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при $k \rightarrow \infty$. Итак,

$$\frac{V_k + W_k}{\sqrt{n_k B(n_k)}} \rightarrow 0 \quad \text{по вероятности при } k \rightarrow \infty.$$

Теорема доказана.

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BUCKDAHN R.*, ENGELBERT H.-J.**

A BACKWARD STOCHASTIC DIFFERENTIAL EQUATION WITHOUT STRONG SOLUTION¹⁾

В предыдущей статье авторов и А. Рашкану (Теория вероятн. и ее примен., 2004, т. 49, № 1) было введено понятие слабого решения общего обратного стохастического дифференциального уравнения (ОСДУ). Там же был приведен пример слабого решения некоторого ОСДУ, которое не является сильным решением, т.е. решением в классическом смысле. Однако рассмотренное решение не единственно по распределению и, как

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было отмечено, у данного ОСДУ существуют также сильные решения. В настоящей заметке мы устраним этот недостаток и приводим пример ОСДУ, которое имеет слабое решение, но не имеет сильных.

Ключевые слова и фразы: обратные стохастические дифференциальные уравнения, слабые решения, сильные решения, пример Цирельсона.

1. Introduction. The notion of a weak solution of a (forward) stochastic differential equation (SDE) has been well understood for a long time and plays a basic role in the development of the theory in the last decades. In comparison with strong solutions, weak solutions need not be adapted to the filtration generated by the driving process, usually a Brownian motion.

Backward stochastic differential equations (BSDEs) were introduced, in a general form, by E. Pardoux and S. Peng [8], who proved existence and uniqueness of the solution, adapted to the Brownian filtration, under conditions including basically Lipschitz continuity of the driver.

The theory of BSDEs has found important applications and has become a powerful tool in fields as financial mathematics, optimal control and stochastic games, partial differential equations and homogenization, see, e.g., the collected texts [4].

From the beginning, many authors gave improvements of the existence and uniqueness result of E. Pardoux and S. Peng [8], weakening the Lipschitz continuity of the driver, see, e.g., the papers by Bahlali et al. [2], J.-P. Lepeltier and J. San Martin [6], S. Hamadène, J.-P. Lepeltier and S. Peng [5], to mention only a few. F. Antonelli and J. Ma [1] discuss certain examples of what they call weak solutions of forward-backward SDEs.

However, all solutions of BSDEs which were constructed in these papers seem to be adapted to the filtration generated by the driving process. Solutions of this type should be called *strong* solutions.

Only recently in [3], the authors and A. Răşcanu made a first attempt to extend the class of solutions and to introduce so-called weak solutions of a BSDE, not requiring the adaptedness with respect to the filtration generated by the driving process. In particular, it was shown in [3, Proposition 3.1] that there are, indeed, weak solutions for a certain BSDE which are not strong. These examples are constructed from the famous Tsirelson example. Furthermore, in [3, Theorem 4.1], a general existence result for weak solutions of BSDEs was proved. For this, the main assumptions on the driver are the boundedness and the continuity on the Skorokhod space in the Meyer–Zheng topology (cf. [7]).

Similar to the Yamada–Watanabe theorem [11], in [3, Section 5] it is also concluded that pathwise uniqueness implies uniqueness in law and that pathwise uniqueness and existence of a weak solution ensure the existence of a (pathwise unique) strong solution. As for SDEs, this conclusion provides an efficient tool for dividing the often hard problem of the existence of a strong solution into two subtasks: To prove existence of a weak solution and to prove pathwise uniqueness.

However, the examples of weak solutions given in [3], which are not strong, suffer from the deficiency that the same BSDE also possesses strong solutions, although they are different in law (cf. [3, Proposition 3.4]). But a real justification of the notion of a weak solution requires to find a BSDE admitting a weak solution but *no* strong solution.

In the present note, we will deal with this problem and provide an example of a BSDE which has a weak solution unique in law but does not admit a strong solution.

2. The concept of weak solutions for BSDEs. This short section is devoted to a brief recall of some basic definitions and notation related to strong and weak solutions for BSDEs. Let $T > 0$ be a real number (the terminal time). For any natural number $p \geq 1$, by $D(\mathbf{R}^p) := D([0, T]; \mathbf{R}^p)$ we denote the Skorokhod space of càdlàg (right-continuous with left-hand limits) functions x on $[0, T]$ taking values in $\mathbf{R}^p$ such that

$$x(T-) := \lim_{t \uparrow T} x(t) = x(T),$$

and, by convention, $x(0-) = 0$. Let ζ be the coordinate mapping on $D(\mathbf{R}^p)$ defined by

$$\zeta_t(x) = x(t), \quad t \in [0, T], \quad x \in D(\mathbf{R}^p).$$

We introduce the σ -algebras of subsets of $D(\mathbf{R}^p)$,

$$\begin{aligned}\mathcal{D}_t^s &:= \mathcal{D}_t^s(\mathbf{R}^p) := \sigma(\{\zeta_u: u \in [s, t]\}), & 0 \leq s \leq t \leq T, \\ \mathcal{D} &:= \mathcal{D}(\mathbf{R}^p) := \mathcal{D}_T^0(\mathbf{R}^p).\end{aligned}$$

