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NOWAK E.*

ABSOLUTE CONTINUITY BETWEEN A GIBBS MEASURE AND ITS TRANSLATE¹⁾

Наша первая цель — получить оценку для расстояния по вариации между мерой Гиббса на пространстве $\mathbf{R}^{\mathbf{Z}^d}$ и ее сдвигом на вектор из этого пространства. Это может быть сделано, если контролировать взаимозависимость между спинами в различных позициях, т.е. налагать некоторые ограничения на соответствующий потенциал. Мы можем затем заключить, в точно описанных случаях, что исходная мера и ее сдвиг эквивалентны.

Ключевые слова и фразы: случайные поля, расстояние по вариации, меры Гиббса, эквивалентность мер.

1. Introduction. Given a measure μ on a measurable space $(\Omega, \mathcal{F})$ and b an element of Ω , we define its translate by $\mu^{(b)}(B) = \mu(B - b)$ for any B in $\mathcal{F}$. The question of distinguishing μ and $\mu^{(b)}$ has been introduced by Shepp [18] in the following case: let $X = \{X_k, k \in \mathbf{N}\}$ be a sequence of independent and identically distributed (i.i.d.) random variables whose Fisher information is finite, and let μ be the measure induced by X . Then Shepp has proven that μ and $\mu^{(b)}$ are equivalent if and only if b belongs to $\ell^2(\mathbf{N})$. Since, the problem of distinguishing a sequence of i.i.d. random variables from a translate of itself has been widely studied (see, for example, [8] or [13]).

The aim of this paper is to remove this independence condition. The normal case is also well known: see Feldman [5] Rozanov [14], [15], Skorokhod and Yadrenko [21], Skorokhod [20, Chapter 3], Ibragimov and Rozanov [7, Chapter 3], and Lifshits [10, Section 9] for general questions of absolute continuity and singularity of Gaussian measures. Differentiability of measures corresponding to Markov processes has also been studied by Skorokhod [19, Chapter 4] and Sato [17], and Noquet [11] has recently given results for homogeneous Markov chains.

Gaussian and Markov random fields can be considered, in some cases, as Gibbs random fields: first results on this topic have been given by Rozanov [16] followed by Dobrushin [3], Künsch [9] and Georgii [6, Chapters 10–13]. We focus here on Gibbs measures on $\mathbf{R}^{\mathbf{Z}^d}$, using a method based on the distance in total variation. At last, let us point out that the study of the distance in total variation between the law of an i.i.d. random variables sequence and its translate has led Davydov and Lifshits [1] to a local invariance principle

* Laboratoire de Mathématiques Paul Painlevé, U.M.R. CNRS 8524, Université des Sciences et Technologies de Lille, U.F.R. de Mathématiques, 59655 Villeneuve d'Ascq Cedex, France; e-mail: e.nowak@isa-lille.fr

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for this sequence. The same method can be used for Gibbs random fields and one of the most difficult step is to obtain a majoration between the Gibbs measure and its translate.

The rest of this paper is organized as follows. The end of Section 1 is dedicated to notation and definitions. Section 2 gives inequalities for the distance we are studying. In Section 3, we can then conclude to the equivalence of the Gibbs measure and its translate, for precise cases. In Section 4, we give examples for which the previous results apply.

Let us now introduce some notation. Let μ be a probability measure on $(\Omega, \mathcal{F}) = (\mathbf{R}^{\mathbf{Z}^d}, \mathcal{B}_{\mathbf{R}^{\mathbf{Z}^d}})$, where $d \geq 1$, and suppose that $\xi = (\xi_k, k \in \mathbf{Z}^d)$ is a random field on $\mathbf{Z}^d$ with distribution μ . For any $\Lambda \subset \mathbf{Z}^d$, let

$$\sigma_\Lambda: \Omega \rightarrow \mathbf{R}^\Lambda$$

denote the projection onto the coordinates in Λ . Thus $\sigma_\Lambda(\omega) = \omega_\Lambda = (\omega_k, k \in \Lambda)$, when ω belongs to Ω and $\Omega_\Lambda = \mathbf{R}^\Lambda$. Inversely, when Λ and Δ are two disjoint subsets of $\mathbf{Z}^d$ and $\omega_\Lambda \in \Omega_\Lambda$, $\omega_\Delta \in \Omega_\Delta$, then $(\omega_\Lambda, \omega_\Delta)$ denotes the concatenation of ω_Λ and ω_Δ , that is, the element $(\omega_k, k \in \Lambda \cup \Delta)$ of $\Omega_{\Lambda \cup \Delta}$.

Further we put

$$\mathcal{S} = \{\Lambda \subset \mathbf{Z}^d: 0 < |\Lambda| < \infty\},$$

where $|\Lambda|$ is the cardinality of a set Λ and for any $\Lambda \subset \mathbf{Z}^d$, $\mathcal{F}_\Lambda$ is the σ -algebra which is generated by the sets $\sigma_\Lambda^{-1}(A)$, $\Delta \in \mathcal{S}$, $\Delta \subset \Lambda$, $A \in \mathcal{B}_{\mathbf{R}^\Delta}$.

The definition of a Gibbs measure can now be introduced. An interaction potential is a family $\phi = \{\phi_A, A \in \mathcal{S}\}$ of functions from Ω to $\mathbf{R}$ such that for each $A \in \mathcal{S}$, ϕ_A is $\mathcal{F}_{A^c}$ -measurable, that is, $\phi_A(\omega)$ depends only on ω_{A^c} . For finite subset $\Lambda \in \mathcal{S}$, the quantity

$$H_\Lambda^\phi = \sum_{A \in \mathcal{S}, A \cap \Lambda \neq \emptyset} \phi_A$$

is called the Hamiltonian in Λ for ϕ . We suppose that there exists a nonempty subset $\tilde{\Omega}$ of Ω such that

$$\forall \Lambda \in \mathcal{S}, \quad \forall \omega \in \tilde{\Omega}, \quad \sum_{A \in \mathcal{S}, A \cap \Lambda \neq \emptyset} |\phi_A(\omega)| < \infty.$$

Then let us introduce by setting $\exp[-H_\Lambda^\phi(\omega_\Lambda, \omega_{\Lambda^c})] = 0$ if the Hamiltonian is not defined, the partition function

$$Z_\Lambda^\phi(\omega) = \int_{\Omega_\Lambda} \exp[-H_\Lambda^\phi(\omega_\Lambda, \omega_{\Lambda^c})] \lambda^\Lambda(d\omega_\Lambda),$$

where λ^Λ is the Lebesgue measure on $\mathbf{R}^\Lambda$. We suppose that ϕ is admissible, in sense that for any $\Lambda \in \mathcal{S}$ and any $\omega \in \tilde{\Omega}$, we have $Z_\Lambda^\phi(\omega) < \infty$. As $Z_\Lambda^\phi(\omega)$ depends only on ω_{Λ^c} , we sometimes denote it $Z_\Lambda^\phi(\omega_{\Lambda^c})$.

