat{x}_p^\dagger$, and

$$\hat{x}_p = \frac{1}{\sqrt{|\Lambda|}} \sum_{\alpha \in \Lambda} e^{-ip\alpha} \hat{x}_\alpha,$$

with $|\Lambda|$ representing the number of sites in Λ .

So replacing the terms in Eq. (11), we obtain

$$-\beta \left(\frac{t}{2} \sum_j |1 - e^{-ip \cdot \delta_j}|^2 \right) + \beta^2 \left[\frac{t}{2} \sum_j |1 - e^{-ip \cdot \delta_j}|^2 \right]^2 (\hat{x}_{-p}, \hat{x}_p) \leq 0.$$

This leads to the inequality ($p \neq 0$). Also,

$$(\hat{x}_p^\dagger, \hat{x}_p) \leq \frac{1}{\beta t \lambda_p}, \quad (12)$$

where $\lambda_p = \nu - \sum_j \cos(p \cdot \delta_j) \geq 0$.

On the other hand, in momentum representation, the selection rule mentioned above can be expressed as

$$\langle a_p^\dagger \rangle = \langle a_p \rangle = 0,$$

with

$$a_p = \frac{1}{\sqrt{|\Lambda|}} \sum_{\alpha \in \Lambda} e^{-ip\alpha} a_\alpha.$$

Using (12) and the selection rules of the system, it follows that for $p \neq 0$,

$$(a_p^\dagger, a_p) \leq \frac{1}{\beta t \lambda_p}. \quad (13)$$

3. BOSE CONDENSATE

The upper bound for the two point temperature average $\langle a_p^\dagger a_p \rangle$ can be obtained from the upper bound of Duhamel's two point function calculated in Section 2 by using the following theorems [3]:

THEOREM 1. *Let h be the function from $[0, \infty)$ to $[0, 1)$ determined by the relation*

$$h(x \tanh x) = x^{-1} \tanh x.$$

Moreover, assume that

$$b(A) \equiv (A^\dagger, A), \quad g(A) \equiv \frac{1}{2} \langle A^\dagger A + A A^\dagger \rangle, \quad c(A) \equiv \langle [A^\dagger, [\beta H, A]] \rangle.$$

Then for any operator A and any selfadjoint operator H ,

$$b(A) \geq g(A) h\left(\frac{c(A)}{4g(A)}\right). \quad (14)$$

THEOREM 2. *Assume $b > gh(\frac{c}{4g})$, where h is the above defined function, and $b, g, c > 0$. Assume that $b \leq b_0$ and $c \leq c_0$. Then $g \leq g_0$, where*

$$g_0 = \frac{1}{2} \sqrt{c_0 b_0} \coth \sqrt{\frac{c_0}{4b_0}}. \quad (15)$$

From Eq. (13) and the above theorems, taking $A = a_p$, we obtain the following upper bound for $g(a_p)$:

$$g(a_p) \leq \frac{1}{2} \sqrt{c_0(p) b_0(p)} \coth \sqrt{\frac{c_0(p)}{4b_0(p)}},$$

where

$$b_0(p) = \frac{1}{\beta t \lambda_p}, \quad c_0(p) \equiv \beta Q_{p,\Lambda},$$

with

$$Q_{p,\Lambda} \equiv t \lambda_p - \mu + (\nu u + \lambda)(1 + 4n_\Lambda) \geq 0, \quad n_\Lambda \equiv \frac{1}{|\Lambda|} \sum_{\alpha} \langle a_{\alpha}^{\dagger} a_{\alpha} \rangle.$$

Evidently, we consider the values of μ which satisfy

$$\mu \leq (\nu u + \lambda)(1 + 4n_\Lambda).$$

Now, using (15) we have for $p \neq 0$:

$$\begin{aligned} \langle a_p^{\dagger} a_p \rangle &\leq \frac{1}{2} \left[\sqrt{\frac{Q_{p,\Lambda}}{t \lambda_p}} \coth \frac{\beta}{2} \sqrt{(t \lambda_p) Q_{p,\Lambda} - 1} - 1 \right], \\ &= \frac{1}{2} \left[\sqrt{1 + \frac{AS - \mu}{t \lambda_p}} \coth \frac{\beta}{2} \sqrt{(t \lambda_p) Q_{p,\Lambda} - 1} - 1 \right], \end{aligned} \quad (16)$$

where

$$A = \lambda + \nu u, \quad S = 1 + 4n_\Lambda.$$

The condition of Bose condensate is

$$n_0 \equiv \lim_{|\Lambda| \rightarrow \infty} \frac{1}{|\Lambda|} \langle a_0^{\dagger} a_0 \rangle \neq 0. \quad (17)$$

