Math-Net.Ru

Общероссийский математический портал

É. Mordecki, Ruin probabilities for Lévy processes with mixed-exponential negative jumps, *Теория вероятн. и ее примен.*, 2003, том 48, выпуск 1, 188–194

DOI: <https://doi.org/10.4213/tvp310>

Использование Общероссийского математического портала Math-Net.Ru подразумевает, что вы прочитали и согласны с пользовательским соглашением

<http://www.mathnet.ru/rus/agreement>

Параметры загрузки:

IP: 34.229.108.108

26 апреля 2023 г., 10:36:04

14. Kallsen J., Shiryaev A. N. Time change representation of stochastic integrals. — Теория вероятн. и ее примен., 2001, т. 46, в. 3, с. 579–585.
15. Kramkov D., Schachermayer W. The asymptotic elasticity of utility functions and optimal investment in incomplete markets. — Ann. Appl. Probab., 1999, v. 9, № 3, p. 904–950.

Поступила в редакцию
6.IX.2002

© 2003 г.

MORDECKI E.*

RUIN PROBABILITIES FOR LÉVY PROCESSES WITH MIXED-EXPONENTIAL NEGATIVE JUMPS¹⁾

To José Luis Massera (1915–2002)
In Memoriam

Найден явный вид вероятности разорения для процессов Леви (возможно, «убиваемых» с постоянной скоростью), отрицательные скачки которых имеют смешанное экспоненциальное распределение без ограничений на положительные скачки.

Ключевые слова и фразы: вероятность разорения, явный вид, процесс Леви, смешанное экспоненциальное распределение.

1. Introduction.

1.1. Let $X = \{X_t\}_{t \geq 0}$ be a real-valued stochastic process defined on a stochastic basis $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t \geq 0}, \mathbf{P})$ that satisfies the usual conditions. Assume that X is càdlàg, adapted, $X_0 = 0$, and for $0 \leq s < t$ the random variable $X_t - X_s$ is independent of the σ -field $\mathcal{F}_s$ with a distribution that only depends on the difference $t - s$. The stochastic process X is a process with stationary independent increments (PIIS) or a Lévy process. For $q \in \mathbf{R}$, $\psi(q)$ denotes the characteristic exponent of X given by the Lévy–Khinchine formula

$$\psi(q) = \frac{1}{t} \ln \mathbf{E} e^{iqX_t} = ibq - \frac{1}{2} \sigma^2 q^2 + \int_{\mathbf{R}} (e^{iqy} - 1 - iqy \mathbf{1}_{\{|y| < 1\}}) \Pi(dy),$$

where b and $\sigma \geq 0$ are real constants and Π is a positive measure on $\mathbf{R} - \{0\}$ such that $\int (1 \wedge y^2) \Pi(dy) < \infty$, called the Lévy measure. The function $\psi(q)$, $q \in \mathbf{R}$, completely determines the law of the process. For general reference on the subject see [4], [11], or [3].

1.2. Consider now a Lévy process with measure Π given by

$$\Pi(dy) = \begin{cases} \pi(dy) & \text{if } y > 0, \\ \lambda \sum_{k=1}^n a_k \alpha_k e^{\alpha_k y} dy & \text{if } y < 0, \end{cases} \quad (1)$$

where π is an arbitrary Lévy measure concentrated on $(0, \infty)$, $0 < \alpha_1 < \dots < \alpha_n$, $a_k > 0$, for $k = 1, \dots, n$ and $\sum_{k=1}^n a_k = 1$. The magnitude of the negative jumps of X is mixed exponentially distributed, with parameter α_k chosen with probability a_k , and they occur at Poissonian times with rate λ . As the process considered has a finite number of negative jumps on $[0, t]$, we consider a truncation function

$$h(y) = y \mathbf{1}_{\{0 < y < 1\}}. \quad (2)$$

* Facultad de Ciencias, Centro de Matemática, Iguá 4225, CP 11400, Montevideo, Uruguay; e-mail: mordecki@cmat.edu.uy

¹⁾ The research was partially supported by Project CONICYT-BID 325/95.

