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ON THE WEAK CONVERGENCE OF
VECTOR-VALUED CONTINUOUS RANDOM PROCESSES¹⁾

В статье устанавливаются некоторые результаты о слабой сходимости непрерывных векторнозначных случайных процессов, таких, например, как векторнозначные субгауссовские процессы, векторнозначные процессы, удовлетворяющие условию Липшица, и случайные ряды Фурье с векторнозначными коэффициентами. Полученные результаты могут рассматриваться как обобщение соответствующих результатов на вещественной прямой. Наши основные идеи берут начало в [3] и [4].

Ключевые слова и фразы: слабая сходимость, векторнозначные гауссовские процессы, метрическая энтропия, пространство типа 2.

1. Introduction and preliminary statements. Suppose that (T, d) is a compact metric or pseudometric space, $(B, \|\cdot\|)$ is a separable real or complex Banach space with its topological dual B' , $\langle \cdot, \cdot \rangle$ denoting their natural bilinear form. Let $C(T, B)$ be the linear space of all continuous functions on (T, d) with values in B . In particular, $C(T)$ represents either $C(T, \mathbf{R})$ or $C(T, \mathbf{C})$. Then $C(T, B)$ is a separable Banach space with the natural sup-norm $\|x\| = \sup_{t \in T} \|x(t)\|$. Our aim of this article is to discuss the weak convergence of a sequence of random processes with sample paths in $C(T, B)$. Fernique [4] proved the following characterization of relatively compact subsets in $C(T, B)$.

Proposition 1.1. *Suppose that F is a separating set of real continuous functions on B . For a subset C of $C(T, B)$ to be relatively compact it is sufficient and necessary that the set C satisfies*

- (i) *there is a compact set K of B such that $\{x(t), t \in T, x \in C\} \subset K$,*
- (ii) *for each $f \in F$, the set $f \circ C = \{f \circ x, x \in C\}$ is relatively compact in $C(T)$.*

We endow $C(T, B)$ with a Borel σ -field $\mathcal{B}(C(T, B))$ induced by the uniform topology, then the well-known Prokhorov theorem holds for any family of measures on $(C(T, B), \mathcal{B}(C(T, B)))$ and from Proposition 1.1 we easily obtain

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Proposition 1.2. *Let $\mathcal{M}$ be a family of probability measures on the space $C(T, B)$. Suppose that for any positive ε there exists a compact set K_ε of B such that*

$$\mu\{x: \exists t \in T, x(t) \notin K_\varepsilon\} \leq \varepsilon, \quad \forall \mu \in \mathcal{M}. \quad (1.1)$$

Then $\mathcal{M}$ is relatively compact if and only if there exists a separating set F of real continuous functions on B such that for any $f \in F$ the family

$$f \circ \mathcal{M} = \{f \circ \mu, \mu \in \mathcal{M}\}$$

is relatively compact on $C(T)$. Here $f \circ \mu(A) = \mu\{x \in C(T, B), f \circ x \in A\}$ for each Borel subset A in $C(T)$.

Since it is easy for us to deal with the relative compactness of a family of probability measures on $C(T)$, the key to the relative compactness of a family of probability measures on $C(T, B)$ is to verify the first condition of Proposition 1.2. We will base our discussions on Proposition 1.2 in two ways. One way is to use the following facts: a subset K of a separable Banach space B is relatively compact if and only if K is bounded and for any $\eta > 0$ there is a finite-dimensional closed subspace F such that $q_F(x) \leq \eta$ for all $x \in K$, where q_F denotes the quotient norm, i.e., $q_F(x) = \inf_{y \in F} \|x - y\|$. Thus, we can rewrite (1.1) of Proposition 1.2 as follows.

For any $\varepsilon > 0$ and $\eta > 0$ there are a positive constant M and a finite-dimensional closed subspace F such that

$$\mu\{x: \exists t \in T, \|x(t)\| > M\} < \varepsilon, \quad (1.2)$$

and

$$\mu\{x: \exists t \in T, q_F(x(t)) > \eta\} < \varepsilon \quad (1.3)$$

for any $\mu \in \mathcal{M}$.

The second way needs something more. Let C be a convex symmetric closed set in B with its gauge $N_C(x) = \inf\{\lambda: x \in \lambda C\}$ and polar set $C^0 = \{f \in B': \langle f, x \rangle \leq 1, \text{ for all } x \in C\}$. If C is convex and $\sigma(B, B')$ -closed, we have $C = \bigcap_{y \in C^0} A(y)$ where $A(y) = \{x: \langle y, x \rangle \leq 1\}$. Clearly, $C^c = \bigcup_{y \in C^0} A(y)^c$. Thus there exists a sequence of y_n such that for any $x \in B$,

$$N_C(x) = \inf\{\lambda > 0: x \in \lambda C\} = \sup_n \langle y_n, x \rangle.$$

On the other hand, suppose that b is a closed and convex symmetric bounded subset of B , then b is the intersection of a sequence of decreasing convex symmetric and closed neighborhood of origin in B , and hence there exists a sequence of increasing continuous seminorms N_k , converging simply to N_b , in B . B with such a continuous seminorm N_k defines a separable Banach space $B_k = (B, N_k)$ by means of quotient and completion. We are also to work with these Banach spaces as well as $(B, \|\cdot\|)$.

Proposition 1.2 concerns only the relative compactness, the following Singer theorem can usually be used to identify the candidate for the limit in weak convergence of a family of probability measures.

Theorem (I. Singer, [11]). *The topological dual of $C(T, B)$ is isometrically isomorphic to the space of all the completely additive and regular functions f_e defined on the Borel set $e \subset T$ with values in B' . The isomorphism $\Phi \leftrightarrow f_e$ between the two spaces is given by the relation*

$$\Phi(x) = \int \langle x(t), df_t \rangle, \quad \forall x \in C(T, B). \quad (1.4)$$

As in the scalar setting, let $\mathcal{W} = \{\sum_{i=1}^n f_i \delta_{t_i}, f_i \in B', t_i \in T, 1 \leq i \leq n, n \geq 1\}$, where δ_t denotes the unit point mass at t , then $\mathcal{W}$ is sequentially w^* -dense in the topological dual $(C(T, B))'$ of $C(T, B)$.

