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A NOTE ON DIFFUSION-TYPE APPROXIMATION TO BRANCHING PROCESSES IN RANDOM ENVIRONMENTS¹⁾

Известные результаты об аппроксимации диффузионного типа распространяются на ветвящиеся процессы в случайных средах. В частности, допускается гораздо более широкий разброс начальных значений процессов, более общими являются условия на моменты и аппроксимирующий процесс может быть разрывным. Доказательство основано на полученных автором оценках для диффузионной аппроксимации ветвящихся процессов в изменяющихся средах.

Ключевые слова и фразы: ветвящийся процесс в случайной среде, диффузионная аппроксимация.

It is well known that in the near-critical case, Galton–Watson branching processes can be approximated (after proper scaling) by the Feller–Jirina branching diffusion process $\eta_x(t)$ [5], [8] being the solution to the stochastic differential equation

$$\eta_x(t) = (\eta_x(t))^{1/2} dw(t), \quad \eta_x(0) = x > 0, \quad (1)$$

provided that the first two moments vary in a regular way, and the numbers of the direct offspring are uniformly square integrable [13]; see also [10], [4], [7]. In [2] the author used a Skorokhod-type embedding to establish, together with the estimates of the convergence rate, conditions ensuring that the approximation holds for branching processes in varying environments. For branching processes in random environments, a diffusion approximation was suggested by Keiding [9] and proved, in the case of independent environments for each generation, in [6] (being an illustration to general martingale-type theorems on diffusion approximation). Kurtz [11] used a semigroup approach to establish such an approximation under more general assumptions on the environments. The case of «very large» initial values of the branching process requiring first centering rather than just scaling in order to get a nontrivial diffusion approximation (then we basically transfer from (1) to the Wiener process as a suitable approximant) was dealt with in [12].

In the present note we extend the Kurtz's result. We use a more general and natural scaling and drop the assumption that the limit of the «cumulative variances» processes (our $b_n(t)$ in (3) below) must be absolutely continuous and strictly increasing. Thus the limiting process actually does not need to be a diffusion (cf. our example below). Furthermore, our moment conditions are also weaker. The proof is based on the estimates derived in [2].

Let $\{Z_k^{(n)}, k = 0, 1, \dots, n\}$, $n \geq 1$, be a sequence of branching processes in random environments defined as follows. For each $n \geq 1$, $p_j^{(n)} = (p_{j0}^{(n)}, p_{j1}^{(n)}, p_{j2}^{(n)}, \dots)$, $j = 1, \dots, n$, are random distributions on $\mathbf{Z}_+$. There exist σ -algebras $\mathcal{E}^{(n)}$ («random environments») such that the random elements $p_1^{(n)}, \dots, p_n^{(n)}$ are $\mathcal{E}^{(n)}$ -measurable and, given $\mathcal{E}^{(n)}$, the sequence $Z_j = Z_j^{(n)}$, $j = 0, 1, \dots, n$ (the superindex $^{(n)}$ will be suppressed in what follows for convenience's sake) is a (time-inhomogeneous) Markov branching process with the offspring distribution $p_j^{(n)}$ at time j . That is,

$$Z_j = \sum_{i \leq Z_{j-1}} \xi_{j,i}, \quad Z_0 = m, \quad (2)$$

where, given the environments, the sequence $\{\xi_{j,i}\}_{i \geq 1}$ consists of i.i.d. random variables following the distribution p_j , while for different j 's the sequences are independent, and the

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initial value m is also an $\mathcal{E}$ -measurable random variable (initial value is, in a sense, a part of random environments as well). By $\mathbf{P}_{\mathcal{E}}$ and $\mathbf{E}_{\mathcal{E}}$ we denote the conditional probability and expectation given the environments respectively.