Simple computations give

$$\psi(q) = iaq - \frac{1}{2}\sigma^2 q^2 + \int_0^\infty (e^{iqy} - 1 - iqh(y)) \pi(dy) - \lambda \sum_{k=1}^n a_k \frac{iq}{\alpha_k + iq} \quad (3)$$

with $a = b + \lambda \sum_{k=1}^n (a_k/\alpha_k) [1 - (1 + \alpha_k)/e^{\alpha_k}]$. We introduce now the Laplace exponent of X . For $p \in \mathbf{R}$, $p \leq 0$, $p \neq -\alpha_k$, for $k = 1, \dots, n$, denote

$$\kappa(p) = ap + \frac{1}{2}\sigma^2 p^2 + \int_0^\infty (e^{py} - 1 - ph(y)) \pi(dy) - \lambda \sum_{k=1}^n a_k \frac{p}{\alpha_k + p}. \quad (4)$$

Considered as a function with complex domain, the characteristic exponent $iq \mapsto \psi(q)$ in (3) can be extended analytically to the complex strip $\{z = p + iq: p \in (-\alpha_1, 0]\}$ and, for $-\alpha_1 < p \leq 0$, we have $\kappa(p) = \psi(p)$. As the Laplace transform of X_t when $-\alpha_1 < p \leq 0$ is given by $\mathbf{E}e^{pX_t} = e^{t\kappa(p)}$, following [3] we call

$$\kappa(p) = t^{-1} \ln \mathbf{E}e^{pX_t} \quad (5)$$

the Laplace exponent of X .

Let us examine the roots of the equation $\kappa(p) = 0$, for $p \leq 0$. Since the integrand in (4) is convex in p for fixed y , we obtain that $\kappa(p)$ is convex on $(-\alpha_1, 0]$. From this it follows when $\sigma > 0$, that under the condition

$$\kappa'(0-) = \lim_{p \rightarrow 0-} \frac{1}{p} \kappa(p) = a + \int_1^\infty y \pi(dy) - \lambda \sum_{k=1}^n \frac{a_k}{\alpha_k} > 0, \quad (6)$$

(where the integral can take the value ∞) the equation $\kappa(p) = 0$ has $n + 1$ negative roots $-p_j$, $j = 1, \dots, n + 1$, that satisfy

$$0 < p_1 < \alpha_1 < p_2 < \dots < p_n < \alpha_n < p_{n+1}. \quad (7)$$

Furthermore, observe that when $\gamma > 0$, equation $\kappa(p) = \gamma$ has always $n + 1$ negative roots $-p_j$ satisfying (7).

Denote by $\tau(\gamma)$ an exponential random variable with parameter $\gamma > 0$, independent of X , and for $\gamma = 0$, put $\tau(0) = \infty$. We are interested in the random variables

$$M = \sup_{0 \leq t < \tau(\gamma)} X_t \quad \text{and} \quad I = \inf_{0 \leq t < \tau(\gamma)} X_t \quad (8)$$

called the supremum and infimum of the process, killed at rate γ if $\gamma > 0$ (see [3]).

The following result gives a closed formula for the ruin probability,

$$R(x) = \mathbf{P}\{\exists t \in [0, \tau(\gamma)): x + X_t \leq 0\} = \mathbf{P}\{I \leq -x\}, \quad x \geq 0, \quad (9)$$

for the process with characteristic exponent given by (3).

Theorem 1.1. *Let X be a Lévy process with characteristic exponent given by (3) and $\sigma > 0$.*

(a) *Assume that condition (6) holds. Then $\mathbf{P}\{\lim_{t \rightarrow \infty} X_t = \infty\} = 1$, i.e., the process drifts to infinity.*

(b) *Let $\gamma \geq 0$. Assume that condition (6) holds when $\gamma = 0$. Then the ruin probability (9) is given by*

$$R(x) = \sum_{j=1}^{n+1} A_j e^{-p_j x}, \quad x \geq 0, \quad (10)$$

where $-p_1, \dots, -p_{n+1}$ are the negative roots of the equation $\kappa(p) = \gamma$ and the constants $A_1, \dots, A_{n+1}$ are given by

$$A_j = \frac{\prod_{k=1}^n (1 - p_j/\alpha_k)}{\prod_{k=1, k \neq j}^{n+1} (1 - p_j/p_k)}, \quad j = 1, \dots, n + 1. \quad (11)$$

R e m a r k. As noticed by the referee (and can be confirmed reading the proof), the existence of a real negative root in the interval $(-\infty, -\alpha_n)$ ensures the result (10) also in the case $\sigma = 0$.