Next let us introduce some notation. Throughout we assume that $(\Omega, \mathcal{F}, \mathbf{P})$ is a complete probability space on which all random variables are defined. A random process with values in B is a family $X = (X(t), t \in T)$ of measurable mappings from $(\Omega, \mathcal{F}, \mathbf{P})$ to $(B, \mathcal{B})$ indexed by T . Two random processes X and Y are modifications each other if for each $t \in T$, $\mathbf{P}\{X(t) = Y(t)\} = 1$. $X = (X(t), t \in T)$ is said to be Gaussian if the family $\{\langle X(t), y \rangle, t \in T, y \in B'\}$ is a family of zero-mean real Gaussian random variables. Thus a B -valued Gaussian process X is uniquely determined by its covariance structure $\mathbf{E}\langle X(t), y \rangle \langle X(s), z \rangle$. As an example we give the construction of B -valued Wiener process generated by a given probability measure μ . Let $T = [0, T_0]$; for any $t \in T$, $R_t = R_t(y, z)$ is a symmetric semi-positive bilinear form on B' . Assume that for each $y \in B'$, the mapping $t \rightarrow R_t(y, y)$ is increasing and right-continuous. We also assume that there exists a Gaussian probability measure μ on B such that

$$\int \langle x, y \rangle^2 d\mu(x) = R_{T_0}(y, y) \quad \forall y \in B'. \quad (1.5)$$

Then, under these conditions, there really exists a Gaussian random process W indexed by T with values in B such that for all $s, t \in T$ and $y, z \in B'$,

$$\mathbf{E}\langle W(s), y \rangle \langle W(t), z \rangle = R_{s \wedge t}(y, z). \quad (1.6)$$

This assertion can be proved as follows.

For any pair (s, t) with $s < t$, the difference $R_t - R_s$ is the covariance of a Gaussian measure $\mu_{s,t}$ on B ; we denote by $(G_{s,t}, s \leq t, t \in T)$ a family of independent Gaussian vectors with respective laws $\mu_{s,t}$. Given any finite number of points $\theta = \{0 = t_0 < t_1 < \dots < t_n = T_0\}$ of T , we denote by μ^θ the law of Gaussian random vectors

$$Y^\theta = \left(\sum_{t_k \leq t} G_{t_{k-1}, t_k}, t_k \leq t, t \in \theta \right).$$

In view of independence, for all $s, t \in \theta$ and $y, z \in B'$ we have

$$\int \langle x(s), y \rangle \langle x(t), z \rangle d\mu^\theta(x) = R_{s \wedge t}(y, z). \quad (1.7)$$

This shows that the family (μ^θ) where θ is finite subsets of T is consistent in the sense of Kolmogorov's consistency theorem. So the family (μ^θ) defines a Gaussian probability μ on B^T with the required covariance structure. As usual, we can realize W by the identity mapping from B^T into B^T .

In particular, if $R_t(y, y) = tR_1(y, y) = t \int \langle x, y \rangle^2 d\mu(x)$, the B -valued Wiener process obtained above is called homogeneous and normalized by μ .

This example and its proof is due to Fernique [4]. He has made investigations at length about the problem of existence of continuous versions of some vector-valued Gaussian processes.

As many authors point out that, in a compact metric or pseudometric space (T, d) , there are two important quantities measuring the size of T in the d : metric entropy and majorising measure, which are closely related to the continuity and weak convergence for random processes. We denote by $N(T, d, \varepsilon)$ the minimal number of open balls of radius ε for d that cover T . If there exists a probability m in (T, d) such that

$$\limsup_{\eta \rightarrow 0} \sup_{t \in T} \int_0^\eta \left(\log \frac{1}{m(B_d(t, \varepsilon))} \right)^{1/2} d\varepsilon = 0, \quad (1.8)$$

where $B_d(t, \varepsilon)$ is a ball with center at t and radius ε , then m is called a continuous majorising measure.

Finally, for simplicity and clarity we always assume that c is a positive constant which may vary from line to line and depend on some appropriate parameters.

The present paper is arranged as follows. Section 2 is devoted to the weakly relative compactness of a family of Gaussian measures in $C(T, B)$ and some simple remarks on invariance principle for a sequence of i.i.d. random vectors. In Sections 3 and 4 our subjects will be vector-valued subgaussian processes in the weak sense and vector-valued random processes satisfying the Lipschitz condition, respectively. At the end of this paper, random Fourier series with vector-valued coefficients are taken into account although there is a little gap between our result and one in the scalar setting.

2. Relative compactness of Gaussian measures on $C(T, B)$ and invariance principle. Let T be a compact metric space and x a Gaussian random process on T with continuous sample paths. Fernique [2] proved that the set of separable Gaussian processes y on T such that

$$\mathbf{E}(y(t) - y(s))^2 \leq \mathbf{E}(x(t) - x(s))^2, \quad \forall s, t \in T, \quad (2.1)$$

and

$$\mathbf{E}y^2(t_0) \leq 1, \quad \text{for some } t_0 \in T, \quad (2.2)$$

is relatively compact for the weak convergence of probability measures on the space $C(T)$.

Now we extend the above result to the vector situation. If X is a Gaussian random process with values in a separable Banach space B , we denote its covariance by $d_X(f, t; g, s) = (\mathbf{E}(\langle f, X(t) \rangle - \langle g, X(s) \rangle)^2)^{1/2}$ for $s, t \in T$ and $f, g \in B'$. Gaussian process X is completely specified by d_X .