To get a meaningful functional approximation to the processes $Z^{(n)}$ using the Feller–Jirina diffusion (1), one has to use scaling. While the natural scaling for time reduces the time interval $\{0, 1, \dots, n\}$ (somehow) to $[0, 1]$, the state space is best transformed by scaling it down with the factor $Z_0 = m$, so that the initial value becomes just one. With that choice, we avoid the standard restrictive (and actually irrelevant) assumption that $m \asymp n$ and can assume that m is random with a much wider range than just values of the order n (in particular, our result holds even if $m \not\asymp \infty$).

Set $A_j = \mathbf{E}_{\mathcal{E}} \xi_j$ (ξ_j follows p_j), $\sigma_j^2 = \mathbf{E}_{\mathcal{E}} (\xi_j - A_j)^2$, $A(j) = A_1 \cdots A_j$, $j = 1, \dots, n$, $A(0) = 1$; $D[0, 1]$ will stand for the Skorokhod space (of right-continuous functions having left limits everywhere) with the standard (first Skorokhod) topology (see, e.g., [1]), and $\lfloor x \rfloor$ will be used to denote the integral part of x .

Introduce the following conditions (C1) and (C2).

(C1) The sequence of the two-dimensional processes $(1/a_n(\cdot), b_n(\cdot))$ defined by

$$a_n(t) = A(\lfloor nt \rfloor), \quad b_n(t) = m^{-1} \sum_{j \leq \lfloor nt \rfloor} \sigma_j^2, \tag{3}$$

converges weakly (in the space $D[0, 1] \times D[0, 1]$ endowed with the respective product-topology) as $n \rightarrow \infty$ to a process $(1/a(\cdot), b(\cdot))$, $1/a(t) > 0$ being a continuous process.

Note that the process b does not need to be continuous. Verifying condition (C1) when the environments are i.i.d., stationary weakly dependent or satisfy some other reasonable sets of conditions is a matter of applying known functional limit results from the theory of summation of random variables. Our aim is to show what happens when we superimpose a branching structure on the «backbone» process $(a_n(\cdot), b_n(\cdot))$.

(C2) *Lyapunov type condition.* For some $\delta > 0$,

$$m^{-1-\delta/2} \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} |\xi_j - A_j|^{2+\delta} \xrightarrow{\mathbf{P}} 0 \quad \text{as } n \rightarrow \infty.$$

Before stating our main result, recall that for any constant $c > 0$, the process

$$c\eta_x(\cdot/c) \stackrel{d}{=} \eta_{cx}(\cdot); \tag{4}$$

we set $\eta \equiv \eta_1$.

Theorem. *If conditions (C1) and (C2) hold, then the processes $\{Z_{\lfloor nt \rfloor}/Z_0, t \in [0, 1]\}$ converge weakly to $\{a(t)\eta(\tau(t)), t \in [0, 1]\}$, where*

$$\tau(t) = \int_0^t \frac{db(s)}{a(s)} \tag{5}$$

and the processes (a, b) and η are mutually independent.

We do not exclude the case when $\tau(1) = 0$ with a positive probability.

Remark 1. In [11], it was assumed that the space and time scaling factors must be of the same order (i.e., $m \asymp n$), and that the limiting process $b(t)$ is absolutely continuous and strictly increasing. Moreover, condition (C2) was assumed to hold for $\delta = 1$.

Remark 2. A possible weaker replacement for (C2) is the following condition.

(C3) *Lindeberg-type condition.* For any $\varepsilon > 0$,

$$m^{-1} \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} \left((\xi_j - A_j)^2; \xi_j - A_j > \varepsilon m \right) \xrightarrow{\mathbf{P}} 0 \quad \text{as } n \rightarrow \infty. \tag{6}$$

This condition is easily seen to be a consequence of (C2). If $m \xrightarrow{\mathbf{P}} \infty$, one can clearly replace (6) with

$$m^{-1} \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} (\xi_j^2; \xi_j > \varepsilon m) \xrightarrow{\mathbf{P}} 0 \quad \text{as } n \rightarrow \infty, \tag{7}$$

condition of that type was used in the case of independent environments in [6].