We should always have in mind that the quantity n_Λ remains constant in the limit $|\Lambda| \rightarrow \infty$, i.e.,

$$n \equiv \lim_{|\Lambda| \rightarrow \infty} n_\Lambda = \text{const.}$$

Using the condition in Eq. (17), and passing to the thermodynamic limit $|\Lambda| \rightarrow \infty$, we obtain the following equation in β whose solution gives a lower estimation for the critical temperature of condensate for the model (1):

$$n = \frac{1}{(2\pi)^\nu} \int_{|p_i| \leq \pi} d^\nu p \frac{1}{2} \left[\sqrt{1 + \frac{AS - \mu}{t \lambda_p}} \coth \frac{\beta}{2} \sqrt{(t \lambda_p) Q_{p,\infty} - 1} - 1 \right].$$

This equation has the unique solution for $\nu \geq 3$ and

$$n \geq \frac{1}{(2\pi)^\nu} \int_{|p_i| \leq \pi} d^\nu p \frac{1}{2} \left(\sqrt{1 + \frac{AS - \mu}{t \lambda_p}} - 1 \right).$$

When $A = 0$, we have the well-known equation for the critical inverse temperature of phase transition in the case of a free boson lattice system.

4. A LOWER BOUND FOR THE STATIC STRUCTURE FACTOR

In the previous paper [2] we obtained an upper bound for the two-point correlator $\langle \rho_p^\dagger \rho_p \rangle$, where $\rho_p^\dagger, \rho_p$ are the density operators defined by

$$\rho_p^\dagger = \frac{1}{\sqrt{|\Lambda|}} \sum_k a_k^\dagger a_{k-p}, \quad \rho_p = \frac{1}{\sqrt{|\Lambda|}} \sum_k a_k^\dagger a_{k+p}. \quad (18)$$

THEOREM 3. *For the lattice Boson-model (1), the following bounds on the correlation $\langle \rho_r^\dagger \rho_r \rangle$ hold (see [2]):*

$$\langle \rho_r^\dagger \rho_r \rangle \leq \sqrt{\frac{n_\Lambda t}{2u}} \coth\left(\beta \sqrt{\frac{n_\Lambda u t}{2}} \lambda_r\right) \equiv R_{r,\Lambda}, \quad (19)$$

$$\langle \rho_r^\dagger \rho_r \rangle \geq \frac{\beta^{-1} \left[\frac{2}{r|\Lambda|} \sum_p V_{p,r} n_p \right]^2}{2u \lambda_{r,\hat{r}}^2 \langle n_\Lambda^2 \rangle + \frac{1}{|\Lambda|} \left(t \sum_p d_{p,r} \langle a_p^\dagger a_p \rangle + u \sum_{p \neq 0} f_{p,r} R_{p,\Lambda} \right)} \quad (r \neq 0), \quad (20)$$

where

$$\begin{aligned} V_{p,r} &= \sum_i \cos(p \cdot \delta_i) (1 - \cos(r \cdot \delta_i)), \\ d_{p,r} &= \frac{1}{r^2} \left| (\lambda_{(p+r) \cdot \hat{r}} - \lambda_{p \cdot \hat{r}})^3 - (\lambda_{p \cdot \hat{r}} - \lambda_{(p-r) \cdot \hat{r}})^3 \right|, \\ f_{p,r} &= \left| 2\lambda_{p \cdot \hat{r}}^2 + \lambda_{(p-r) \cdot \hat{r}}^2 + \lambda_{(p+r) \cdot \hat{r}}^2 \right|. \end{aligned}$$

PROOF. The translational invariance of the Hamiltonian (1) guarantees the conservation of the total momentum, i.e.,

$$[\hat{P}, H] = 0,$$

where the discrete version of the total momentum operator is:

$$\hat{P} = \sum_{p,i} \sin(p \cdot \delta_i) \hat{p} a_p^\dagger a_p,$$

where $\hat{p} = \frac{p}{\sqrt{p \cdot p}}$.