Theorem 2.1. *Given a continuous Gaussian random process X on T with values in B , let $\mathcal{M}$ be the set of the law μ_Y of Gaussian random process Y on T satisfying $d_Y(f, t; g, s) \leq d_X(f, t; g, s)$ for any $s, t \in T$ and $f, g \in B'$, and for each $f \in B'$ there is an element a in T such that $\mathbf{E}\langle f, Y(a) \rangle^2 \leq 1$. Then $\mathcal{M}$ is relatively compact for the weak convergence of probability measures on $C(T, B)$.*

Proof. It is obvious that for each $f \in B'$ (B' is of course a separating set of real continuous functions on B) $f \circ \mathcal{M}$ is relatively compact on $C(T)$. Thus by Proposition 1.2 it is enough to show that for any $\varepsilon > 0$ there is a compact subset $K \subset B$ such that for each $\mu_Y \in \mathcal{M}$,

$$\mu_Y(x: \exists t \in T, x(t) \notin K) < \varepsilon,$$

i.e., $\mathbf{P}\{\omega: \exists t \in T, Y(\omega, t) \notin K\} < \varepsilon$. For this we need only consider $\mathbf{E} \sup_{t \in T} N_K(Y(t))$. The continuity of Gaussian random process X implies that for any positive ε there exists a convex symmetric and compact set K such that

$$\mathbf{E} \sup_{t \in T} N_K(X(t)) < \varepsilon.$$

As we noticed in Section 1, for such a convex symmetric and compact set $K \subset B$ there is a sequence of $f_n \in B'$ such that $N_K(x) = \sup_{n \geq 1} \langle f_n, x \rangle$ for all $x \in B$. Since $d_Y(f, t; g, s) \leq d_X(f, t; g, s)$, the well-known Slepian's lemma shows

$$\begin{aligned} \mathbf{E} \sup_{t \in T} N_K(Y(t)) &= \mathbf{E} \sup_{t \in T} \sup_{n \geq 1} \langle f_n, Y(t) \rangle \leq \mathbf{E} \sup_{t \in T} \sup_{n \geq 1} \langle f_n, X(t) \rangle \\ &= \mathbf{E} \sup_{t \in T} N_K(X(t)), \end{aligned}$$

from which we obtain the required result.

Before concluding this section, we should mention Kuelbs' invariance principle (cf. Kuelbs [8]) for a sequence of i.i.d. random vectors.

Theorem 2.2. *Let $\{X_n, n \geq 1\}$ be a sequence of independent and symmetric random vectors with the common distribution μ . Suppose*

$$\frac{S_n}{\sqrt{n}} \Rightarrow G. \quad (2.3)$$

Construct the partial sum process

$$X_n(t) = \frac{S_{[nt]}}{\sqrt{n}} + \frac{nt - [nt]}{\sqrt{n}} X_{[nt]+1}, \quad 0 \leq t \leq 1. \quad (2.4)$$

Then $X_n \Rightarrow W$, W is a vector-valued Wiener process generated by μ .

P r o o f. Obviously, the classical Donsker invariance principle can be applied to $\langle f, X_n(t) \rangle$ for each $f \in B'$. Thus by Proposition 1.2 it suffices to prove that for any $\varepsilon > 0$ there is a compact set $K \subset B$ such that

$$\sup_n \mathbf{P} \left\{ \omega: \exists t \in [0, 1], X_n(t) \notin K \right\} < \varepsilon. \quad (2.5)$$

In fact, since $S_n/\sqrt{n} \Rightarrow G$, there must be a compact and convex symmetric set $K \subset B$ such that

$$\sup_{n \geq 1} \mathbf{P} \left\{ \omega: \frac{S_n}{\sqrt{n}} \notin \frac{1}{2} K \right\} < \varepsilon,$$

from which we deduce by using Levy's inequality

$$\begin{aligned} \sup_{n \geq 1} \mathbf{P} \left\{ \omega: \sup_{t \in [0, 1]} N_K(X_n(t)) > 1 \right\} &\leq \sup_{n \geq 1} \mathbf{P} \left\{ \omega: \max_{1 \leq k \leq n} N_K \left(\frac{S_k}{\sqrt{n}} \right) > \frac{1}{2} \right\} \\ &\quad + \sup_{n \geq 1} \mathbf{P} \left\{ \omega: \max_{1 \leq k \leq n} N_K \left(\frac{X_k}{\sqrt{n}} \right) > \frac{1}{2} \right\} \\ &\leq 4 \sup_{n \geq 1} \mathbf{P} \left\{ \omega: N_K \left(\frac{S_n}{\sqrt{n}} \right) > \frac{1}{2} \right\} < 4\varepsilon. \end{aligned} \quad (2.6)$$

Theorem 2.2 now easily follows.

Comparing with Kuelbs' direct method, Lemma 1.2 pays more attention to the topology on any compact set $K \subset B$: the topology induced on K by the norm topology coincides with the weak topology on K , and so has more extensive applications.

3. Vector-valued subgaussian processes. Let us first recall that a real random process $x = (x(t), t \in T)$ is called subgaussian if

$$\|x(t) - x(s)\|_{\phi_2} \leq d(s, t)$$

for any $s, t \in T$, where $\phi_2(x) = \exp |x|^2 - 1$ and $\|\cdot\|_{\phi_2}$ denotes its Orlicz norm. Heinkel [6] showed that the continuous majorising measure condition is sufficient for x to satisfy the central limit theorem in $C(T)$. (As for the general theory on central limit theorems, see [1], we write CLT for central limit theorem later.)

We can similarly define a vector-valued subgaussian process $X = (X(t), t \in T)$ in the strong sense, that is,

$$\|X(t) - X(s)\|_{\phi_2} \leq d(s, t), \quad \forall s, t \in T. \quad (3.1)$$

Although the continuous majorising measure condition can still guarantee the existence of modification with continuous sample paths, the CLT does not hold for X any longer.