Proof. First note that since a_n converge to a continuous process,

$$\max_{j \leq n} |A_j - 1| \xrightarrow{\mathbf{P}} 0 \quad \text{as } n \rightarrow \infty. \tag{8}$$

Next put $T_k = \sum_{j \leq k} \sigma_j^2 / (A_j A(j))$ and note that, due to (C1) and (8), since the limiting process $1/a$ is continuous and b_n 's are nondecreasing,

$$m^{-1} T_{[nt]} \Rightarrow \tau(t), \quad (9)$$

$\Rightarrow$ standing for the weak convergence of the respective distributions in $D[0, 1]$ (this can be seen, for example, using the Skorokhod almost sure representation theorem).

1. We will show now that, without loss of generality, one can assume that $m \xrightarrow{\mathbf{P}} \infty$ as $n \rightarrow \infty$. Indeed, on the event $\max_{j \leq n} |A_j - 1| < \frac{1}{2}$, of which the probability tends to 1 due to (8), one has

$$\begin{aligned} \sum_{j \leq n} |A_j - 1| &\leq \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} |\xi_j - 1| = \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} \left(|\xi_j - 1|; |\xi_j - A_j| > \frac{1}{2} \right) \\ &\leq 2 \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} \left(|\xi_j - A_j|; |\xi_j - A_j| > \frac{1}{2} \right) \leq 2^{2+\delta} \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} |\xi_j - A_j|^{2+\delta} \end{aligned} \quad (10)$$

But, for any fixed $C < \infty$, condition (C2) implies that

$$\mathbf{1}(m < C) \sum_{j=1}^n \mathbf{E}_{\mathcal{E}} |\xi_j - A_j|^{2+\delta} \xrightarrow{\mathbf{P}} 0. \quad (11)$$

Therefore $\mathbf{1}(m < C)(a_n(\cdot) - 1) \xrightarrow{\mathbf{P}} 0$. Similarly, $b_n(t)$ converges to zero, so that the respective limits are then $\tau(t) \equiv 0$ and $a(t)\eta(\tau(t)) \equiv 1$.

On the other hand, given the environments, the sequence $Z_k/A(k) - m$ is clearly a martingale. Since

$$\begin{aligned} \mathbf{E}_{\mathcal{E}} \left(Z_n - A(n)m \right)^2 &= mA(n-1)\sigma_n^2 + A_n^2 \mathbf{E}_{\mathcal{E}} \left(Z_{n-1} - A(n-1)m \right)^2 \\ &= m(A(n))^2 \sum_{j \leq n} \frac{\sigma_j^2}{A_j A(j)} = O(mT_n), \end{aligned} \quad (12)$$

we have from the Kolmogorov inequality that for any $\varepsilon > 0$,

$$\begin{aligned} \mathbf{P}_{\mathcal{E}} \left\{ \max_{0 \leq t \leq 1} \left| \frac{Z_{[nt]}}{m} - a_n(t) \right| > \varepsilon \right\} &\leq \mathbf{P}_{\mathcal{E}} \left\{ \max_{j \leq n} \left| \frac{Z_j}{A(j)} - m \right| > \frac{m\varepsilon}{\max_{j \leq n} A(j)} \right\} \\ &= O\left(m^{-2} \mathbf{E}_{\mathcal{E}} (Z_n - A(n)m)^2\right) = O\left(m^{-1} T_n\right) \end{aligned} \quad (13)$$

is $o_{\mathbf{P}}(1)$ on $\{m < C\}$ from (11) similarly to (10). (Here and in what follows, we write $\alpha_n = o_{\mathbf{P}}(\beta_n)$ and $\alpha_n = O_{\mathbf{P}}(\beta_n)$ when

$$\frac{\alpha_n}{\beta_n} \xrightarrow{\mathbf{P}} 0 \quad \text{as } n \rightarrow \infty \quad \text{and} \quad \lim_{x \rightarrow \infty} \limsup_{n \rightarrow \infty} \mathbf{P} \left\{ \frac{\alpha_n}{\beta_n} > x \right\} = 0$$

respectively.) In other words, in that case $\{Z_{[nt]}/m\}$ is asymptotically also just 1, so that we have the claimed convergence.