1.3. Example. In the following example, the distributions of the supremum and infimum are obtained simultaneously, when jumps are completely specified. Let the process $X = \{X_t\}_{t \geq 0}$ be given by

$$X_t = at + \sigma B_t + \sum_{k=1}^{N_t} Y_k, \quad (12)$$

where $B = \{B_t\}_{t \geq 0}$ is a standard Brownian motion, $N = \{N_t\}_{t \geq 0}$ is a Poisson process with parameter λ , $Y = \{Y_k\}_{k \in \mathbb{N}}$ is a sequence of independent and identically distributed random variables with density

$$d(y) = \begin{cases} \sum_{k=1}^m b_k \beta_k e^{-\beta_k y} & \text{if } y > 0, \\ \sum_{k=1}^n a_k \alpha_k e^{\alpha_k y} & \text{if } y < 0, \end{cases} \quad (13)$$

with $0 < \alpha_1 < \dots < \alpha_n$, $0 < \beta_1 < \dots < \beta_m$, a_k and b_k strictly positive for all k , and $\sum_{k=1}^m b_k + \sum_{k=1}^n a_k = 1$. The processes B , N , and Y are independent. The Laplace exponent (5) is given by

$$\kappa(p) = ap + \frac{1}{2} \sigma^2 p^2 + \lambda \sum_{k=1}^m b_k \frac{p}{\beta_k - p} - \lambda \sum_{k=1}^n a_k \frac{p}{\alpha_k + p}.$$

A simple application of Theorem 1.1 to X and $-X$ gives the following corollary.

Corollary 1.1. *Let X be given in (12).*

(a) *For $\gamma > 0$, the infimum I in (8) has a density given by*

$$f_I(x) = \sum_{k=1}^{n+1} A_k p_k e^{p_k x}, \quad x \leq 0, \quad (14)$$

where $-p_1, \dots, -p_{n+1}$ are the negative roots of the equation $\kappa(p) = \gamma$ and $A_1, \dots, A_{n+1}$ are given by (11).

(b) *For $\gamma > 0$, the supremum M in (8) has density given by*

$$f_M(x) = \sum_{k=1}^{m+1} B_k r_k e^{-r_k x}, \quad x \geq 0, \quad (15)$$

where $r_1, \dots, r_{m+1}$ are the positive roots of the equation $\kappa(p) = \gamma$ and the coefficients are given by

$$B_j = \frac{\prod_{k=1}^m (1 - r_j / \beta_k)}{\prod_{k=1, k \neq j}^{m+1} (1 - r_j / r_k)}, \quad j = 1, \dots, m+1.$$

(c) *If $\kappa'(0) > 0$ (or $\kappa'(0) < 0$), then the process drifts to ∞ (or to $-\infty$), and the density of I (of M) in (8) with $\gamma = 0$ is given by (14) (or (15)).*

R e m a r k. (a) and (b) in Corollary 1 can also be obtained from [10] by fractional expansion. In case of Theorem 1.1, in order to use this factorization methods, complex variable results must be considered in order to count and locate the roots of $\psi(z) = \gamma$.

1.4. Related results to Theorem 1.1 and applications can be found in [1]. The distribution of the infimum of a process with no negative jumps was found by Zolotarev [13], see also [8] or [3] and the references therein. Results giving double Laplace transform of finite time ruin can be found in [2], based on combinatorial methods and complex analysis. Rogozin [10] gives a factorization concerning the ruin of a killed process, extending combinatorial identities for random walks by Spitzer [12]. Theorem 1.1 generalizes previous work in the case of a Wiener process with negative exponential jumps ([5]) and of a Lévy process with negative exponential jumps ([6]). Within the possible applications, the results are used to give closed solutions to optimal stopping problems for Lévy processes ([7]).

The method of proof is direct, along the lines of the barrier problem for simple random walks, consisting in finding a martingale with value 1 in the stopping region that vanishes if this region is not attained.