Lastly, we can define a kernel from $\mathcal{F}_{\Lambda^c}$ to $\mathcal{F}$ by

$$\forall \eta \in \Omega_{\Lambda^c}, \quad \pi_\Lambda(d\omega, \eta) = \frac{1}{Z_\Lambda^\phi(\omega)} \exp[-H_\Lambda^\phi(\omega)] (\lambda^\Lambda \otimes \delta_\eta)(d\omega),$$

where δ_η is a Dirac measure in η .

If a probability measure μ on Ω is such that $\mu(\tilde{\Omega}) = 1$, we say that μ is a Gibbs measure for ϕ and we write $\mu \in \mathcal{G}(\phi)$ if

$$\forall \Lambda \in \mathcal{S}, \quad \mu(\cdot) = \int_{\Omega_{\Lambda^c}} \pi_\Lambda(\cdot, \eta) \mu_{\Lambda^c}(d\eta), \quad (1)$$

where $\mu_\Delta = \mu \sigma_\Delta^{-1}$. We say that a random field $\xi = (\xi_k, k \in \mathbf{Z}^d)$ is Gibbsian if its distribution μ is a Gibbs measure on Ω .

From now, we always make the following regularity hypothesis on the Hamiltonian.

Hypothesis (H). For all $\Lambda \in \mathcal{S}$ and $\eta \in \tilde{\Omega}_{\Lambda^c}$, the mapping

$$\omega_\Lambda \longmapsto H_\Lambda^\phi(\omega_\Lambda, \eta)$$

from $\tilde{\Omega}_\Lambda$ to $\mathbf{R}$ is locally Lipschitzian.

Of course, we only consider potentials for which $\mathcal{G}(\phi)$ is nonempty. At last, we will sometimes omit, when no confusion is possible, the symbol $\{\cdot\}$: for instance we will write H_i^ϕ instead of $H_{\{i\}}^\phi$.

If ν is a signed measure on $(\Omega, \mathcal{F})$, its total variation is defined by

$$\|\nu\| = \sup_{(A_n)} \sum_n |\nu(A_n)|,$$

where the supremum is taken over all finite partitions (A_n) of Ω . The distance for this norm is called *distance in variation* and the convergence for it is denoted by $\xrightarrow{\text{var}}$.

2. Translated law: inequalities for the distance in variation. Let μ be a Gibbs measure on $(\Omega, \mathcal{F})$ and b an element which belongs to Ω . We are going to study the distance in variation between μ and its translate $\mu^{(b)}$ defined by $\mu^{(b)}(B) = \mu(B - b)$ for any B in $\mathcal{F}$.

2.1. Local translation. First we study the distance in variation between μ and $\mu^{(b^\Lambda)}$, where $\Lambda \in \mathcal{S}$ and b^Λ is the finite number of nonzero coordinates translation obtained from b and defined by

$$b^\Lambda = (b_i \mathbb{I}_\Lambda(i))_{i \in \mathbb{Z}^d}.$$

The aim is to find a result which allows us to pass to the limit when Λ goes to $\mathbb{Z}^d$.

2.1.1. Potentials fulfilling hypothesis (H).

Theorem 1. *Let $\mu \in \mathcal{G}(\phi)$, where ϕ is a potential which fulfill hypothesis (H). Let*

$$Y_i = \frac{\partial H_i^\phi}{\partial \omega_i}$$

denote the partial derivative of H_i^ϕ in the i -th direction, that we suppose integrable. Then we have

$$\|\mu - \mu^{(b^\Lambda)}\|^2 \leq \sum_{i,j \in \Lambda} b_i b_j \mathbf{E}_\mu(Y_i Y_j). \quad (2)$$

P r o o f. For any B in $\mathcal{F}$ we have

$$\mu(B) = \int_{\Omega_{\Lambda^c}} \pi_\Lambda(B, \eta) \mu_{\Lambda^c}(d\eta), \quad \mu^{(b^\Lambda)}(B) = \int_{\Omega_{\Lambda^c}} \pi_\Lambda(B - b^\Lambda, \eta) \mu_{\Lambda^c}(d\eta).$$

Let $\pi_\Lambda^{(b^\Lambda)}(B, \eta) = \pi_\Lambda(B - b^\Lambda, \eta)$, for $B \in \mathcal{F}$. According to [1, Chapter 1], we have

$$\|\mu - \mu^{(b^\Lambda)}\| = \int_{\Omega_{\Lambda^c}} \|\pi_\Lambda(\cdot, \eta) - \pi_\Lambda^{(b^\Lambda)}(\cdot, \eta)\| \mu_{\Lambda^c}(d\eta). \quad (3)$$

Now

$$\begin{aligned} \pi_\Lambda(B - b^\Lambda, \eta) &= \int_{B - b^\Lambda} \frac{1}{Z_\Lambda^\phi(\eta)} \exp[-H_\Lambda^\phi(\omega)] \lambda^\Lambda \otimes \delta_\eta(d\omega) \\ &= \int_B \frac{1}{Z_\Lambda^\phi(\eta)} \exp[-H_\Lambda^\phi(\omega - b^\Lambda)] \lambda^\Lambda \otimes \delta_\eta(d\omega) \end{aligned}$$

which means that

$$\pi_\Lambda^{(b^\Lambda)}(d\omega, \eta) = \frac{1}{Z_\Lambda^\phi(\eta)} \exp[-H_\Lambda^\phi(\omega - b^\Lambda)] \lambda^\Lambda \otimes \delta_\eta(d\omega).$$