Our next observation is that without loss of generality we can now assume that $T_n \xrightarrow{\mathbf{P}} \infty$ as well. Indeed, given $\{T_n < C\}$, the sequence $b_n(\cdot)$ again converges to zero, so that the limiting process stated in the theorem is just $a(t)$. On the other hand, the right-hand side of (13) is now again $o_{\mathbf{P}}(1)$ since $T_n = o(m)$, so that we do have the desired result.

Thus from now on we can assume that both m and T_n tend to infinity in probability and

$$m = O_{\mathbf{P}}(T_n) \quad (14)$$

as $n \rightarrow \infty$ (if the latter is not true, (13) implies the desired approximation).

2. Next we will show that it suffices to establish the stated convergence for the processes with truncated offspring distributions. Introduce the process $\hat{Z}_k$ which differs from Z_k only in that the ξ -variables are now truncated versions of the original ones:

$$\hat{\xi}_j = \begin{cases} \xi_j & \text{if } \xi_j \leq m, \\ 0 & \text{if } \xi_j > m. \end{cases}$$

Set $\hat{A}_j = \mathbf{E}_{\mathcal{E}} \hat{\xi}_j$, $\hat{A}(j) = \hat{A}_1 \cdots \hat{A}_j$ (all the variables related to the process $\{\hat{Z}_k\}$ will be marked with a hat). On the event $C_n = \{m > 2, \max_{j \leq n} A_j \leq 2\}$, of which the probability tends to 1 due to (8), one has

$$\begin{aligned} 0 &\leq A_j - \hat{A}_j = \mathbf{E}_{\mathcal{E}}(\xi_j; \xi_j > m) \leq m^{-(1+\delta)} \mathbf{E}_{\mathcal{E}}(\xi_j^{2+\delta}; \xi_j > m) \\ &\leq 3^{2+\delta} m^{-(1+\delta)} \mathbf{E}_{\mathcal{E}}|\xi_j - A_j|^{2+\delta}, \end{aligned} \quad (15)$$

so that for all $k \leq n$,

$$0 \leq A(k) - \hat{A}(k) \leq \max_{k \leq n} A(k) \sum_{j=1}^n A_j^{-1} (A_j - \hat{A}_j) = o_{\mathbf{P}}(m^{-\delta/2}) \quad (16)$$

from (C1) and (C2) since $\max_{k \leq n} A(k) = O_{\mathbf{P}}(1)$.

Since $(Z_k - \hat{Z}_k)/A(k) \geq 0$ can easily be seen to be a submartingale given $\mathcal{E}$, we have by the Kolmogorov inequality and (16) that for any $\varepsilon > 0$ and $\delta > 0$,

$$\begin{aligned} &\mathbf{P}\left\{\max_{k \leq n} (Z_k - \hat{Z}_k) > \varepsilon \max_{k \leq n} A(k)\right\} \\ &\leq \mathbf{E}\left(\mathbf{P}_{\mathcal{E}}\left\{\max_{k \leq n} \frac{Z_k - \hat{Z}_k}{A(k)} > \varepsilon\right\}; A(n) - \hat{A}(n) \leq \varepsilon^2, A(n) \geq \delta\right) \\ &+ \mathbf{P}\{A(n) - \hat{A}(n) > \varepsilon^2\} + \mathbf{P}\{A(n) < \delta\} \leq \frac{\varepsilon}{\delta} + o_{\mathbf{P}}(1) + \mathbf{P}\{A(n) < \delta\}. \end{aligned}$$