For example, let $(x_i, i \geq 1)$ be a sequence of elements in $C(T, B)$ and $(\varepsilon_i, i \geq 1)$ a Rademacher sequence, define $X(t) = \sum_{i=1}^{\infty} \varepsilon_i x_i(t)$, for $t \in T$. By [9] we have

$$\|X(s) - X(t)\|_{\phi_2} \leq 8 \left(\mathbf{E} \left\| \sum_{i=1}^{\infty} \varepsilon_i (x_i(s) - x_i(t)) \right\|^2 \right)^{1/2} =: d(s, t). \quad (3.2)$$

Only when $Y(t) = \sum_{i=1}^{\infty} g_i x_i(t)$, $t \in T$, is also a.s. continuous can X as a random vector in $C(T, B)$ satisfy the CLT.

Now a vector-valued random process $X = (X(t), t \in T)$ with values in B is said to be subgaussian in the weak sense if for each $f \in B'$ there is a continuous distance $d_f(s, t)$ with respect to d on T such that

$$\left\| \langle f, X(s) \rangle - \langle f, X(t) \rangle \right\|_{\phi_2} \leq d_f(s, t), \quad \forall s, t \in T. \quad (3.3)$$

Obviously, $\|\langle f, X(s) \rangle - \langle f, X(t) \rangle\|_{\phi_2} \leq \|f\| \|X(s) - X(t)\|_{\phi_2}$.

On the other hand, $X = (X(t), t \in T)$ is called uniformly γ -subgaussian if there is a Gaussian vector G in B such that

$$\mathbf{E} \exp \langle f, X(t) \rangle \leq \mathbf{E} \exp \langle f, G \rangle, \quad \forall t \in T, f \in B'. \quad (3.4)$$

See [6] for details on γ -subgaussian and subgaussian vectors.

Our main result on vector-valued subgaussian process is as follows.

Theorem 3.1. Suppose that $X = (X(t), t \in T)$ is both a subgaussian in the weak sense and a uniformly γ -subgaussian process. If there exists a probability measure m on (T, d) such that

$$\lim_{\eta \rightarrow 0} \sup_{f \in B'} \sup_{t \in T} \int_0^{\eta} \left(\log \frac{1}{m(B_{d_f}(t, \varepsilon))} \right)^{1/2} d\varepsilon = 0 \quad (3.5)$$

and $\sup_{t \in T} \int_0^{\infty} (\log(m(B_{d_f}(t, \varepsilon)))^{-1})^{1/2} d\varepsilon$ is continuous with respect to the topology of the compact convergence on B , then X is a.s. continuous on T and satisfies the CLT in $C(T, B)$.

P r o o f. We first note that for any $s, t \in T$ and $f, g \in B'$,

$$\begin{aligned} d_X(f, t; g, s) &= \left\| \langle f, X(t) \rangle - \langle g, X(s) \rangle \right\|_{\phi_2} \leq \left\| \langle f, X(s) \rangle - \langle g, X(s) \rangle \right\|_{\phi_2} \\ &\quad + \left\| \langle f, X(s) \rangle - \langle f, X(t) \rangle \right\|_{\phi_2} \\ &\leq 5(\mathbf{E} \langle f - g, G \rangle^2)^{1/2} + d_f(s, t) =: d_G(f, g) + d_f(s, t). \end{aligned} \quad (3.6)$$

So, for any $\varepsilon > 0$, $B_{d_G}(f, \varepsilon/2) \times B_{d_f}(t, \varepsilon/2) \subset B_{d_X}((f, t); \varepsilon)$. Since G is a Gaussian vector with values in B and B'_1 is $\sigma(B, B')$ compact and metrizable,

by Talagrand's theorem [9] there is a probability measure μ on B'_1 such that

$$\sup_{f \in B'_1} \int_0^\infty \left(\log \frac{1}{\mu(B_{d_G}(f, \varepsilon))} \right)^{1/2} d\varepsilon \leq cE \|G\|. \quad (3.7)$$

Let $\pi = \mu \times m$, then $m(B_{d_f}(t, \varepsilon/2)) \mu(B_{d_G}(f, \varepsilon/2)) \leq \pi(B_{d_X}((f, t); \varepsilon))$, and hence

$$\begin{aligned} & \sup_{f \in B'_1} \sup_{t \in T} \int_0^\infty \left(\log \frac{1}{\pi(B_{d_X}((f, t); \varepsilon))} \right)^{1/2} d\varepsilon \\ & \leq \sup_{f \in B'_1} \sup_{t \in T} \int_0^\infty \left(\log \frac{1}{m(B_{d_f}(t, \frac{\varepsilon}{2}))} \right)^{1/2} d\varepsilon \\ & \quad + \sup_{f \in B'_1} \int_0^\infty \left(\log \frac{1}{\mu(B_{d_G}(f, \frac{\varepsilon}{2}))} \right)^{1/2} d\varepsilon \\ & \leq c \left(E \|G\| + \sup_{f \in B'_1} \sup_{t \in T} \int_0^\infty \left(\log \frac{1}{m(B_{d_f}(t, \varepsilon))} \right)^{1/2} d\varepsilon \right). \end{aligned} \quad (3.8)$$

Now we can use Fernique's theorem [2] to obtain

$$\begin{aligned} E \sup_{t \in T} \|X(t)\| &= E \sup_{t \in T} \sup_{f \in B'_1} \langle f, X(t) \rangle \\ &\leq c \left(E \|G\| + \sup_{f \in B'_1} \sup_{t \in T} \int_0^\infty \left(\log \frac{1}{m(B_{d_f}(t, \varepsilon))} \right)^{1/2} d\varepsilon \right). \end{aligned}$$

On the other hand, for any $\varepsilon > 0$ there is a convex symmetric and compact set $K \subset B$ such that

$$EN_K(G) + \sup_{f \in K^0} \sup_{t \in T} \int_0^\infty \left(\log \frac{1}{m(B_{d_f}(t, \varepsilon))} \right)^{1/2} d\varepsilon < \varepsilon. \quad (3.9)$$

For such a compact set K there exists an increasing sequence N_k of continuous seminorms, converging simply to N_K in B , and (B, N_k) is a separable Banach space.