This law of conservation leads to the following selection rules:

$$\langle a_{p_1}^\dagger \dots a_{p_j}^\dagger a_{q_1} \dots a_{q_j} \rangle = 0,$$

for $p_1 + \dots + p_j \neq q_1 + \dots + q_j$. In other words, $\langle \rho_p^\dagger \rangle = \langle \rho_p \rangle = 0$ for $p \neq 0$.

On the other hand, to get the lower bound for the static structure factor, we use Bogoliubov's inequality [11]

$$|\langle [A^\dagger, \beta[H, A]] \rangle| \langle B^\dagger B + B B^\dagger \rangle \geq 2 |\langle [A, B] \rangle|^2, \quad (21)$$

where H is the Hamiltonian of the system and A, B are arbitrary operators.

Taking into account the selection rules described above for the proof of Theorem 3, we make use of the following operator J_r , defined as:

$$J_r = \frac{1}{\sqrt{|\Lambda|}} \sum_p \left(\frac{\lambda_{(p+r) \cdot \hat{r}} - \lambda_{p \cdot \hat{r}}}{r} \right) a_p^\dagger a_{p+r}.$$

So, defining

$$B^\dagger = \rho_r^\dagger, \quad A^\dagger = J_r,$$

we obtain

$$\langle \rho_r^\dagger \rho_r \rangle \geq \frac{\beta^{-1} \left[\frac{2}{r|\Lambda|} \sum_p V_{p,r} n_p \right]^2}{2u\lambda_{r \cdot \hat{r}}^2 \langle n_\Lambda^2 \rangle + \frac{1}{|\Lambda|} \left| t \sum_p d_{p,r}^* \langle a_p^\dagger a_p \rangle + u \sum_{p \neq 0} f_{p,r}^* \langle \rho_p^\dagger \rho_p \rangle \right|}, \quad (22)$$

where

$$\begin{aligned} d_{p,r}^* &= \frac{1}{r^2} [(\lambda_{(p+r) \cdot \hat{r}} - \lambda_{p \cdot \hat{r}})^3 - (\lambda_{p \cdot \hat{r}} - \lambda_{(p-r) \cdot \hat{r}})^3], \\ f_{p,r}^* &= 2\lambda_{p \cdot \hat{r}}^2 - \lambda_{(p-r) \cdot \hat{r}}^2 - \lambda_{(p+r) \cdot \hat{r}}^2. \end{aligned}$$

Evidently, we have

$$|d_{k,r}^*| \leq d_{k,r}, \quad |f_{k,r}^*| \leq f_{k,r}. \quad (23)$$

Finally,

$$|\langle [\rho_r, J_r] \rangle|^2 = \left[\frac{2}{r|\Lambda|} \sum_p V_{p,r} n_p \right]^2.$$

Note that replacing λ_p by $p^2 = p \cdot p$ in Eq. (22), we exactly obtain the inequality derived in [11] for the correlation $\langle \rho_p^\dagger \rho_p \rangle$.

The so-called static structure factor $S(p)$ is defined as

$$S(r) = \frac{1}{N} \left\langle \sum_{k_1, k_2} a_{k_1}^\dagger a_{k_1+r} a_{k_2}^\dagger a_{k_2-r} \right\rangle,$$

where $N = \langle \hat{N} \rangle$ is the number of particles.

The power spectrum of the density fluctuations in the p -space is $S(p)$. In this case, we have

$$S(r) = \frac{|\Lambda|}{N} \langle \rho_r^\dagger \rho_r \rangle = \frac{\langle \rho_r^\dagger \rho_r \rangle}{n_\Lambda}. \quad (24)$$

So, theorem 3 and Eq. (24) lead to the following upper and lower bounds for the static structure factor $S(r)$:

$$\begin{aligned} S(r) &\leq \sqrt{\frac{t}{2un_\Lambda}} \coth \left(\beta \sqrt{\frac{n_\Lambda u t}{2}} \lambda_r \right), \\ S(r) &\geq \frac{\frac{\beta^{-1}}{n_\Lambda} \left[\frac{2}{r|\Lambda|} \sum_p V_{p,r} n_p \right]^2}{2u\lambda_{r \cdot \hat{r}}^2 \langle n_\Lambda^2 \rangle + \frac{1}{|\Lambda|} \left(t \sum_p d_{p,r} \langle a_p^\dagger a_p \rangle + u \sum_{p \neq 0} f_{p,r} R_{p,\Lambda} \right)} \quad (r \neq 0). \end{aligned}$$