From (C1) the last term vanishes uniformly in n as $\delta \rightarrow \infty$, and therefore we can infer that

$$\max_{k \leq n} |Z_k - \hat{Z}_k| \xrightarrow{\mathbf{P}} 0. \quad (17)$$

On the other hand,

$$\sigma_j^2 - \hat{\sigma}_j^2 = \mathbf{E}_{\mathcal{E}}(\xi_j^2; \xi_j > m) - (A_j - \hat{A}_j)(A_j + \hat{A}_j), \quad (18)$$

so that similarly to (15) and (16), we have

$$\max_{k \leq n} \left| \sum_{j=1}^k \sigma_j^2 - \sum_{j=1}^k \hat{\sigma}_j^2 \right| = o_{\mathbf{P}}(m^{1-\delta/2}). \quad (19)$$

Hence the processes $(1/\hat{a}_n, \hat{b}_n)$ will also converge to the same $(1/a, b)$. One consequence of that is that from (9) and (C2) it follows that the processes

$$m^{-1} \hat{T}_{[nt]} \equiv m^{-1} \sum_{j=1}^{[nt]} \frac{\hat{\sigma}_j^2}{\hat{A}_j \hat{A}(j)} \Rightarrow \tau(t), \quad \text{and} \quad T_n \sim \hat{T}_n. \quad (20)$$

3. Having noted all that, we will proceed now to proving the desired convergence for the truncated processes.

Set $y(t) = \hat{Z}_k / (\hat{T}_n \hat{A}(k))$ for $t \in [\hat{T}_k / \hat{T}_n, \hat{T}_{k+1} / \hat{T}_n)$, $k = 0, 1, \dots, n-1$. We know from [2] that, for the fixed environments, one can construct the processes y and $\eta_{m/\hat{T}_n}$ on a common probability space so that for $x_1 > 0$, $q \in (0, 1)$ and $\delta \in (0, 1]$,

$$\begin{aligned} &\mathbf{P}_{\mathcal{E}}\left\{\sup_{0 \leq t \leq 1} |y(t) - \eta_{m/\hat{T}_n}| > 12 |\ln U|^{1/2} x_1 + x_2\right\} \\ &\leq 5U^q + \frac{2^{3/2} q^{1/2}}{x_1 U^{(1-q)/(2+\delta)}} \exp\left(-\frac{x_1^2}{q U^{(1-q)/(1+\delta/2)}}\right) + 5.3 \left(\frac{m}{\hat{T}_n}\right)^{8/27} H^{1/27} x_2^{-10/27} \end{aligned} \quad (21)$$

provided that

$$2q |\ln U| \geq \frac{5m}{\hat{T}_n} \quad \text{and} \quad U^{(1-q)/(1+\delta/2)} + \hat{T}_n^{-1} \max_{j \leq n} \frac{\hat{\sigma}_j^2}{\hat{A}_j \hat{A}(j)} \leq \frac{1}{2}, \quad (22)$$

where

$$U = \frac{44}{\hat{T}_n^{1+\delta/2}} \sum_{j=1}^n \frac{\mathbf{E}_{\mathcal{E}}|\hat{\xi}_j - \hat{A}_j|^{2+\delta}}{(\hat{A}_j \hat{A}(j))^{1+\delta/2}} \quad (23)$$

and

$$H = \frac{3}{\widehat{T}_n^2} \left[m^{1/2} \sum_{j=1}^n \frac{\widehat{\sigma}_j^3}{(\widehat{A}_j \widehat{A}(j))^3} + \frac{1}{2} \sum_{j=1}^n \frac{\mathbf{E}_{\mathcal{E}} |\hat{\xi}_j - \widehat{A}_j|^3}{(\widehat{A}_j \widehat{A}(j))^{3/2} (\widehat{A}(j-1))^{1/2}} \right]. \quad (24)$$