By repeating the previous proof for (B, N_k) and taking the limit, we also have

$$E \sup_{t \in T} N_K(X(t)) < c\varepsilon. \quad (3.10)$$

Thus, according to Theorem 4.2.1 in [4], X has a modification with continuous sample paths since each real-valued random process $f(X)$ is a.s. continuous on T for $f \in B'_1$.

To prove that X satisfies the CLT, without loss of generality we can assume that X is symmetric and $\{X, X_n, n \geq 1\}$ are independent identically distributed. Consider the normed sums $Z_n = n^{-1/2} \sum_{i=1}^n X_i$.

It is not hard to verify that

$$\mathbf{E} \exp \langle f, Z_n(t) \rangle \leq \mathbf{E} \exp \langle f, G \rangle, \quad f \in B'_1, \quad t \in T, \quad (3.11)$$

and

$$\left\| \langle f, Z_n(t) \rangle - \langle f, Z_n(s) \rangle \right\|_{\phi_2} \leq c d_f(s, t). \quad (3.12)$$

Thus one can similarly prove that for any $\varepsilon > 0$ there is a convex symmetric and compact $K \subset B$ such that

$$\sup_{n \geq 1} \mathbf{E} \sup_{t \in T} N_K(Z_n(t)) < \varepsilon. \quad (3.13)$$

For each $f \in B'$, Heinkel [7] tells us that $f(X)$ satisfies the CLT in $C(T)$. Hence we deduce that $\{Z_n, n \geq 1\}$ is relatively compact by using Proposition 1.2 again.

As for the uniqueness of the limit, it is enough to notice Singer Theorem and the CLT in finite-dimensional spaces. The proof of Theorem 3.1 is completed.

4. Vector-valued random Lipschitz processes. We will begin our arguments with the following two lemmas. The first lemma is Lemma 7.13 in [1] and the second is just the formula (4.12) in [9].

Lemma 4.1. *Suppose that (T, d) is a compact metric space and*

$$\int_0^\infty (\log N(T, d, \varepsilon))^{1/2} d\varepsilon < \infty.$$

Then there is another continuous pseudometric d' such that $d \leq d', d/d' \rightarrow 0$ and $\int_0^\infty (\log N(T, d', \varepsilon))^{1/2} d\varepsilon < \infty$.

Lemma 4.2. *Suppose that X is a subgaussian vector in B . Then for any $t > 0$,*

$$\mathbf{P}\{\|X\| > t\} \leq 2 \exp \left(- \frac{t^2}{32 \mathbf{E}\|X\|^2} \right). \quad (4.1)$$

Our object in this section is to establish the following

Theorem 4.1. *Suppose that B is the space of type 2, $X = (X(t), t \in T)$ is a mean zero B -valued random process satisfying Lipschitz condition, i.e.,*

$$\|X(t) - X(s)\| \leq M d(s, t), \quad (4.2)$$

where M is a real-valued random variable with finite variance.

If $\int_0^\infty (\log N(T, d, \varepsilon))^{1/2} d\varepsilon < \infty$, then X satisfies the CLT on $C(T, B)$.

Proof. Our hypotheses guarantee that $f(X)$ satisfies the CLT for each $f \in B'$ and multi-dimensional CLT remains true. This is a consequence of Theorem 14.2 in [9].

The key to the proof of Theorem 4.1 is to prove that for each $\varepsilon > 0$ and $\eta > 0$ there are a positive constant a and a finite-dimensional closed subspace F such that

$$\sup_{n \geq 1} \mathbf{P} \left\{ \omega: \sup_{t \in T} \left\| \frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i X_i(t) \right\| > a \right\} < \varepsilon \quad (4.3)$$

and

$$\sup_{n \geq 1} \mathbf{P} \left\{ \omega: \sup_{t \in T} q_F \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i X_i(t) \right) > \eta \right\} < \varepsilon. \quad (4.4)$$

Here $\{X, X_n, n \geq 1\}$ is a sequence of independent identically distributed random processes and $\{\varepsilon_n, n \geq 1\}$ is a Rademacher sequence independent of $\{X_n, n \geq 1\}$. The two statements will be proved in analogous ways: using truncation.

In fact, since $\mathbf{E}M^2 < \infty$, for any $\varepsilon > 0$ there is a positive constant a_0 such that

$$\sup_{n \geq 1} \mathbf{P} \left\{ \omega: \frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{\|X_i(s) - X_i(t)\|^2}{a_0^2 d^2(s, t)} \geq 1 \right\} \leq \frac{\mathbf{E}M^2}{a_0^2} < \frac{\varepsilon}{2}. \quad (4.5)$$

For such a fixed a_0 it easily follows from Lemma 4.2 that

$$\begin{aligned} & \mathbf{E}_X \mathbf{P}_\varepsilon \left\{ \left\| \frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i (X_i(t) - X_i(s)) \right\| \right. \\ & \quad \times \left. I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{\|X_i(s) - X_i(t)\|^2}{a_0^2 d^2(s, t)} \leq 1 \right) \right\| > u a_0 d(s, t) \Big\} \\ & \leq 2 \mathbf{E}_X \exp \left(-u^2 \left[32 \mathbf{E}_\varepsilon \left\| \frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i (X_i(t) - X_i(s)) \right\| \right. \right. \\ & \quad \times \left. \left. I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{\|X_i(s) - X_i(t)\|^2}{a_0^2 d^2(s, t)} \leq 1 \right) \right\|^2 \right]^{-1} \right) \\ & \leq 2 \exp \left(-\frac{u^2}{32c} \right), \end{aligned}$$

where c is an absolute constant in the definition of the space of type 2. In other words,

$$\begin{aligned} & \left\| \frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i (X_i(t) - X_i(s)) I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{\|X_i(s) - X_i(t)\|^2}{a_0^2 d^2(s, t)} \leq 1 \right) \right\|_{\phi_2} \\ & \leq c a_0 d(s, t). \end{aligned}$$