The two probability measures $\pi_\Lambda(\cdot, \eta)$ and $\pi_\Lambda^{(b^\Lambda)}(\cdot, \eta)$ having, for fixed η , a density with respect to $\lambda^\Lambda \otimes \delta_\eta$ it comes:

$$\begin{aligned} \|\pi_\Lambda(\cdot, \eta) - \pi_\Lambda^{(b^\Lambda)}(\cdot, \eta)\| &= \int_\Omega \frac{1}{Z_\Lambda^\phi(\eta)} \left| \exp[-H_\Lambda^\phi(\omega)] - \exp[-H_\Lambda^\phi(\omega - b^\Lambda)] \right| \lambda^\Lambda \otimes \delta_\eta(d\omega) \\ &= \int_{\Omega_\Lambda} \frac{1}{Z_\Lambda^\phi(\eta)} \left| \exp[-H_\Lambda^\phi(\omega_\Lambda, \eta)] - \exp[-H_\Lambda^\phi(\omega_\Lambda - b_\Lambda, \eta)] \right| \lambda^\Lambda(d\omega_\Lambda). \end{aligned} \quad (4)$$

Then let us define the function $f_\eta: [0, 1] \times \tilde{\Omega}_\Lambda \mapsto \mathbf{R}$ as follows:

$$f_\eta(\alpha, \omega_\Lambda) = \frac{1}{Z_\Lambda^\phi(\eta)} \exp[-H_\Lambda^\phi(\omega_\Lambda - \alpha b_\Lambda, \eta)].$$

First case. Let the mapping $H_\Lambda^\phi(\cdot, \eta)$ be differentiable almost everywhere on the segment $\{\omega_\Lambda - \alpha b_\Lambda, \alpha \in [0, 1]\}$, for λ^Λ -almost every $\omega_\Lambda \in \tilde{\Omega}_\Lambda$. Considered as a function of α , f_η is almost everywhere differentiable and its derivative is

$$\frac{\partial f_\eta}{\partial \alpha}(\alpha, \omega_\Lambda) = \frac{1}{Z_\Lambda^\phi(\eta)} \sum_{i \in \Lambda} b_i \frac{\partial H_\Lambda^\phi}{\partial \omega_i}(\omega_\Lambda - \alpha b_\Lambda, \eta) \exp[-H_\Lambda^\phi(\omega_\Lambda - \alpha b_\Lambda, \eta)].$$

So we get

$$\begin{aligned} \left\| \pi_\Lambda(\cdot, \eta) - \pi_\Lambda^{(b^\Lambda)}(\cdot, \eta) \right\| &= \int_{\Omega_\Lambda} \left| \int_0^1 \frac{\partial f_\eta}{\partial \alpha}(\alpha, \omega_\Lambda) d\alpha \right| \lambda^\Lambda(d\omega_\Lambda) \\ &\leq \int_0^1 \int_{\Omega_\Lambda} \frac{1}{Z_\Lambda^\phi(\eta)} \left| \sum_{i \in \Lambda} b_i \frac{\partial H_\Lambda^\phi}{\partial \omega_i}(\omega_\Lambda - \alpha b_\Lambda, \eta) \right| \exp[-H_\Lambda^\phi(\omega_\Lambda - \alpha b_\Lambda, \eta)] \lambda^\Lambda(d\omega_\Lambda) d\alpha \\ &= \int_0^1 \int_{\Omega_\Lambda} \frac{1}{Z_\Lambda^\phi(\eta)} \left| \sum_{i \in \Lambda} b_i \frac{\partial H_\Lambda^\phi}{\partial \omega_i}(\omega_\Lambda, \eta) \right| \exp[-H_\Lambda^\phi(\omega_\Lambda, \eta)] \lambda^\Lambda(d\omega_\Lambda) d\alpha \\ &= \int_{\Omega_\Lambda} \frac{1}{Z_\Lambda^\phi(\eta)} \left| \sum_{i \in \Lambda} b_i \frac{\partial H_\Lambda^\phi}{\partial \omega_i}(\omega_\Lambda, \eta) \right| \exp[-H_\Lambda^\phi(\omega_\Lambda, \eta)] \lambda^\Lambda(d\omega_\Lambda). \end{aligned} \quad (5)$$

And so we have

$$\begin{aligned} \|\mu - \mu^{b^\Lambda}\| &\leq \int_{\Omega_{\Lambda^c}} \frac{1}{Z_\Lambda^\phi(\eta)} \int_{\Omega_\Lambda} \left| \sum_{i \in \Lambda} b_i \frac{\partial H_\Lambda^\phi}{\partial \omega_i}(\omega_\Lambda, \eta) \right| \exp[-H_\Lambda^\phi(\omega_\Lambda, \eta)] \lambda^\Lambda(d\omega_\Lambda) \mu_{\Lambda^c}(d\eta) \\ &= \mathbf{E}_\mu \left| \sum_{i \in \Lambda} b_i \frac{\partial H_\Lambda^\phi}{\partial \omega_i} \right|. \end{aligned} \quad (6)$$

To get the expected result, one only has to remark that

$$H_\Lambda^\phi(\omega) = \sum_{\substack{A \cap \Lambda \neq \emptyset \\ A \in \mathcal{S}, A \ni i}} \phi_A(\omega) + \sum_{\substack{A \in \mathcal{S} \\ A \ni i}} \phi_A(\omega).$$

Since $\phi_A(\omega)$ depends only on ω_A , we have for all $i \in \Lambda$:

$$\frac{\partial H_\Lambda^\phi}{\partial \omega_i}(\omega) = \frac{\partial H_i^\phi}{\partial \omega_i}(\omega) = Y_i(\omega).$$

Finally,

$$\|\mu - \mu^{(b^\Lambda)}\|^2 \leq \left(\mathbf{E}_\mu \left| \sum_{i \in \Lambda} b_i \frac{\partial H_i^\phi}{\partial \omega_i} \right| \right)^2 \leq \sum_{i, j \in \Lambda} b_i b_j \mathbf{E}_\mu(Y_i Y_j).$$

Second case. Let us now suppose that there exists a segment $\{\omega_\Lambda - \alpha b_\Lambda, \alpha \in [0, 1]\}$, $\omega_\Lambda \in \tilde{\Omega}_\Lambda$ on which $H_\Lambda^\phi(\cdot, \eta)$ is not almost everywhere differentiable.