Put $q = \frac{1}{2}$. We see from (14) that conditions (22) will be met with probability tending to one once we have $U = o_{\mathbf{P}}(1)$ and

$$\max_{j \leq n} \widehat{\sigma}_j^2 = o_{\mathbf{P}}(\widehat{T}_n). \quad (25)$$

The latter follows immediately from (15), (18), (C2) and (14). Further,

$$\mathbf{E}_{\mathcal{E}} |\hat{\xi}_j - \widehat{A}_j|^{2+\delta} \leq 3^{1+\delta} (\mathbf{E}_{\mathcal{E}} (\xi_j^{2+\delta}; \xi_j > m) + \mathbf{E}_{\mathcal{E}} |\xi_j - A_j|^{2+\delta} + (A_j - \widehat{A}_j)^{2+\delta}),$$

and from (15) we derive that on the event C_n one has

$$\sum_{j \leq n} \mathbf{E}_{\mathcal{E}} |\hat{\xi}_j - \widehat{A}_j|^{2+\delta} = O \left(\sum_{j \leq n} \mathbf{E}_{\mathcal{E}} |\xi_j - A_j|^{2+\delta} \right) = o_{\mathbf{P}}(m^{1+\delta/2}) \quad (26)$$

from (C2). Therefore $U = o_{\mathbf{P}}(1)$ due to (20) and (14).

Now we can see from (21) that to show that the process $y(\cdot)$ will be approximated by $\eta_{m/\widehat{T}_n}(\cdot)$, it remains to verify that $H = o_{\mathbf{P}}(1)$. Due to (20) and (19), the first term in (24) is just

$$O_{\mathbf{P}} \left(\left(\frac{m}{T_n} \right)^{1/2} \max_{j \leq n} \left(\frac{\sigma_j^2}{T_n} \right)^{1/2} \right) = o_{\mathbf{P}}(1)$$

from (14) and (25).

As for the second term in (24), we have similarly to (26) that

$$\sum_{j \leq n} \mathbf{E}_{\mathcal{E}} |\hat{\xi}_j - \widehat{A}_j|^3 = o_{\mathbf{P}} \left(m^{3-(2+\delta)} \times m^{1+\delta/2} \right) = o_{\mathbf{P}}(m^2),$$

and hence that term also vanishes.

To complete the proof, it remains to notice that what we have just proved and (4) imply that one can construct the processes on a common probability space in such a way that

$$\frac{\widehat{Z}_{[nt]}}{m} \equiv \frac{\widehat{T}_n}{m} \widehat{A}_{[nt]} y \left(\frac{\widehat{T}_{[nt]}}{\widehat{T}_n} \right) = \widehat{A}_{[nt]} \left(\eta \left(\frac{\widehat{T}_{[nt]}}{m} \right) + \theta_n(t) \right),$$

where $\sup_{0 \leq t \leq 1} |\theta_n(t)| = o_{\mathbf{P}}(1)$ and η is independent of the environments. Now the desired assertion follows from (17), (C1), (16), (20) and continuity of the process η .

To illustrate how our theorem works in situations where the results of Helland [6] and Kurtz [11] are inapplicable, consider the following simple example.

Example. Assume that for the n -th process $\{Z_j^{(n)}\}$, the environments are governed by a two-state Markov chain $X_j = X_j^{(n)}$, $X_0 = 0$, with the transition probabilities $\pi_{01} = 1 - \pi_{00} = \lambda/n$, $\pi_{10} = 1 - \pi_{11} = \mu/m$, where $\lambda, \mu > 0$ are some constants, $m \rightarrow \infty$ and $m = o(n)$. If $X_j = 0$, there is no branching: $p_{j1} = 1$ (and hence $A_j = 1$, $\sigma_j^2 = 0$). If $X_j = 1$, the offspring law is given by $p_{j0} = p_{j2} = \frac{1}{2}$ (so that $A_j = 1$, $\sigma_j^2 = 1$). Thus, there are relatively long «hibernation» periods (when $X_j = 0$) alternating with shorter «bursts of activity» (when $X_j = 1$).