2. Proofs. First introduce some notation and necessary facts. The infinitesimal generator of the process X with exponent in (3) is

$$(L^X f)(x) = af'(x) + \frac{1}{2} \sigma^2 f''(x) + \int_0^\infty \left(f(x+y) - f(x) - f'(x)h(y) \right) \pi(dy) \\ + \lambda \sum_{k=1}^n a_k \int_{-\infty}^0 \left(f(x+y) - f(x) \right) \alpha_k e^{\alpha_k y} dy \quad (16)$$

with h in (2). The jump measure of X is

$$\mu^X = \mu^X(\omega, dt, dy) = \sum_s \mathbf{1}_{\{\Delta X_s \neq 0\}} \delta_{(s, \Delta X_s)}(dt, dy).$$

Its compensator is a deterministic measure given by $\nu = \nu(dt, dy) = dt \Pi(dy)$ with Π in (1). The process $M = \{M_t\}_{t \geq 0}$ given by

$$M_t = \sigma B_t + \int_{[0, t] \times \mathbf{R}} h(y)(\mu^X - \nu),$$

where $B = \{B_t\}_{t \geq 0}$ is a standard Brownian motion, is a martingale with no negative jumps, and X has the decompositions

$$X_t = M_t + at + \int_{[0, t] \times \mathbf{R}} (y - h(y)) \mu^X \quad (17)$$

$$= M_t + at + \sum_{0 < s \leq t} \Delta X_s \mathbf{1}_{\{\Delta X_s \geq 1\}} - \sum_{k=1}^{N_t} Y_k, \quad (18)$$

where $N = \{N_t\}_{t \geq 0}$ is a Poisson process with parameter λ , $Y = \{Y_k\}_{k \in \mathbf{N}}$ is a sequence of positive identically distributed random variables with density $d(y) = \sum_{i=1}^n a_i \alpha_i e^{-\alpha_i y}$, and B , N and Y are independent.

The following result gives (20), the integro-differential equation that the ruin probability must satisfy.

Lemma 2.1. (a) Let $0 < p_1 < \alpha_1 < p_2 < \dots < p_n < \alpha_n < p_{n+1}$, and define the constants

$$A_j = \frac{\prod_{k=1}^n (1 - p_j/\alpha_k)}{\prod_{k=1, k \neq j}^{n+1} (1 - p_j/p_k)}, \quad j = 1, \dots, n+1.$$

Then

$$\begin{cases} \sum_{j=1}^{n+1} A_j = 1, \\ \sum_{j=1}^{n+1} A_j \frac{p_j}{p_j - \alpha_k} = 0, \quad k = 1, \dots, n. \end{cases} \quad (19)$$

(b) For the infinitesimal generator L^X in (16), the function $R(x)$ in (10), where now $-p_1, \dots, -p_{n+1}$ are the negative roots of the equation $\kappa(p) = \gamma$ (such that (7) holds), and $x > 0$, we have

$$(L^X R)(x) = \gamma R(x) \quad (20)$$

with $R(x) = 1$ if $x < 0$.

P r o o f. (a) By the theorem on simple fractional expansion, there exist unique constants $A_1, \dots, A_{n+1}$ such that

$$\frac{\prod_{j=1}^n (1 + p/\alpha_j)}{\prod_{j=1}^{n+1} (1 + p/p_j)} = \sum_{j=1}^{n+1} A_j \frac{p_j}{p_j + p}. \quad (21)$$

Each equation of the linear system (19) is equation (21) evaluated at $p = 0$ and $p = -\alpha_k$ for $k = 1, \dots, n$, respectively. On the other hand, the relation for A_j ($j = 1, \dots, n+1$) is obtained by standard methods, i.e., multiply (21) by $(p_j + p)/p_j$ and substitute $p = p_j$.

(b) Denote $f_p(x) = e^{-px^+}$, where $x^+ = \max(x, 0)$. As $\sum_{j=1}^{n+1} A_j = 1$, we can write $R(x) = \sum_{j=1}^{n+1} A_j f_{p_j}(x)$. For $x > 0$,

$$\begin{aligned} (L^X f_p)(x) &= e^{-px} \left(\frac{1}{2} \sigma^2 p^2 - ap + \int_0^\infty (e^{-py} - 1 + ph(y)) \pi(dy) \right) \\ &\quad + \lambda \sum_{k=1}^n a_k \int_{-\infty}^0 (e^{-p(x+y)^+} - e^{-px}) \alpha_k e^{\alpha_k y} dy \\ &= e^{-px} \left(\frac{1}{2} \sigma^2 p^2 - ap + \int_0^\infty (e^{-py} - 1 + ph(y)) \pi(dy) + \lambda \sum_{k=1}^n a_k \frac{p}{\alpha_k - p} \right) \\ &\quad + \lambda \sum_{k=1}^n a_k e^{-\alpha_k x} \frac{p}{p - \alpha_k} = e^{-px} \kappa(-p) + \lambda \sum_{k=1}^n a_k e^{-\alpha_k x} \frac{p}{p - \alpha_k}. \end{aligned}$$