Lemma. Let $S_\Lambda = \{r_\Lambda \in \mathbf{R}^\Lambda, \|r_\Lambda\| = \|b_\Lambda\|\}$ be the centered sphere in $\mathbf{R}^\Lambda$ with radius $\|b_\Lambda\|$. For almost every $r_\Lambda \in S_\Lambda$ and for almost every $\omega_\Lambda \in \mathbf{R}^\Lambda$, the mapping $H_\Lambda^\phi(\cdot, \eta)$ is almost everywhere differentiable on the segment $\{\omega_\Lambda + \alpha r_\Lambda, \alpha \in [0, 1]\}$.

P r o o f. According to hypothesis (H) and Rademacher theorem (see [4, p. 216]), $H_\Lambda^\phi(\cdot, \eta)$ is differentiable in λ^Λ -almost every point ω_Λ of $\mathbf{R}^\Lambda$. Let ω_Λ be such a point and let B_{ω_Λ} be the ball in $\mathbf{R}^\Lambda$ with center ω_Λ and radius $\|b_\Lambda\|$. Then $H_\Lambda^\phi(\cdot, \eta)$ is almost everywhere differentiable on the segment $\{\omega_\Lambda + \alpha r_\Lambda, \alpha \in [0, 1]\}$ and this for almost every $r_\Lambda \in S_\Lambda$. Indeed, if $\mathcal{R}_\Lambda$ denotes the set of points of the ball B_{ω_Λ} where $H_\Lambda^\phi(\cdot, \eta)$ is not differentiable, we have $\lambda^\Lambda(\mathcal{R}_\Lambda) = 0$. The result can be obtained by switching to polar coordinates and applying Fubini's theorem.

We can then choose a sequence $(b_\Lambda^{(n)}, n \in \mathbf{N})$ which tends to b_Λ such that $H_\Lambda^\phi(\cdot, \eta)$ is almost everywhere differentiable on the segment $\{\omega_\Lambda + \alpha b_\Lambda^{(n)}, \alpha \in [0, 1]\}$, for λ^Λ -almost every $\omega_\Lambda \in \bar{\Omega}_\Lambda$, for each $n \in \mathbf{N}$. According to first case, we have then

$$\|\mu - \mu^{(b^\Lambda, n)}\| \leq \mathbf{E}_\mu \left| \sum_{i \in \Lambda} b_i^{(n)} \frac{\partial H_i^\phi}{\partial \omega_i} \right|, \quad \text{where } (b^\Lambda, n) = (b_i^{(n)} \mathbb{1}_\Lambda(i))_{i \in \mathbf{Z}^d}.$$

Now if (μ_n) and (ν_n) are two sequences of measures such that μ_n weakly converges to a measure μ (we note $\mu_n \Rightarrow \mu$) and $\nu_n \Rightarrow \nu$, then $\|\mu - \nu\| \leq \liminf \|\mu_n - \nu_n\|$ (see [2, Theorem 2.7]). We get then

$$\|\mu - \mu^{(b^\Lambda)}\| \leq \liminf_{n \rightarrow \infty} \mathbf{E}_\mu \left| \sum_{i \in \Lambda} b_i^{(n)} \frac{\partial H_i^\phi}{\partial \omega_i} \right|$$

and according to Lebesgue's dominated convergence theorem:

$$\|\mu - \mu^{(b^\Lambda)}\| \leq \mathbf{E}_\mu \left| \sum_{i \in \Lambda} b_i \frac{\partial H_i^\phi}{\partial \omega_i} \right|.$$

2.1.2. Potentials with finite range. We say that a potential ϕ has finite range if $\sup\{d(A), A \in \mathcal{S}, \phi_A \neq 0\} = r < \infty$, where $d(A)$ denotes the diameter of A for the uniform norm on $\mathbf{Z}^d$.

A potential ϕ is translation invariant if $\phi_{A+j} \circ \tau_j = \phi_A$ for all $A \in \mathcal{S}$, $j \in \mathbf{Z}^d$, where $A+j$ is the set $\{i+j, i \in A\}$ and $\tau_j: \omega \mapsto (\omega_{i-j})_{i \in \mathbf{Z}^d}$.

Theorem 2. Let $\mu \in \mathcal{G}(\phi)$, where ϕ is a finite range (with range r), translation invariant potential such that $\mathbf{E}_\mu(Y_i^2) = \sigma_Y^2 < \infty$. Then we have

$$\|\mu - \mu^{(b^\Lambda)}\|^2 \leq C_r \sum_{i \in \Lambda} b_i^2 \quad (7)$$

with $C_r = (2r+1)^d \sigma_Y^2$.

P r o o f. We will first prove that $\mathbf{E}_\mu(Y_i Y_j) = 0$ if $|i-j| > r$. Setting $\Delta = \{i, j\}$, we have

$$\mathbf{E}_\mu(Y_i Y_j) = \int_{\Omega_{\Delta^c}} \frac{1}{Z_\Delta^\phi(\omega)} \int_{\Omega_\Delta} \frac{\partial H_i^\phi}{\partial \omega_i}(\omega) \frac{\partial H_j^\phi}{\partial \omega_j}(\omega) \exp[-H_\Delta^\phi(\omega)] \lambda^2(d\omega_\Delta) \mu_{\Delta^c}(d\omega_{\Delta^c}). \quad (8)$$

Now $\phi_A = 0$ if $d(A) > r$, which gives $H_\Delta^\phi(\omega) = H_i^\phi(\omega) + H_j^\phi(\omega)$. Moreover, $H_i^\phi(\omega)$ (respectively, $H_j^\phi(\omega)$) depends only on the ω_k for which $|k-i| \leq r$. This allows us to write

$$\begin{aligned} \mathbf{E}_\mu(Y_i Y_j) &= \int_{\Omega_{\Delta^c}} \frac{1}{Z_\Delta^\phi(\omega)} \left(\int_{\Omega_{\{i\}}} \frac{\partial H_i^\phi}{\partial \omega_i}(\omega) \exp[-H_i^\phi(\omega)] \lambda(d\omega_i) \right. \\ &\quad \times \left. \int_{\Omega_{\{j\}}} \frac{\partial H_j^\phi}{\partial \omega_j}(\omega) \exp[-H_j^\phi(\omega)] \lambda(d\omega_j) \right) \mu_{\Delta^c}(d\omega_{\Delta^c}). \end{aligned}$$