Let us assume that there are given a measurable $\mathbf{R}^m$ -valued functional H on the space $(D(\mathbf{R}^d), \mathcal{D}(\mathbf{R}^d))$ and a measurable $\mathbf{R}^m$ -valued functional f on the product space

$$([0, T] \times D(\mathbf{R}^d) \times D(\mathbf{R}^m), \mathcal{B}([0, T]) \otimes \mathcal{D}(\mathbf{R}^d) \otimes \mathcal{D}(\mathbf{R}^m)),$$

where $\mathcal{B}([0, T])$ is the σ -algebra of Borel subsets of $[0, T]$.

Now we recall the definition of strong and weak solutions of a BSDE given in [3].

Definition 1. Let $X = (X_t)_{t \in [0, T]}$ be an $\mathbf{R}^d$ -valued càdlàg process defined on a complete probability space $(\Omega, \mathcal{F}, \mathbf{P})$, i.e., X is a random variable with values in the Skorokhod space $(D(\mathbf{R}^d), \mathcal{D}(\mathbf{R}^d))$ defined on $(\Omega, \mathcal{F}, \mathbf{P})$. By $\mathbb{F}^X = (\mathcal{F}_t^X)_{t \in [0, T]}$ we denote the smallest of all filtrations $\mathbb{G}$ satisfying the usual conditions and being such that X is $\mathbb{G}$ -adapted. An $\mathbf{R}^m$ -valued càdlàg process Y is called a *strong solution of BSDE* (H, f, X) if the following conditions are satisfied.

(S.i) It holds

$$\mathbf{E}[|H(X)|] < +\infty, \quad \mathbf{E}\left[\int_0^T |f(s, X, Y)| ds\right] < +\infty.$$

(S.ii) The equation

$$Y_t = \mathbf{E}\left[H(X) + \int_t^T f(s, X, Y) ds \middle| \mathcal{F}_t^X\right], \quad t \in [0, T],$$

is fulfilled.

The triplet (H, f, X) is called the *data set* of BSDE (H, f, X) , it determines the *terminal value* $H(X)$. The functional f is called the *driver* and X the *driving process*.

Clearly, a strong solution Y of BSDE (H, f, X) is always $\mathbb{F}^X$ -adapted. Usually the driving process X is a d -dimensional Brownian motion. For a (noncoupled) forward-backward SDE, X is to be chosen as a (strong) solution of a (forward) SDE.

Now we come to the definition of a weak solution of the BSDE.

Definition 2. Let μ be a probability measure on the space $(D(\mathbf{R}^d), \mathcal{D}(\mathbf{R}^d))$. Suppose that $(\Omega, \mathcal{F}, \mathbf{P})$ is a complete probability space, $\mathbb{F} = (\mathcal{F}_t)_{t \in [0, T]}$ is a filtration satisfying the usual conditions, and X, Y are càdlàg processes defined on $(\Omega, \mathcal{F}, \mathbf{P})$ with values in $\mathbf{R}^d$ and $\mathbf{R}^m$, respectively. Then $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P}, X, Y)$ is called a *weak solution of BSDE* (H, f, μ) if the following conditions are satisfied.

(W.i) The distribution of X on $D(\mathbf{R}^d)$ is μ .

(W.ii) X is $\mathbb{F}$ -adapted.

(W.iii) It holds

$$\mathbf{E}[|H(X)|] < +\infty, \quad \mathbf{E}\left[\int_0^T |f(s, X, Y)| ds\right] < +\infty.$$

(W.iv) Every $\mathbb{F}^X$ -martingale is also an $\mathbb{F}$ -martingale.

(W.v) The equation

$$Y_t = \mathbf{E}\left[H(X) + \int_t^T f(s, X, Y) ds \middle| \mathcal{F}_t\right], \quad t \in [0, T],$$

is fulfilled.