In conclusion, for $x > 0$,

$$\begin{aligned} L^X R(x) &= \sum_{k=1}^{n+1} A_j (L^X f_{p_j})(x) \\ &= \sum_{j=1}^{n+1} A_j e^{-p_j x} \kappa(-p_j) + \lambda \sum_{k=1}^n a_k e^{-\alpha_k x} \sum_{j=1}^{n+1} A_j \frac{p_j}{p_j - \alpha_k} = \gamma R(x), \end{aligned}$$

since $\kappa(-p_j) = \gamma$ for $j = 1, \dots, n+1$, and (19).

Next lemma summarizes the application of the Meyer-Itô formula to the process X with ψ in (3) and the function R in (10). Denote $Y = \{Y_t = x + X_t\}_{t \geq 0}$.

Lemma 2.2. Let $\tau_0 = \inf\{t \geq 0: x + X_t \leq 0\}$. For R in (10)

$$R(Y_{t \wedge \tau_0}) - R(x) = \int_0^{t \wedge \tau_0} (L^X R)(Y_{s-}) ds + M(R)_{t \wedge \tau_0} \quad (22)$$

with L^X in (16) and $M(R) = \{M(R)_t\}_{t \geq 0}$ given by

$$M(R)_t = \int_0^t R'(Y_{s-}) dM_s + \int_{[0,t] \times \mathbf{R}} W(s, y) (\mu^X - \nu), \quad (23)$$

is a local martingale, where $W(s, y) = R(Y_{s-} + y) - R(Y_{s-}) - R'(Y_{s-})h(y)$.

P r o o f. Since the function R is not $C^2(\mathbf{R})$ we will apply the Meyer-Itô formula ([9, IV.51]) denoting by $m(da)$ the signed measure that is the second derivative of R in the sense of distributions, when restricted to compacts:

$$\begin{aligned} R(Y_t) - R(x) &= \int_0^t R'(Y_{s-}) dY_s + \sum_{0 < s \leq t} \left(R(Y_s) - R(Y_{s-}) - R'(Y_{s-}) \Delta Y_s \right) \\ &\quad + \frac{1}{2} \int_{\mathbf{R}} l^Y(a, t) m(da), \end{aligned} \quad (24)$$

where $l^Y(a, t)$ is the local time of the process Y at level a and time t .

First, observe that $m(da) = R''(a) da + R'(0+) \delta_0(da)$ with $R''(a)$ the second derivative of R if $a \neq 0$, $R'(0+)$ the right derivative of R at the point $x = 0$ and $\delta_0(da)$ the point mass at $x = 0$. Applying Corollary 1 to Theorem IV.51 in [9],

$$\int_{\mathbf{R}} l^Y(a, t) m(da) = \int_0^t R''(Y_{s-}) d\langle Y^c, Y^c \rangle_s + R'(0+) l^Y(0, t). \quad (25)$$

In view of (17) and (25), as $d\langle Y^c, Y^c \rangle_s = \sigma^2 ds$, (24) gives

$$\begin{aligned} R(Y_t) - R(x) &= \int_0^t R'(Y_{s-}) dM_s + \int_0^t \left(\frac{1}{2} \sigma^2 R''(Y_{s-}) + a R'(Y_{s-}) \right) ds \\ &\quad + \frac{1}{2} R'(0+) l^Y(0, t) + \sum_{0 < s \leq t} W(s, \Delta X_s). \end{aligned} \quad (26)$$