Just remark that for any $k \in \mathbf{Z}^d$, we have

$$\int_{\mathbf{R}} \frac{\partial H_k^\phi}{\partial \omega_k}(\omega) \exp[-H_k^\phi(\omega)] \lambda(d\omega_k) = 0 \quad (9)$$

to obtain that $\mathbf{E}_\mu(Y_i Y_j) = 0$ if $|i-j| > r$. Indeed we have supposed that

$$Z_{\{k\}}^\phi(\omega) = \int_{\mathbf{R}} \exp[-H_k^\phi(\omega)] \lambda(d\omega_k) < \infty,$$

and so we can always find two real sequences, (α_n) decreasing to $-\infty$ and (β_n) increasing to $+\infty$, such that

$$\lim_{n \rightarrow \infty} \exp[-H_k^\phi(\alpha_n, \omega_{\{k\}^c})] = \lim_{n \rightarrow \infty} \exp[-H_k^\phi(\beta_n, \omega_{\{k\}^c})] = 0.$$

This leads to

$$\begin{aligned} & \int_{\mathbf{R}} \frac{\partial H_k^\phi}{\partial \omega_k}(\omega) \exp[-H_k^\phi(\omega)] \lambda(d\omega_k) \\ &= \lim_{n \rightarrow \infty} \int_{\alpha_n}^{\beta_n} \frac{\partial H_k^\phi}{\partial \omega_k}(\omega) \exp[-H_k^\phi(\omega)] \lambda(d\omega_k) = 0 \quad \mu_{\{k\}^c}\text{-a.s.} \end{aligned}$$

Let us point out that we have shown that Y_i is centered for any $i \in \mathbf{Z}^d$. On the other hand, $|\mathbf{E}_\mu(Y_i Y_j)| \leq (\mathbf{E}_\mu Y_i^2)^{1/2} (\mathbf{E}_\mu Y_j^2)^{1/2} = \sigma_Y^2$ as soon as $|i - j| \leq r$. The expected result follows then from Theorem 1:

$$\begin{aligned} \|\mu - \mu^{(b^\Lambda)}\|^2 &\leq \sum_{i \in \Lambda} \sum_{\substack{k \in \mathbf{Z}^d \\ |k| \leq r}} |b_i^\Lambda| |b_{i+k}^\Lambda| \sigma_Y^2 \leq \sum_{\substack{k \in \mathbf{Z}^d \\ |k| \leq r}} \left(\sum_{i \in \Lambda} b_i^{\Lambda^2} \right)^{1/2} \left(\sum_{i \in \Lambda} b_{i+k}^{\Lambda^2} \right)^{1/2} \sigma_Y^2 \\ &\leq (2r+1)^d \left(\sum_{i \in \Lambda} b_i^2 \right) \sigma_Y^2. \end{aligned}$$

We would like to have a similar inequality for infinite range potential, but C_r tends to infinity when r tends to infinity. So we have to impose some supplementary conditions on decreasing rate of the potential at infinity.

2.1.3. Potentials with infinite range. We are going to use a technique due to Dobrushin by introducing a matrix $C(\pi) = (C_{ij}(\pi))_{i,j \in \mathbf{Z}^d}$ defined by

$$C_{ij}(\pi) = \sup \|\pi_i(\cdot, \zeta) - \pi_i(\cdot, \eta)\|,$$

where the supremum is taken over the ζ, η such that $\zeta_{\{j\}^c} = \eta_{\{j\}^c}$. It describes the interdependence between the spins at distinct sites and will allow us to control $\mathbf{E}_\mu(Y_i Y_j)$.

In the following, $|A|$ denotes the cardinality of a set A of $\mathbf{Z}^d$, $|i|$ the uniform norm of an element i of $\mathbf{Z}^d$, and $|b_i|$ the absolute value of an element b_i of $\mathbf{R}$. Lastly, we define the oscillation $\delta(f)$ of a real function f on Ω as follows:

$$\delta(f) = \sup \{ |f(\xi) - f(\xi')|, \xi, \xi' \in \Omega \}.$$

Theorem 3. *Let $\mu \in \mathcal{G}(\phi)$, where ϕ is translation invariant. We suppose that there is a translation invariant semi-metric s on $\mathbf{Z}^d$ (we note $s(i, j) = s(i - j)$) such that*

$$\begin{aligned} \text{(i)} \quad & C_1 = \sum_{j \in \mathbf{Z}^d} C_{0j}(\pi) e^{s(j)} < 1, \\ \text{(ii)} \quad & C_2 = \sum_{A \in \mathcal{S}, A \ni 0} |A|^{1/2} \delta\left(\frac{\partial \phi_A}{\partial \omega_0}\right) \exp[s(A)] < \infty, \\ \text{(iii)} \quad & C_3 = \sum_{k \in \mathbf{Z}^d} \exp[-s(k)] < \infty, \end{aligned}$$

where $s(A)$ is the s -diameter of the set A . Then there is a finite constant C_ϕ such that

$$\|\mu - \mu^{(b^\Lambda)}\|^2 \leq C_\phi \sum_{i \in \Lambda} b_i^2. \quad (10)$$

P r o o f. We are going to study the behavior of $\mathbf{E}_\mu(Y_i Y_j)$ and show that it tends to zero with the same speed as $\exp[-s(i - j)]$ when $s(i - j)$ tends to infinity

$$\mathbf{E}_\mu(Y_i Y_j) = \text{cov}(Y_i, Y_j) = \sum_{\substack{A \ni i, A' \ni j \\ A, A' \in \mathcal{S}}} \text{cov}\left(\frac{\partial \phi_A}{\partial \omega_i}, \frac{\partial \phi_{A'}}{\partial \omega_j}\right). \quad (11)$$