5. CRITICAL BEHAVIOR OF A HUANG–DAVIES TYPE BOSON SYSTEM

Finally, we will study the boson model whose energy is governed by the Hamiltonian

$$H = \sum_p (\epsilon_p - \mu) a_p^\dagger a_p + H_1, \quad (25)$$

where

$$H_1 = \frac{\nu}{2|\Lambda|} \hat{N}^2, \quad \hat{N} = \sum_p a_p^\dagger a_p.$$

Using the inequality

$$(a_p^\dagger, \hat{n}^{\frac{1}{2k}} a_p) \geq 2n^{\frac{1}{2k+1}} (a_p^\dagger, \hat{n}^{\frac{1}{2k+1}} a_p) - n^{\frac{1}{2k}} (a_p^\dagger, a_p)$$

recursively and noting that

$$\beta(\epsilon_p - \mu) (a_p^\dagger, a_p) + \beta(a_p^\dagger, [a_p, H_1]) = 1,$$

we prove that for $p \neq 0$

$$(a_p^\dagger, \hat{n} a_p) \geq n (a_p^\dagger, a_p),$$

where n is as always the density of particles of the system and $\hat{n}$ represents the density operator.

For our model,

$$\mu \leq \nu n, \quad \epsilon_p \geq 0.$$

It is easy to see that for $p \neq 0$

$$(a_p^\dagger, a_p) \leq \frac{1}{\beta(\epsilon_p + \nu n - \mu)}.$$

By using the usual procedure of the infrared bounds method, the above bound enables us to obtain the following upper bound for the correlation function $\langle a_p^\dagger a_p \rangle$ ($p \neq 0$):

$$\langle a_p^\dagger a_p \rangle \leq \frac{1}{2} \left[\coth \frac{\beta}{2} (\epsilon_p + \nu n - \mu) - 1 \right].$$

This equation leads to an equation in β which gives the lower estimate for the critical temperature of (25)

$$n_c = \frac{1}{(2\pi)^\nu} \int \frac{d^\nu p}{e^{\beta_c \epsilon_p} - 1},$$

where $\mu_c = \nu n_c$.

For the continuous case, the results of this section are an improvement of our previous results [8].

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References

- [1] *J. Fröhlich* // Bull. Amer. Math. Soc. 1978. V. 84. P. 165.
- [2] *N.N. Bogoliubov, Jr., M. Corgini, D.P. Sankovich* // Mod. Phys. Lett. 1992. V. B6. P. 215.
- [3] *J. Fröhlich, B. Simon, T. Spencer* // Commun. Math. Phys. 1976. V. 50. P. 79.
- [4] *F. Dyson, E. Lieb, B. Simon* // Phys. Rev. Lett. 1976. V. 37. P. 79.
- [5] *F. Dyson, E. Lieb, B. Simon* // J. Stat. Phys. 1978. V. 18. P. 335.
- [6] *J. Fröhlich, E. Lieb* // Commun. Math. Phys. 1978. V. 60. P. 233.
- [7] *J. Fröhlich, R. Israel* // Commun. Math. Phys. 1978. V. 62. P. 1.
- [8] *D.P. Sankovich* // Theor. and Math. Phys. 1989. V. 70. P. 460.
- [9] *N. N. Bogoliubov, Jr., D.P. Sankovich* // Phys. Lett. 1989. V. A137. P. 179.
- [10] *N. N. Bogoliubov, Jr., D.P. Sankovich* // Mod. Phys. Lett. 1991. V. B5. P. 51.
- [11] *N. N. Bogoliubov, Jr., B.I. Sadovnikov, A.S. Schumovski*. Mathematical Methods in Statistical Mechanics of Model Systems. M.: Nauka, 1989.

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МЕТОД ИНФРАКРАСНЫХ ОЦЕНОК В ТЕОРИИ БОЗОННЫХ СИСТЕМ

Изучается термодинамическое равновесие решеточного бозе-газа с одноузельным отталкиванием и притяжением ближайших соседей. Доказано существование бозе-конденсата, и получено уравнение для нижней оценки критической температуры. Даны нижняя и верхняя оценки структурного фактора. В рамках метода инфракрасных оценок изучена модель бозонной системы типа Хуанга–Дэвиса.