For a detailed discussion of the notion of a weak solution of BSDE (H, f, μ) , see [3]. We only remark that, if $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P}, X, Y)$ is a weak solution of BSDE (H, f, μ) such that Y is $\mathbb{F}^X$ -adapted, Y is clearly a strong solution of BSDE (H, f, X) on $(\Omega, \mathcal{F}, \mathbf{P})$. On the other side, every strong solution Y of BSDE (H, f, X) on $(\Omega, \mathcal{F}, \mathbf{P})$ defines a weak solution $(\Omega, \mathcal{F}, \mathbb{F}^X, \mathbf{P}, X, Y)$ of BSDE (H, f, μ) , where μ is the distribution of X .

We emphasize the importance of condition (W.iv) of the above definition. It means that the filtration $\mathbb{F}$ is not too large in comparison with $\mathbb{F}^X$. More precisely, this condition is easily shown to be equivalent to the following condition:

(W.iv') $\mathcal{F}_t$ and $\mathcal{F}_T^X$ are conditionally independent given $\mathcal{F}_t^X$, $t \in [0, T]$.

This amounts to saying that the filtration $\mathbb{F}$ consists, additionally to $\mathbb{F}^X$, only of randomizations (i.e., independent experiments).

3. A Tsirelson type example for BSDEs. In his famous example, in 1975 B. S. Tsirelson [10] gave an SDE for which, although a weak solution can be constructed, there does not exist any strong solution (also see [9, IX.3.6]). In [3], the Tsirelson equation was adopted in order to construct a BSDE which admits a weak solution which is not strong. But there is even an infinite number of weak (but not strong) solutions different in law. Moreover, it may be surprising that, contrary to the Tsirelson SDE, there can also be found strong solutions.

Our main motivation is to remove this major deficiency of that example and to construct a BSDE which admits a weak solution unique in law but *no* strong solution.

Following the idea of the construction of the example for an SDE given by B. S. Tsirelson [10], which admits a weak solution unique in law but no strong solution, we restrict ourselves to one-dimensional equations ($d = m = 1$). We choose $T := 1$ and a strictly decreasing sequence of real numbers $\frac{1}{2} = t_0 > t_1 > \dots > t_n > \dots > 0$ such that $t_n \rightarrow 0$ as $n \rightarrow +\infty$. For every $(t, y) \in [0, \frac{1}{2}] \times D(\mathbf{R})$, where $D(\mathbf{R}) := D([0, 1]; \mathbf{R})$, we put

$$b(t, y) = \begin{cases} \left\{ \frac{y(t_j) - y(t_{j+1})}{t_j - t_{j+1}} \right\}, & \text{if } t_j < t \leq t_{j-1}, j \geq 1, \\ 0, & \text{if } t = 0, \end{cases}$$

where $\{r\}$ denotes the fractional part of the real r , being always a number from $[0, 1)$. We now extend b to $[0, 1]$, setting

$$b(t, y) = -b((1-t)+, y) = -\left\{ \frac{y(t_j) - y(t_{j+1})}{t_j - t_{j+1}} \right\}, \quad \text{if } 1 - t_{j-1} < t \leq t_j, j \geq 1,$$

for all $(t, y) \in (\frac{1}{2}, 1] \times D(\mathbf{R})$. This extension is motivated by the desire to have the property

$$\int_0^1 b(s, y) ds = 0, \quad y \in D(\mathbf{R}). \quad (1)$$

Obviously, the functional b is measurable on $([0, 1] \times D(\mathbf{R}), \mathcal{B}([0, 1]) \otimes \mathcal{D})$ and adapted to the filtration $(\mathcal{D}_t^0)_{t \in [0, 1]}$. Furthermore, $b(\cdot, y)$ is left-continuous for all $y \in D(\mathbf{R})$. This yields that b is $(\mathcal{D}_t^0)_{t \in [0, 1]}$ -predictable. Next we define the functional H , putting

$$H(x) = x(1), \quad x \in D(\mathbf{R}),$$

which, clearly, is measurable on $(D(\mathbf{R}), \mathcal{D})$. Finally, let μ be the Wiener measure lifted to the space $(D(\mathbf{R}), \mathcal{D})$.