Now, we compensate the jumps in the last term. As R is bounded, the left-hand side in (26) is a special semimartingale. On the right-hand side, the first term is a local martingale and the second and third are continuous processes, and in consequence, locally integrable. We can apply II.1.28 in [4], obtaining

$$\begin{aligned} \sum_{0 < s \leq t} W(s, \Delta X_s) &= \int_{[0, t] \times \mathbf{R}} W \mu^X = \int_{[0, t] \times \mathbf{R}} W(\mu^X - \nu) + \int_{[0, t] \times \mathbf{R}} W \nu \\ &= \int_{[0, t] \times \mathbf{R}} W(\mu^X - \nu) + \int_0^t \left[\int_{\mathbf{R}} W(s, y) \Pi(dy) \right] ds. \end{aligned} \quad (27)$$

In order to evaluate (26) at τ_0 observe that $l^Y(0, \tau_0) = 0$ because $x + X_t > 0$ on the set $[0, \tau_0]$ and the local time is continuous in t (see [9, p. 165]). In view of (26) and (27), (22) follows.

P r o o f o f T h e o r e m 1.1. (a) In order to see that

$$\mathbf{P}\{\lim_{t \rightarrow \infty} X_t = \infty\} = 1 \quad (28)$$

consider the decomposition (18). Denote $d = \lambda \sum_{k=1}^n a_k / \alpha_k$. By (6) there exists $\varepsilon > 0$ such that

$$a + \int_1^\infty y \pi(dy) - (d + \varepsilon) > 0. \quad (29)$$

Consider $X^1 = \{X_t^1\}_{t \geq 0}$ and $X^2 = \{X_t^2\}_{t \geq 0}$ with

$$X_t^1 = at + M_t + \sum_{0 < s \leq t} \Delta X_s \mathbf{1}_{\{\Delta X_s \geq 1\}} - (d + \varepsilon)t, \quad X_t^2 = (d + \varepsilon)t - \sum_{k=1}^{N_t} Y_k.$$

On the one hand, as X^1 has only positive jumps, by (29) and VII.1.2 in [3] we deduce $\mathbf{P}\{\lim_{t \rightarrow \infty} X_t^1 = \infty\} = 1$. On the other hand, as $\mathbf{E}(X_t^2) = \varepsilon t > 0$, we have $\mathbf{P}\{\lim_{t \rightarrow \infty} X_t^2 = \infty\} = 1$ and conclude (28).

Proof of (b). According to Lemma 2.2, $\{R(Y_{t \wedge \tau_0})\}$ is a semimartingale. So the quadratic covariation $[R(Y_{t \wedge \tau_0}), e^{-\gamma t}] = 0$ since the deterministic process $\{e^{-\gamma t}\}$ is continuous and has bounded variation. This gives

$$\begin{aligned} e^{-\gamma(t \wedge \tau_0)} R(Y_{t \wedge \tau_0}) - R(x) &= \int_0^{\tau_0 \wedge t} e^{-\gamma s} dR(Y_{s \wedge \tau_0}) - \int_0^{\tau_0 \wedge t} R(Y_{s \wedge \tau_0-}) \gamma e^{-\gamma s} ds \\ &= \int_0^{\tau_0 \wedge t} e^{-\gamma s} [L^X R(Y_{s-}) - \gamma R(Y_{s-})] ds + \int_0^{\tau_0 \wedge t} e^{-\gamma s} dM(R)_s \\ &= \int_0^{\tau_0 \wedge t} e^{-\gamma s} dM(R)_s \end{aligned}$$

with $M(R)$ in (23), by Lemma 2.1(b), as $Y_{s-} = x + X_{s-} > 0$ on $[0, \tau_0]$ and (20). Since the function R is bounded, taking expectations and limits as $t \rightarrow \infty$

$R(x) = \mathbf{E}(e^{-\gamma \tau_0} R(x + X_{\tau_0})) = \mathbf{E}e^{-\gamma \tau_0} = \mathbf{P}\{\tau_0 < \tau(\gamma)\} = \mathbf{P}\{\exists t \in [0, \tau(\gamma)): x + X_t \leq 0\}$ concludes the proof.

Acknowledgments. The author is indebted to A. Abella for helpful discussion and to an anonymous referee for helpful comments.

REFERENCES

1. *Asmussen S.* Stationary distributions via first passage times. — Advances in Queueing Theory, Methods and Open Problems. Ed. by J. H. Dshalalow. Boca Raton: CRC Press, 1995, p. 79–102.
2. *Baxter G., Donsker M. D.* On the distribution of the supremum functional for processes with stationary independent increments. — Trans. Amer. Math. Soc., 1952, v. 85, p. 73–87.
3. *Bertoin J.* Lévy Processes. Cambridge: Cambridge Univ. Press, 1996, 265 p.