It is easy to see that the processes $b_n(t)$ converge in distribution to the compound Poisson process $b(t)$ with the jump intensity λ and compounding exponential law with parameter μ , and clearly $\tau(t) \equiv b(t)$. Thus condition (C1) is met. Since $m^{-3/2} \sum_{j \leq n} \mathbf{E}_{\mathcal{E}} |\xi_j - A_j|^3 = m^{-1/2} b_n(1)$, condition (C2) is also satisfied (with $\delta = 1$), and hence we conclude from our theorem that the processes $\{Z_{[nt]}/m, t \in [0, 1]\}$ converge to a pure jump Markov process with the initial state one and constant jump intensity λ , the transition probabilities $P(x, B)$ of the embedded Markov chain having the Laplace transform

$$\int_0^\infty e^{-sx} P(x, dy) = \mu \int_0^\infty \exp \left(-\mu t - \frac{sx}{1 + st/2} \right) dt, \quad s > 0.$$

The last formula is a simple consequence of the relation $Ee^{-s\eta_x(t)} = \exp(-sx/(1+st/2))$, $s > -2/t$ (see, e.g., [4]).

In conclusion note that the one-dimensional distributions of the process $b(t)$ are of the same type as those of the process η ; the general finite-dimensional distributions are, of course, quite different (for more detail see, e.g., [3]).

REFERENCES

1. Биллингсли П. Сходимость вероятностных мер. М.: Наука, 1977, 352 с.
2. Боровков К. А. О скорости сходимости ветвящихся процессов к диффузионному. — Теория вероятн. и ее примен., 1985, т. 30, в. 3, с. 468–477.
3. Боровков К. А. Аппроксимация ветвящихся процессов и случайных полей. — Сиб. матем. журн., 1991, т. 32, № 4, с. 36–47.
4. Чистяков В. П. О сходимости ветвящихся процессов к диффузионным. — Теория вероятн. и ее примен., 1970, т. 15, в. 4, с. 727–731.
5. Feller W. Diffusion processes in genetics. — Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. Berkeley: Univ. of California Press, 1951, p. 227–246.
6. Helland I. S. Minimal conditions for weak convergence to a diffusion process on the line. — Ann. Probab., 1981, v. 9, p. 429–452.
7. Jagers P. Diffusion approximation of branching processes. — Ann. Math. Statist., 1971, v. 42, p. 2074–2078.
8. Jiřina M. On Feller's branching diffusion processes. — Časop. Peřtov. Mat., 1969, v. 94, p. 84–90.
9. Keiding N. Extinction and exponential growth in random environments. — Theoret. Population Biol., 1975, v. 8, p. 49–63.
10. Колмогоров А. Н. Переход ветвящихся процессов в диффузионные и примыкающие задачи генетики. — Теория вероятн. и ее примен., 1959, т. 4, в. 2, с. 233–236.
11. Kurtz T. G. Diffusion approximations for branching processes. — Branching Processes. Ed. by A. Joffe and P. Ney. 1978, p. 269–292. (Adv. Probab. Relat. Topics, v. 5.)
12. Лебедев Е. А. Предельные теоремы для критических ветвящихся процессов в случайной среде. — Теория вероятн. и матем. статист., 1979, т. 20, с. 68–76.
13. Lindvall T. Convergence of critical Galton–Watson branching processes. — J. Appl. Probab., 1972, v. 9, p. 445–450.

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MULTISTABILITY IN QUEUES WITH RETRANSMISSION AND ITS RELATIONSHIP WITH LARGE DEVIATIONS IN BRANCHING PROCESSES

Характерный для современных коммуникационных сетей алгоритм повторной передачи данных может привести к ситуации, когда система массового обслуживания имеет много псевдостационарных состояний. Только одно из этих состояний соответствует высоким характеристикам сети, остальные состояния соответствуют плохим характеристикам

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