Setting $f = -b$, we now look for a weak solution of BSDE (H, f, μ) . To this end, we start from a Brownian motion $(Y, \mathbf{F})$ on a filtered probability space $(\Omega, \mathcal{F}, \mathbf{Q}, \mathbb{F})$,

$$Y = (Y_t)_{t \in [0, 1]}, \quad \mathbb{F} = (\mathcal{F}_t)_{t \in [0, 1]},$$

and define

$$M_1 = \exp \left\{ \int_0^1 b(s, Y) dY_s - \frac{1}{2} \int_0^1 b^2(s, Y) ds \right\}.$$

Since b is bounded, it is well known from Girsanov's theorem (cf. [9, Chapter VIII]) that $\mathbf{E}M_1 = 1$, hence

$$\mathbf{P}(A) = \int_A M_1 d\mathbf{Q}, \quad A \in \mathcal{F},$$

defines a new probability measure on $(\Omega, \mathcal{F})$, and the process $(W, \mathbb{F})$,

$$W_t := Y_t - \int_0^t b(s, Y) ds, \quad t \in [0, 1], \quad (2)$$

is a Brownian motion on the filtered probability space $(\Omega, \mathcal{F}, \mathbf{P}, \mathbb{F})$.

We claim that $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P}, W, Y)$ is a weak solution of BSDE (H, f, μ) . Obviously, conditions (W.i)–(W.iii) for $X = W$ are satisfied. Condition (W.iv) for $X = W$ also holds, because $(W, \mathbb{F})$ is a Brownian motion and, in view of the representation property of $(W, \mathbb{F}^W)$, every $\mathbb{F}^W$ -martingale N admits a representation

$$N_t = N_0 + \int_0^t Z_s dW_s, \quad t \in [0, 1],$$

with an $\mathbb{F}^W$ -predictable process Z and is therefore an $\mathbb{F}$ -martingale, too. To verify (W.v), first we notice that in our example the driver f does not depend on the variable corresponding to the driving process $X = W$. From (1) and (2) we obtain

$$Y_t = \int_0^t b(s, Y) ds + W_t = W_1 + \int_t^1 f(s, Y) ds - (W_1 - W_t), \quad t \in [0, 1]. \quad (3)$$

Taking the conditional expectation with respect to $\mathcal{F}_t$ yields

$$Y_t = \mathbf{E} \left[W_1 + \int_t^1 f(s, Y) ds \middle| \mathcal{F}_t \right], \quad t \in [0, 1],$$

verifying (W.v). Hence $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P}, W, Y)$ is a weak solution of BSDE (H, f, μ) .

We are now going to investigate existence of strong solutions and uniqueness in law of BSDE (H, f, μ) . For this purpose, we assume that $(Y, \mathbb{F})$ is an arbitrary process defined on some filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P})$ equipped with a Brownian motion $(W, \mathbb{F}^W)$. (All processes are considered with time parameter $t \in [0, 1]$.)

Proposition 1. *The following conditions are equivalent:*

- (i) $(Y, \mathbb{F})$ is a weak solution of SDE (3). (In particular, this includes that $(W, \mathbb{F})$ is a Brownian motion.)
- (ii) $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P}, W, Y)$ is a weak solution of BSDE (H, f, μ) .
- (iii) $(W, \mathbb{F})$ is a Brownian motion and there exists an $\mathbb{F}$ -predictable process Z with

$$\mathbf{E} \left[\int_0^1 Z_s^2 ds \right] < \infty \quad (4)$$

such that the equation

$$Y_t = W_1 + \int_t^1 f(s, Y) ds - \int_t^1 Z_s dW_s, \quad t \in [0, 1], \quad (5)$$

holds.

P r o o f. The implication (i) $\Rightarrow$ (ii) is exactly shown as above. To verify (ii) $\Rightarrow$ (iii), we assume that $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P}, W, Y)$ is a weak solution of BSDE (H, f, μ) . From (W.iv) it follows immediately that $(W, \mathbb{F})$ is a martingale and hence $(W, \mathbb{F})$ is a Brownian motion. Using (1), we then obtain

$$\begin{aligned} Y_t &= \mathbf{E} \left[W_1 + \int_t^1 f(s, Y) ds \middle| \mathcal{F}_t \right] = \mathbf{E} \left[W_1 - \int_0^t f(s, Y) ds \middle| \mathcal{F}_t \right] \\ &= W_t - \int_0^t f(s, Y) ds = W_1 + \int_t^1 f(s, Y) ds - \int_t^1 Z_s dW_s \end{aligned}$$

with $Z \equiv 1$. In the last but one equality we used the martingale property of $(W, \mathbb{F})$ and the $\mathcal{F}_t$ -measurability of $\int_0^t f(s, Y) ds$, following from the property that $f(\cdot, Y)$ is $\mathbb{F}^Y$ -predictable (and hence $\mathbb{F}^Y$ -progressively measurable) and from $\mathbb{F}^Y \subseteq \mathbb{F}$. Finally, we show (iii) $\Rightarrow$ (i). If Y satisfies (5), using (1) and the martingale property of $(W, \mathbb{F})$ and of $(\int_0^\bullet Z_s dW_s, \mathbb{F})$, we obtain

$$\begin{aligned} Y_0 &= \mathbf{E}[Y_0 | \mathcal{F}_0] = \mathbf{E}\left[W_1 + \int_0^1 f(s, Y) ds - \int_0^1 Z_s dW_s \middle| \mathcal{F}_0\right] \\ &= \mathbf{E}\left[W_1 - \int_0^1 Z_s dW_s \middle| \mathcal{F}_0\right] = 0. \end{aligned}$$

Hence

$$W_1 = \int_0^1 Z_s dW_s$$

and from (4) and the uniqueness of this integral representation it follows $Z \equiv 1$. But this implies (3), completing the proof of the proposition.