Let D be the $\mathbf{Z}^d \times \mathbf{Z}^d$ matrix defined by

$$D = (D_{ij})_{i,j \in \mathbf{Z}^d} \stackrel{\text{def}}{=} \sum_{n \geq 0} C^n(\pi),$$

where $C^n(\pi)$ denotes the n -th power of the interaction matrix $C(\pi)$. According to [6, Proposition 8.34], we have

$$\begin{aligned} \left| \text{cov} \left(\frac{\partial \phi_A}{\partial \omega_i}, \frac{\partial \phi_{A'}}{\partial \omega_j} \right) \right| &\leq \frac{1}{4} \sum_{i,j \in \mathbf{Z}^d} \delta_i \left(\frac{\partial \phi_A}{\partial \omega_i} \right) \delta_j \left(\frac{\partial \phi_{A'}}{\partial \omega_j} \right) D_{ij} \\ &= \frac{1}{4} \sum_{\substack{i \in A \\ j \in A'}} \delta_i \left(\frac{\partial \phi_A}{\partial \omega_i} \right) \delta_j \left(\frac{\partial \phi_{A'}}{\partial \omega_j} \right) D_{ij} \leq \frac{1}{4} \delta \left(\frac{\partial \phi_A}{\partial \omega_i} \right) \delta \left(\frac{\partial \phi_{A'}}{\partial \omega_j} \right) \sum_{i \in A, j \in A'} D_{ij}, \end{aligned} \quad (12)$$

where $\delta_i(f) = \sup\{|f(\xi) - f(\xi')|, \xi_{\{i\}^c} = \xi'_{\{i\}^c}\}$ is the oscillation at $i \in \mathbf{Z}^d$ of a real function f on Ω . Let us suppose that $\min(|A|, |A'|) = |A|$. Then

$$\exp[s(A, A')] \sum_{\substack{i \in A \\ j \in A'}} D_{ij} \leq \sum_{i \in A} \sum_{j \in A'} \exp[s(i, j)] D_{ij} \leq |A| \sup_{i \in A} \sum_{j \in \mathbf{Z}^d} \exp[s(i, j)] D_{ij}.$$

Now using the triangle inequality for s we have for any $i_0 \in \mathbf{Z}^d$

$$\begin{aligned} \sum_{j \in \mathbf{Z}^d} \exp[s(i_0, j)] D_{i_0 j} &\leq 1 + \sum_{n \geq 1} \sum_{i_1, \dots, i_n \in \mathbf{Z}^d} \prod_{k=1}^n C_{i_{k-1} i_k}(\pi) e^{s(i_{k-1}, i_k)} \\ &\leq 1 + \sum_{n \geq 1} C_1^n = \frac{1}{1 - C_1}. \end{aligned}$$

This leads to

$$\left| \text{cov} \left(\frac{\partial \phi_A}{\partial \omega_i}, \frac{\partial \phi_{A'}}{\partial \omega_j} \right) \right| \leq \min(|A|, |A'|) \frac{(1 - C_1)^{-1}}{4} \delta \left(\frac{\partial \phi_A}{\partial \omega_i} \right) \delta \left(\frac{\partial \phi_{A'}}{\partial \omega_j} \right) \exp[-s(A, A')] \quad (13)$$

and so

$$\begin{aligned} |\mathbf{E}_\mu(Y_i Y_j)| &\leq \frac{(1 - C_1)^{-1}}{4} \sum_{A \ni i, A' \ni j} |A|^{1/2} |A'|^{1/2} \delta \left(\frac{\partial \phi_A}{\partial \omega_i} \right) \delta \left(\frac{\partial \phi_{A'}}{\partial \omega_j} \right) \\ &\quad \times \exp[-s(i - j) + s(A) + s(A')] \\ &= \frac{(1 - C_1)^{-1}}{4} \left(\sum_{A \in \mathcal{S}, A \ni 0} |A|^{1/2} \delta \left(\frac{\partial \phi_A}{\partial \omega_0} \right) \exp[s(A)] \right)^2 \\ &\quad \times \exp[-s(i - j)] = K \exp[-s(i - j)] \end{aligned} \quad (14)$$

with $K = C_2^2 (1 - C_1)^{-1}/4$. According to Theorem 1, we finally obtain

$$\|\mu - \mu^{(b^\Lambda)}\|^2 \leq K \sum_{i \in \Lambda} \sum_{j \in \Lambda} \exp[-s(i - j)] |b_i| |b_j| \leq K \sum_{k \in \mathbf{Z}^d} \exp[-s(k)] \sum_{i \in \Lambda} b_i^2.$$

Remark that the expression of the constant C_ϕ is known: $C_\phi = C_3 C_2^2 (1 - C_1)^{-1}/4$. In other respects, it is shown in [6, p. 158] that

$$C_1 \leq \frac{1}{2} \sum_{A \in \mathcal{S}, A \ni 0} (|A| - 1) \delta(\phi_A) \exp[s(A)]. \quad (15)$$

Hence we can state the following corollary whose hypothesis are more restrictive but easier to check.

Corollary 1. *If we replace condition (i) in Theorem 3 by the following one:*

$$(i') \quad C'_1 = \sum_{A \in \mathcal{S}, A \ni 0} (|A| - 1) \delta(\phi_A) \exp[s(A)] < 2,$$

then there exists a finite constant C'_ϕ such that

$$\|\mu - \mu^{(b^\Lambda)}\|^2 \leq C'_\phi \sum_{i \in \Lambda} b_i^2. \quad (16)$$

P r o o f. We immediately have $C'_\phi = \frac{1}{4} C_3 C_2^2 (1 - C'_1/2)^{-1}$.

2.2. Translation by a vector in $\mathbf{R}^{\mathbf{Z}^d}$. Now we are able to find a majoration of $\|\mu - \mu^{(b)}\|$ for $b \in \Omega$.