4. Жакоб Ж., Ширяев А. Н. Предельные теоремы для случайных процессов. Т. 1, 2. М.: Физматлит, 1994, 544 с; 368 с.
5. Mordecki E. Ruin probabilities and optimal stopping for a diffusion with jumps. — Proceedings of the 4th conference of mathematics «Dr. Antonio A. R. Monteiro» (Bahia Blanca, Argentina, 1997). Ed. by E. L. Fernández Stacco et al., p. 39–48.
6. Mordecki E. Optimal stopping, ruin probabilities and prophet inequalities for Lévy processes. Prepublicaciones Matemáticas del Uruguay 2000/38. Montevideo: Centro de Matemática, 2000 ([http://\(premat.fing.edu.uy/\)](http://(premat.fing.edu.uy/))).
7. Mordecki E. Optimal stopping and perpetual options for Lévy processes. — Finance Stoch., 2002, v. 6, № 4, p. 473–493.
8. Prabhu N. U. Stochastic Storage Processes. New York: Springer-Verlag, 1980, 140 p.
9. Protter P. Stochastic Integration and Differential Equations. Berlin-Heidelberg: Springer-Verlag, 1992.
10. Рогозин Б. А. О распределении некоторых функционалов, связанных с граничными задачами для процессов с независимыми приращениями. — Теория вероятн. и ее примен., 1966, т. 11, в. 4, с. 656–670.
11. Скороход А. В. Случайные процессы с независимыми приращениями. М.: Наука, 1986, 320 с.
12. Spitzer F. A combinatorial lemma and its applications to probability theory. — Trans. Amer. Math. Soc., 1956, v. 82, p. 323–339.
13. Золотарев В. М. Момент первого прохождения уровня и поведение на бесконечности одного класса процессов с независимыми приращениями. — Теория вероятн. и ее примен., 1964, т. 9, в. 4, с. 724–733.

Поступила в редакцию
18.XI.1999

© 2003 г.

DI NUNNO G.*

HÖLDER EQUALITY FOR CONDITIONAL EXPECTATIONS WITH APPLICATION TO LINEAR MONOTONE OPERATORS

Для стандартного пространства $L_p = L_p(\Omega, \mathfrak{A}, \mathbf{P})$, $1 \leq p < \infty$, и заданных множителя f и σ -алгебры $\mathfrak{B} \subseteq \mathfrak{A}$ выводится некоторый критерий представимости непрерывного линейного оператора $x(X)$, $X \in L_p$, в виде условного математического ожидания: $x(X) = \mathbf{E}(Xf | \mathfrak{B})$. В качестве применения указанное представление (с соответствующим множителем $f \geq 0$) рассматривается для общих линейных монотонных операторов $x(X)$, $X \in K$, заданных на произвольном конусе $K \subseteq L_p^+$, где $L_p^+ = \{X \in L_p : X \geq 0\}$.

Ключевые слова и фразы: условные математические ожидания, неравенство Гёльдера, линейные монотонные операторы, линейные монотонные продолжения.

Let $(\Omega, \mathfrak{A}, \mathbf{P})$ be a probability space with a probability measure $\mathbf{P}(A)$, $A \in \mathfrak{A}$, on a σ -algebra $\mathfrak{A}$ of sets $A \subseteq \Omega$, and let

$$L_p = L_p(\Omega, \mathfrak{A}, \mathbf{P}), \quad 1 \leq p < \infty,$$

be a standard L_p -space of real random variables $X = X(\omega)$, $\omega \in \Omega$, with $\|X\| = (\mathbf{E}|X|^p)^{1/p}$. Here we consider the *conditional expectation*

$$x(X) = \mathbf{E}(Xf | \mathfrak{B}), \quad X \in L_p, \quad (1)$$

associated with an appropriate factor $f = f(\omega)$, $\omega \in \Omega$, and a σ -algebra $\mathfrak{B} \subseteq \mathfrak{A}$. Naturally the factor f should be at least such that $\mathbf{E}|Xf| < \infty$ for all $X \in L_p$.

* Dipartimento di Matematica, Università degli Studi di Pavia, via Ferrata, 1, 27100 Pavia, Italy; e-mail: giulia@dimat.unipv.it