R e m a r k 1. Proposition 1 gives the connection of BSDE (H, f, μ) to the traditional form of a BSDE

$$Y_t = H(W) + \int_t^1 f(s, W, Y) ds - \int_t^1 Z_s dW_s, \quad t \in [0, T], \quad (6)$$

$(W, \mathbb{F})$ being a Brownian motion. This holds true more generally: Indeed, under assumptions (W.iii) and (4) a solution (Y, Z) of equation (6) always defines a weak solution $(\Omega, \mathcal{F}, \mathbb{P}, W, Y)$ of BSDE (H, f, μ) , where μ is the law of W . Conversely, if the driver f is nonanticipative and, for a given weak solution $(\Omega, \mathcal{F}, \mathbb{P}, W, Y)$ of BSDE (H, f, μ) , is such that $\int_0^T f(s, W, Y) ds$ is $\mathcal{F}_T^W$ -measurable (e.g., equal to a constant), then there is an $\mathbb{F}^W$ -predictable process Z with the property that (Y, Z) satisfies (6).

Using Proposition 1, now we can easily show that there is no strong solution of BSDE (H, f, W) . Indeed, suppose that Y is a strong solution of BSDE (H, f, W) on $(\Omega, \mathcal{F}, \mathbb{P})$. Then $(\Omega, \mathcal{F}, \mathbb{F}^W, \mathbb{P}, W, Y)$ is a weak solution of BSDE (H, f, μ) . Using Proposition 1, we observe that $(Y, \mathbb{F}^W)$ is a solution to SDE (3). Actually, this means that Y is a strong solution to SDE (3), but this is impossible in view of Tsirelson's result (see [9, IX.3.6]).

Using Girsanov's theorem, it can easily be shown that the (weak) solution of SDE (3) is unique in law, being a well-known result in the theory of SDE. Proposition 1 now implies that the weak solution of BSDE (H, f, μ) is unique in law, too. (Also, the solution of (4) and (5) is unique in law.)

Summarizing, we state the following proposition.

Proposition 2. (i) *There exists a weak solution of BSDE (H, f, μ) unique in law.*
(ii) *The BSDE (H, f, W) does not admit any strong solution.*

R e m a r k 2. The example can be generalized in the following way. Suppose that T and H are defined as above and that there is given a driver f with the following properties.

(i) f is $(\mathcal{D}_0^t)_{t \in [0, T]}$ -progressively measurable and

$$\mathbf{E}\left[\exp\left\{-\int_0^1 f(s, W) dW_s - \frac{1}{2} \int_0^1 f^2(s, W) ds\right\}\right] = 1,$$

where W is a Brownian motion.

(ii) We have

$$\int_0^T f(s, y) ds = 0, \quad y \in D(\mathbf{R}).$$

(iii) The SDE (3) with drift coefficient $b = -f$ does not possess any strong solution. Then the same conclusions hold.

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SOME PROPERTIES OF GENERALIZED PICKANDS CONSTANTS¹⁾

Изучаются свойства обобщенных констант Пикандса $\mathcal{H}_\eta$, которые возникают в теории экстремальных значений гауссовских процессов и определяются следующим образом:

$$\mathcal{H}_\eta = \lim_{T \rightarrow \infty} \frac{\mathcal{H}_\eta(T)}{T},$$

где $\mathcal{H}_\eta(T) = \mathbf{E} \exp(\max_{t \in [0, T]} (\sqrt{2} \eta(t) - D\eta(t)))$ и $\eta(t)$ — центрированный гауссовский процесс со стационарными приращениями.

Даны оценки скорости сходимости $\mathcal{H}_\eta(T)/T$ к $\mathcal{H}_\eta$ и доказано, что если $\eta_{(n)}(t)$ слабо сходится в $C([0, \infty))$ к $\eta(t)$, то при некоторых неограниченных условиях $\lim_{n \rightarrow \infty} \mathcal{H}_{\eta_{(n)}} = \mathcal{H}_\eta$.

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