2.2.1. Fundamental cases.

Theorem 4. Let $\mu \in \mathcal{G}(\phi)$; we suppose that $|\mathbf{E}_\mu(Y_i Y_j)| \leq \alpha(i-j)$, where $\alpha: \mathbf{Z}^d \rightarrow \mathbf{R}^+$ is summable:

$$C_\phi'' = \sum_{k \in \mathbf{Z}^d} \alpha(k) < \infty.$$

Then

$$\|\mu - \mu^{(b)}\|^2 \leq C_\phi'' \sum_{i \in \mathbf{Z}^d} b_i^2. \quad (17)$$

P r o o f. For any $\Lambda \in \mathcal{S}$, we have

$$\|\mu - \mu^{(b^\Lambda)}\|^2 \leq \sum_{i,j \in \Lambda} |b_i b_j| \alpha(i-j) = \sum_{k \in \mathbf{Z}^d} \alpha(k) \sum_{i \in \Lambda} |b_i^\Lambda b_{i+k}^\Lambda| \leq \sum_{k \in \mathbf{Z}^d} \alpha(k) \sum_{i \in \Lambda} b_i^2.$$

Let us define $(\mu_n)_{n \geq 0}$ and $(\nu_n)_{n \geq 0}$ by $\mu_n = \mu$, $\nu_n = \mu^{(b^{\Lambda_n})}$, where $\Lambda_n =]-n, n[{}^d$ ($\Lambda_0 = \emptyset$). We have $\nu_n \Rightarrow \mu^{(b)}$ and so

$$\|\mu - \mu^{(b)}\|^2 \leq \liminf \|\mu - \mu^{(b^{\Lambda_n})}\|^2 \leq \liminf C_\phi'' \sum_{i \in \Lambda_n} b_i^2 = C_\phi'' \sum_{i \in \mathbf{Z}^d} b_i^2.$$

2.2.2. Finite range potentials.

Theorem 5. Let $\mu \in \mathcal{G}(\phi)$, where ϕ is a translation invariant, finite range potential with range r such that $\mathbf{E}_\mu(Y_i^2) = \sigma_Y^2 < \infty$. Then we have

$$\|\mu - \mu^{(b)}\|^2 \leq C_r \sum_{i \in \mathbf{Z}^d} b_i^2, \quad (18)$$

where $C_r = (2r+1)^d \sigma_Y^2$.

P r o o f. With the same notation as in the proof of Theorem 4, we have

$$\|\mu - \mu^{(b)}\|^2 \leq \liminf \|\mu - \mu^{(b^{\Lambda_n})}\|^2 \leq \liminf C_r \sum_{i \in \Lambda_n} b_i^2 = C_r \sum_{i \in \mathbf{Z}^d} b_i^2.$$

2.2.3. Infinite range potentials.

Theorem 6. Let $\mu \in \mathcal{G}(\phi)$, where ϕ is translation invariant. We suppose that there is a translation invariant semi-metric s on $\mathbf{Z}^d$ (we note $s(i, j) = s(i-j)$) such that

- (i) $C_1 = \sum_{j \in \mathbf{Z}^d} C_{0j}(\pi) e^{s(j)} < 1$,
- (ii) $C_2 = \sum_{A \in \mathcal{S}, A \ni 0} |A|^{1/2} \delta\left(\frac{\partial \phi_A}{\partial \omega_0}\right) \exp[s(A)] < \infty$,
- (iii) $C_3 = \sum_{k \in \mathbf{Z}^d} \exp[-s(k)] < \infty$.

Then

$$\|\mu - \mu^{(b)}\|^2 \leq C_\phi \sum_{i \in \mathbf{Z}^d} b_i^2, \quad (19)$$

where C_ϕ is the same finite constante as in Theorem 3.

Corollary 2. If we replace condition (i) in Theorem 6 by the following one:

$$(i') \quad C_1' = \sum_{A \in \mathcal{S}, A \ni 0} (|A| - 1) \delta(\phi_A) \exp[s(A)] < 2,$$

then

$$\|\mu - \mu^{(b)}\|^2 \leq C_\phi' \sum_{i \in \Lambda} b_i^2, \quad (20)$$

where C_ϕ' is the same finite constante as in Corollary 1.

P r o o f. We just have to replace C_r by C_ϕ or C_ϕ' in the proof of Theorem 5.

3. Equivalence of the translated law.

Thanks to the previous study, we are able to find conditions for the measures μ and $\mu^{(b)}$ to be equivalent. It will be one of the following.

(D_α) There is a function $\alpha: \mathbf{Z}^d \rightarrow \mathbf{R}$ such that $|\mathbf{E}_\mu(Y_i Y_j)| \leq \alpha(i - j)$ and

$$\sum_{k \in \mathbf{Z}^d} \alpha(k) < \infty.$$

(F_r) The potential has finite range r and $\mathbf{E}_\mu(Y_i^2) < \infty$, where

$$Y_i = \frac{\partial H_i^\phi}{\partial \omega_i}.$$

(D_s) There is a translation invariant semi-metric s on $\mathbf{Z}^d$ (we write $s(i, j) = s(i - j)$) such that

$$(i) \quad C_1 = \sum_{j \in \mathbf{Z}^d} C_{0j} e^{s(j)} < 1,$$

$$(ii) \quad C_2 = \sum_{A \in \mathcal{S}, A \ni 0} |A|^{1/2} \delta\left(\frac{\partial \phi_A}{\partial \omega_0}\right) \exp[s(A)] < \infty,$$

$$(iii) \quad C_3 = \sum_{k \in \mathbf{Z}^d} \exp[-s(k)] < \infty.$$

3.1. Main theorem.

Theorem 7. Let $\mu \in \mathcal{G}(\phi)$, where ϕ is translation invariant potential and such that one of the conditions (F_r), (D_s), or (D_α) is satisfied. Then we have

$$\mu \sim \mu^{(b)} \quad \text{if} \quad \sum_{i \in \mathbf{Z}^d} b_i^2 < \infty.$$

P r o o f. For $n \geq 0$, let $\Lambda_n =] - n, n[^d$ and let ν_n be the distribution defined this time by

$$\nu_n = \mu^{(b^{\Lambda_n^c})}.$$

We will show that for each $n \geq 0$, ν_{n+1} is absolutely continuous with respect to ν_n (we denote $\nu_{n+1} \ll \nu_n$). To do this, let $A \in \mathcal{B}_{\mathbf{R}^{\mathbf{Z}^d}}$ be such that $\nu_n(A) = 0$. This means that

$$\int_{\mathbf{R}^{\Lambda_{n+1}^c}} \pi_{\Lambda_{n+1}}(A - b^{\Lambda_n^c}, \eta) \mu_{\Lambda_{n+1}^c}(d\eta) = 0.$$

Thus

$$\pi_{\Lambda_{n+1}}(A - b^{\Lambda_n^c}, \eta) = 0 \quad \mu_{\Lambda_{n+1}^c}\text{-a.e.} \quad (21)$$