Thus we have

$$\begin{aligned} & \mathbf{E} \sup_{t \in T} \left\| \frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i X_i(t) I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{\|X_i(s) - X_i(t)\|^2}{a_0^2 d^2(s, t)} \leq 1 \right) \right\| \\ & \leq c a_0 \int_0^\infty (\log N(T, d, \varepsilon))^{1/2} d\varepsilon, \end{aligned}$$

from which there exists a positive constant a such that

$$\begin{aligned} & \sup_{n \geq 1} \mathbf{P} \left\{ \omega: \sup_{t \in T} \left\| \frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i X_i(t) \right. \right. \\ & \quad \times I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{\|X_i(s) - X_i(t)\|^2}{a_0^2 d^2(s, t)} \leq 1 \right) \left. \right\| \geq a \Bigg\} \leq \frac{\varepsilon}{2}. \quad (4.6) \end{aligned}$$

Now, the first statement holds.

The proof of the second one is similar but to use Lemma 4.1. Let d' be as in Lemma 4.1, and define

$$Y(s, t) = \begin{cases} \frac{X(t) - X(s)}{d'(s, t)} & \text{for } s \neq t, \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to see that $Y(s, t)$ is a.s. in $C(T \times T, B)$ since $d/d' \rightarrow 0$.

So, for each $\varepsilon > 0$ and $\eta > 0$ there must be a finite-dimensional closed subspace F such that

$$\mathbf{P} \{ \omega: \exists s \neq t, q_F(Y(s, t)) > \eta \} < \varepsilon. \quad (4.7)$$

Further, by the dominated convergence theorem it is not hard to conclude that for any $\varepsilon > 0$ and $\eta > 0$ there must be a finite-dimensional closed subspace F such that

$$\sup_{n \geq 1} \mathbf{P} \left\{ \omega: \frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{q_F(X_i(s) - X_i(t))^2}{\varepsilon^2 \eta^2 (d'(s, t))^2} \geq 1 \right\} < \frac{\varepsilon}{2}. \quad (4.8)$$

Analogously,

$$\begin{aligned} & \mathbf{E}_X \mathbf{P}_\varepsilon \left\{ q_F \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i (X_i(t) - X_i(s)) \right) \right. \\ & \quad \times I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{q_F(X_i(s) - X_i(t))^2}{\varepsilon^2 \eta^2 (d'(s, t))^2} \leq 1 \right) > u a_0 d(s, t) \Bigg\} \\ & \leq 2 \mathbf{E}_X \exp \left(-u^2 \left(32 \mathbf{E}_\varepsilon q_F \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i (X_i(t) - X_i(s)) \right) \right) \right) \end{aligned}$$

$$\times I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{q_F(X_i(s) - X_i(t))^2}{\varepsilon^2 \eta^2 (d'(s, t))^2} \leq 1 \right) \Big)^{-1} \Big)^{-1} \\ \leq 2 \exp \left(- \frac{u^2}{32c} \right).$$

Here in the last step we use the fact that any quotient space of type 2 remains to be of type 2 and has the same constant.

So, we have

$$\begin{aligned} & \mathbf{E} \sup_{t \in T} q_F \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n \varepsilon_i X_i(t) I \left(\frac{1}{n} \sum_{i=1}^n \sup_{s \neq t} \frac{q_F(X_i(s) - X_i(t))^2}{\varepsilon^2 \eta^2 (d'(s, t))^2} \leq 1 \right) \right) \\ & \leq c \int_0^\infty (\log N(T, \varepsilon \eta d', u))^{1/2} du \\ & \leq c \varepsilon \eta \int_0^\infty (\log N(T, d', u))^{1/2} du. \end{aligned} \quad (4.9)$$

Thus the required result (4.4) can be proved by noting Lemma 4.1. The proof of Theorem 4.3 is concluded.

5. Random Fourier series with vector-valued coefficients. Random Fourier series with real coefficients have been studied in details by Marcus and Pisier [10] and so on. It appears many elegant results and applications to harmonic analysis. Up to now, however, the results concerning random Fourier series with vector-valued coefficients are still fragmentary. Let us first recall some backgrounds.

Let G be a compact Abel group, $\Gamma = \{\gamma_n, n \geq 1\}$ a countable subset of its character group. Suppose that $\{g_n, n \geq 1\}$ is a sequence of independent standard Gaussian random variables, $\{\varepsilon_n, n \geq 1\}$ is a Rademacher sequence, $\{\xi_n, n \geq 1\}$ is a sequence of independent symmetric real random variables with $\mathbf{E} \xi_n^2 = 1$ and $\inf_{n \geq 1} \mathbf{E} |\xi_n| > 0$, $\{a_n, n \geq 1\}$ are real numbers satisfying $\sum_{n=1}^\infty |a_n|^2 < \infty$. Define three Fourier series on G as follows:

$$\begin{aligned} x(t) &= \sum_{n=1}^\infty a_n \xi_n \gamma_n(t), & y(t) &= \sum_{n=1}^\infty a_n g_n \gamma_n(t), \\ z(t) &= \sum_{n=1}^\infty a_n \varepsilon_n \gamma_n(t), \end{aligned} \quad (5.1)$$

and define $d(s, t) = (\sum_{n=1}^\infty |a_n|^2 |\gamma_n(s) - \gamma_n(t)|^2)^{1/2}$ for $s, t \in G$. Main results in [10] show that

$$\mathbf{E} \left\| \sum_{n=1}^\infty a_n \xi_n \gamma_n \right\| \approx \mathbf{E} \left\| \sum_{n=1}^\infty a_n g_n \gamma_n \right\| \approx \mathbf{E} \left\| \sum_{n=1}^\infty a_n \varepsilon_n \gamma_n \right\| \quad (5.2)$$

(all of them are equivalent to $\int_0^\infty (\log N(G, d, \varepsilon))^{1/2} < \infty$) although $C(G)$, the set of all bounded continuous complex functions on G , is of no finite cotype. As an important corollary, x satisfies the CLT on $C(G)$ and the limit is just y .

Similar problems naturally arise when real coefficients $\{a_n, n \geq 1\}$ are replaced by a sequence of vectors $\{x_n, n \geq 1\}$ in a separable Banach space B . If $\sum_{n=1}^\infty x_n \xi_n$, $\sum_{n=1}^\infty x_n g_n$, and $\sum_{n=1}^\infty x_n \varepsilon_n$ a.s. converge respectively, we can define respectively by the comparison convergence theorem for random series

$$\begin{aligned} X(t) &= \sum_{n=1}^\infty x_n \xi_n \gamma_n(t), & Y(t) &= \sum_{n=1}^\infty x_n g_n \gamma_n(t), \\ Z(t) &= \sum_{n=1}^\infty x_n \varepsilon_n \gamma_n(t). \end{aligned} \quad (5.3)$$

Fernique [3] first showed that a vector-valued stationary Gaussian process Y has a modification with continuous sample paths if and only if the mapping $y \rightarrow \mathbf{E} \sup_{t \in G} \langle y, Y(t) \rangle$ from B' to C is continuous with respect to the topology of the compact convergence on B . On the other hand, we have the following relation (cf. [9, Corollary 13.14])

$$\begin{aligned} &c \left(\mathbf{E} \left\| \sum_{n=1}^\infty \varepsilon_n x_n \right\| + \sup_{f \in B'_1} \mathbf{E} \sup_{t \in G} \sum_{n=1}^\infty \varepsilon_n \langle f, x_n \rangle \gamma_n(t) \right) \\ &\leq \mathbf{E} \sup_{t \in G} \left\| \sum_{n=1}^\infty \varepsilon_n x_n \gamma_n(t) \right\| \leq \mathbf{E} \sup_{t \in G} \left\| \sum_{n=1}^\infty g_n x_n \gamma_n(t) \right\| \\ &\leq c \left(\mathbf{E} \left\| \sum_{n=1}^\infty g_n x_n \right\| + \sup_{f \in B'_1} \mathbf{E} \sup_{t \in G} \sum_{n=1}^\infty g_n \langle f, x_n \rangle \gamma_n(t) \right). \end{aligned} \quad (5.4)$$

Thus it is easily seen that if $\sum_{n=1}^\infty g_n x_n < \infty$ a.s., then $\sum_{n=1}^\infty \varepsilon_n x_n \gamma_n(t)$ and $\sum_{n=1}^\infty g_n x_n \gamma_n(t)$ converge uniformly a.s. simultaneously, i.e., they have the same continuity. Of course, when $Y = \sum_{n=1}^\infty x_n g_n \gamma_n(t)$ converges uniformly a.s., Z also satisfies the CLT in $C(G, B)$.

The problems remain open, however, for a sequence of random variables $\{\xi_n, n \geq 1\}$, both Gaussian and Rademacher. Next we give our results at this area.

Theorem 5.1. *Suppose that B is the space of cotype q ($2 \leq q < \infty$), $1/p + 1/q = 1$. $\{\xi_n, n \geq 1\}$ is a sequence of independent symmetric real random variables with $\mathbf{E} \xi_n^2 = 1$ and $\inf_n \mathbf{E} |\xi_n| > 0$, $\sup_n \mathbf{E} |\xi_n|^{2q} < \infty$. Further, assume that $\mathbf{E} \left\| \sum_{n=1}^\infty \xi_n x_n \right\| < \infty$. Define*

$$X(t) = \sum_{n=1}^\infty \xi_n x_n \gamma_n(t)$$

and

$$d(s, t) = \sup_{f \in B'_1} \left(\sum_{n=1}^{\infty} |\langle f, x_n \rangle|^p |\gamma_n(s) - \gamma_n(t)|^{2p} \right)^{1/(2p)}. \quad (5.5)$$

If

$$\int_0^{\infty} (\log N(G, d, \varepsilon))^{1/2} d\varepsilon < \infty,$$

then $X(t)$ is a.s. continuous on G .

In addition, if $\sum_{n=1}^{\infty} \xi_n x_n$ satisfies the CLT in B , then X satisfies the CLT in $C(G, B)$.

Proof. First note that $\sum_{n=1}^{\infty} \|x_n\|^q < \infty$ due to the hypotheses, and for each $f \in B'_1$

$$\begin{aligned} & \left(\sum_{n=1}^{\infty} |\langle f, x_n \rangle|^2 |\gamma_n(s) - \gamma_n(t)|^2 \right)^{1/2} \\ & \leq \left(\sum_{n=1}^{\infty} \|x_n\|^q \right)^{1/(2q)} \left(\sum_{n=1}^{\infty} |\langle f, x_n \rangle|^p |\gamma_n(s) - \gamma_n(t)|^{2p} \right)^{1/(2p)} \\ & = \left(\sum_{n=1}^{\infty} \|x_n\|^q \right)^{1/(2q)} d(s, t) \end{aligned} \quad (5.6)$$

by Hölder inequality.

Since $\int_0^{\infty} (\log N(G, d, \varepsilon))^{1/2} d\varepsilon < \infty$, then $f(X)$ is a.s. continuous on G and satisfies the CLT in $C(G)$ for each $f \in B'_1$.