But

$$\begin{aligned} \pi_{\Lambda_{n+1}}(A - b^{\Lambda_n^c}, \eta) &= \int_{A - b^{\Lambda_n^c}} \frac{1}{Z_{\Lambda_{n+1}}^\phi(\eta)} \exp[-H_{\Lambda_{n+1}}^\phi(\omega)] \lambda^{\Lambda_{n+1}} \otimes \delta_\eta(d\omega) \\ &= \int_A \frac{1}{Z_{\Lambda_{n+1}}^\phi(\eta)} \exp\left[-H_{\Lambda_{n+1}}^\phi(\omega - b^{\Lambda_n^c})\right] \lambda^{\Lambda_{n+1}} \otimes \delta_{(\eta + b_{\Lambda_{n+1}^c})}(d\omega) \end{aligned}$$

since the projection of $b^{\Lambda_n^c}$ on $\Omega_{\Lambda_{n+1}^c}$ is $b_{\Lambda_{n+1}^c}$. We obtain that

$$\lambda^{\Lambda_{n+1}} \otimes \delta_{(\eta + b_{\Lambda_{n+1}^c})}(A) = 0 \quad \mu_{\Lambda_{n+1}^c}\text{-a.e.} \quad (22)$$

This leads to

$$\begin{aligned} \pi_{\Lambda_{n+1}}(A - b^{\Lambda_{n+1}^c}, \eta) &= \int_A \frac{1}{Z_{\Lambda_{n+1}}^\phi(\eta)} \exp\left[-H_{\Lambda_{n+1}}^\phi(\omega - b^{\Lambda_{n+1}^c})\right] \lambda^{\Lambda_{n+1}} \otimes \delta_{(\eta + b_{\Lambda_{n+1}^c})}(d\omega) \\ &= 0 \quad \mu_{\Lambda_{n+1}^c}\text{-a.e.} \end{aligned} \quad (23)$$

This proves that $\nu_{n+1} \ll \nu_n$ and by recurrence, we have for any $n \geq 1$:

$$\nu_n \ll \nu_0 = \mu^{(b)}.$$

On the other hand, $\nu_n \xrightarrow[n \rightarrow \infty]{\text{var}} \mu$ since $\|\nu_n - \mu\| \leq K (\sum_{i \notin \Lambda_n} b_i^2)^{1/2}$, where K is a finite constant. It follows immediately that $\mu \ll \mu^{(b)}$. By the same way, we have $\mu^{(b)} \ll \mu$.

3.2. Examples. We can give some examples to illustrate the previous theorem. Most of them can be found in [12] with the expression of the distance between μ and $\mu^{(b)}$. The first one (range $r = 0$) corresponds to i.i.d. random variables, and is in line with Shepp's result [18].

Example 1. Let ϕ be the potential defined by

$$\phi_A(\omega) = \begin{cases} \varphi(\omega_i) & \text{if } A = \{i\}, \\ 0 & \text{otherwise.} \end{cases}$$

Let

$$p(\cdot) = Z^{-1} \exp[-\varphi(\cdot)], \quad \text{where } Z = \int_{\mathbf{R}} \exp[-\varphi(x)] \lambda(dx),$$

and let I_p be the Fisher quantity associated to p :

$$I_p = \int_{\mathbf{R}} \frac{p'(\omega_0)^2}{p(\omega_0)} \lambda(d\omega_0).$$

Then $\mu \sim \mu^{(b)}$ as soon as $I_p < \infty$ and $\sum b_i^2 < \infty$.

Another case of potential with finite range is noteworthy: $r = 1$ (Markovian specification).

Example 2. If μ corresponds to a strictly stationary Markov chain, with bounded transition probability density $p(\cdot, \cdot) > 0$, then μ is a Gibbs measure (see [6, Chapter 10]) whose potential is defined by

$$\phi_A(\omega) = \begin{cases} -\ln p(\omega_{i-1}, \omega_i) & \text{if } A = \{i-1, i\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\mu \sim \mu^{(b)}$ as soon as $\sum b_i^2 < \infty$ and $I < \infty$, where I is a quantity which is similar to the Fisher quantity (see [12] for the determination of I).

The following example corresponds to a Gaussian specification ($d \geq 1$).

Example 3. Let $\phi^{J,h}$ be a translation invariant quadratic potential:

$$\phi_A^{J,h}(\omega) = \begin{cases} \frac{1}{2} J(i, i) \omega_i^2 + h_i \omega_i & \text{if } A = \{i\}, \\ J(i, j) \omega_i \omega_j & \text{if } A = \{i, j\}, i \neq j, \\ 0 & \text{otherwise,} \end{cases}$$

where $J(i, j) = J(i - j)$. Then $\mu \sim \mu^{(b)}$ as soon as $\sum_{k \in \mathbf{Z}^d} |J(k)| < \infty$ and $\sum b_i^2 < \infty$.

Harmonic potentials are a special case of quadratic potentials.

Example 4. A translation invariant quadratic potential $\phi^{J,h}$ is said to be harmonic if

$$\begin{aligned} J(0) &= \beta \geq 1 & (\beta > 1 \text{ if } d < 3), \\ J(\pm e_p) &= -\frac{1}{2d}, & p = 1, \dots, d \text{ (0 otherwise),} \end{aligned}$$

where e_p is the p -th basis vector of $\mathbf{Z}^d$: $e_p^{(i)} = \delta_{ip}$ for $i = 1, \dots, d$. In this case, $\mu \sim \mu^{(b)}$ as soon as $\sum b_i^2 < \infty$.

Finally, we can give examples of pair potentials fulfilling hypothesis $(\mathbf{D}_s)$.

Example 5. Let ϕ be a potential which have the following form:

$$\phi_A(\omega) = \begin{cases} J(i-j) \varphi(\omega_i, \omega_j) & \text{if } A = \{i, j\}, i \neq j, \\ \psi(\omega_i) & \text{if } A = \{i\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\varphi(x, y) = \varphi(x - y)$ and J are even function, $\delta(\varphi)$ is sufficiently small, $\delta(\varphi')$ and $\delta(\psi')$ are finite, and J decreases at a sufficient rate. Then $\mu \sim \mu^{(b)}$ as soon as $\sum b_i^2 < \infty$.

An appropriate choice of the semi-metric s allows $J(k)$ to decrease with $\exp[-|k|]$ or with $|k|^{-q}$, where $q > d + 1$.

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