Next define

$$d_f(s, t) = \left(\sum_{n=1}^{\infty} \xi_n^2 |\langle f, x_n \rangle|^2 |\gamma_n(s) - \gamma_n(t)|^2 \right)^{1/2}.$$

Then using (5.4) and noting B is of cotype q space, we have

$$\begin{aligned} & \mathbf{E} \sup_{t \in G} \left\| \sum_{n=1}^{\infty} \xi_n x_n \gamma_n(t) \right\| \\ & \leq c \left(\mathbf{E} \left\| \sum_{n=1}^{\infty} \xi_n x_n \right\| + \mathbf{E}_{\xi} \sup_{f \in B'_1} \mathbf{E}_{\varepsilon} \sup_{t \in G} \sum_{n=1}^{\infty} \xi_n \varepsilon_n \langle f, x_n \rangle \gamma_n(t) \right) \\ & \leq c \left(\mathbf{E} \left\| \sum_{n=1}^{\infty} \xi_n x_n \right\| + \mathbf{E}_{\xi} \sup_{f \in B'_1} \int_0^{\infty} (\log N(G, d_f, \varepsilon))^{1/2} d\varepsilon \right) \\ & \leq c \left(\mathbf{E} \left\| \sum_{n=1}^{\infty} \xi_n x_n \right\| + \mathbf{E}_{\xi} \int_0^{\infty} \left(\log N \left(G, \sup_{f \in B'_1} d_f(s, t), \varepsilon \right) \right)^{1/2} d\varepsilon \right). \end{aligned} \quad (5.7)$$

Clearly, $\sup_{f \in B'_1} d_f(s, t)$ is a translation invariant metric, from Lemma 3.11 in [11] it follows that

$$\begin{aligned}
 & \mathbf{E}_\xi \int_0^\infty \left(\log N \left(G, \sup_{f \in B'_1} d_f(s, t), \varepsilon \right) \right)^{1/2} d\varepsilon \\
 & \leq c \left(\mathbf{E}_\xi \sup_{f \in B'_1} \left(\sum_{n=1}^\infty \xi_n^2 |\langle f, x_n \rangle|^2 \right)^{1/2} \right. \\
 & \quad \left. + \int_0^\infty \left(\log N \left(G, \mathbf{E}_\xi \sup_{f \in B'_1} d_f(s, t), \varepsilon \right) \right)^{1/2} d\varepsilon \right) \\
 & \leq c \left(\mathbf{E}_\xi \left(\sum_{n=1}^\infty \|x_n\|^q |\xi_n|^{2q} \right)^{1/(2q)} \sup_{f \in B'_1} \left(\sum_{n=1}^\infty |\langle f, x_n \rangle|^p \right)^{1/(2p)} \right. \\
 & \quad \left. + \int_0^\infty \left(\log N \left(G, \mathbf{E}_\xi \left(\sum_{n=1}^\infty \|x_n\|^q |\xi_n|^{2q} \right)^{1/(2q)} d, \varepsilon \right) \right)^{1/2} d\varepsilon \right) \\
 & \leq c \left(\left(\sum_{n=1}^\infty \|x_n\|^q \right)^{1/(2q)} \sup_{f \in B'_1} \left(\sum_{n=1}^\infty |\langle f, x_n \rangle|^p \right)^{1/(2p)} \right. \\
 & \quad \left. + \int_0^\infty (\log N(G, d, \varepsilon))^{1/2} d\varepsilon \right). \tag{5.8}
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 & \mathbf{E} \sup_{t \in G} \left\| \sum_{n > N}^\infty \xi_n x_n \gamma_n(t) \right\| \\
 & \leq c \left(\mathbf{E} \left\| \sum_{n > N}^\infty \xi_n x_n \right\| + \left(\sum_{n > N}^\infty \|x_n\|^q \right)^{1/(2q)} \left(\sup_{f \in B'_1} \left(\sum_{n > N}^\infty |\langle f, x_n \rangle|^p \right)^{1/(2p)} \right. \right. \\
 & \quad \left. \left. + \int_0^\infty (\log N(G, d_N, \varepsilon))^{1/2} d\varepsilon \right), \tag{5.9}
 \end{aligned}$$

where $d_N(s, t) = \sup_{f \in B'_1} (\sum_{n > N}^\infty |\langle f, x_n \rangle|^p |\gamma_n(s) - \gamma_n(t)|^{2p})^{1/(2p)}$.

Thus $X(t)$ is a.s. continuous on G .

If $\sum_{n=1}^\infty \xi_n x_n$ satisfies the CLT in B , in order to prove that X satisfies the CLT in $C(G, B)$ it suffices to consider $Z_m = m^{-1/2} \sum_{i=1}^m \sum_{n=1}^\infty \xi_n^i x_n \gamma_n$, where $\{\xi_n^i, n \geq 1\}, i = 1, 2, \dots$, is a sequence of independent sequences with the same distributions as $\{\xi_n, n \geq 1\}$. By some simple calculations,

$$\mathbf{E} \left| \frac{1}{\sqrt{m}} \sum_{i=1}^m \xi_n^i \right|^2 = 1, \quad \sup_m \sup_n \mathbf{E} \left| \frac{1}{\sqrt{m}} \sum_{i=1}^m \xi_n^i \right|^{2q} < \infty.$$

Therefore we obtain as above

$$\sup_{m \geq 1} \mathbf{E} \sup_{t \in G} \|Z_m\| < \infty \quad (5.10)$$

and

$$\lim_{N \rightarrow \infty} \sup_{m \geq 1} \mathbf{E} \sup_{t \in G} \left\| \frac{1}{\sqrt{m}} \sum_{i=1}^m \sum_{n > N}^{\infty} \xi_n^i x_n \gamma_n(t) \right\| = 0. \quad (5.11)$$

We now can complete the proof of Theorem 5.1 by using Lemma 1.2 (1.2) and (1.3).

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