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ROOS B.*

KERSTAN'S METHOD IN THE MULTIVARIATE POISSON APPROXIMATION: AN EXPANSION IN THE EXPONENT

Обобщенное полиномиальное распределение аппроксимируется конечными мерами со знаком, возникающими в разложении пуассоновского типа в экспоненте.

В одномерном случае это разложение впервые было использовано в [9] и [11]. Мы применяем метод Керстена и приводим оценку расстояния по вариации с указанием явного вида констант.

Ключевые слова и фразы: разложение в экспоненте, обобщенное полиномиальное распределение, метод Керстена, многомерная пуассоновская аппроксимация, расстояние по вариации.

1. Introduction and results. Let S_n be the sum of independent Bernoulli random vectors $X_1, \dots, X_n$ in $\mathbf{R}^k$ ($k, n \in \mathbf{N} = \{1, 2, \dots\}$) with probabilities

$$\mathbf{P}\{X_j = e_r\} = p_{j,r} \in [0, 1], \quad \mathbf{P}\{X_j = 0\} = 1 - p_j \in [0, 1]$$

for $j \in \{1, \dots, n\}$ and $r \in \{1, \dots, k\}$, where $p_j = \sum_{r=1}^k p_{j,r}$ and e_r denotes the vector in $\mathbf{R}^k$ with entry 1 at position r and 0 otherwise. We assume that $\lambda_r = \sum_{j=1}^n p_{j,r} > 0$ for all r .

In this paper, we are concerned with the approximation of the distribution P^{S_n} of S_n by the finite signed measures $\mathcal{P}_s$ ($s \in \mathbf{N}$) concentrated on $\mathbf{Z}_+^k$ with the generating function

$$\Psi_{\mathcal{P}_s}(z) = \sum_{l \in \mathbf{Z}_+^k} \mathcal{P}_s(\{l\}) z^l = \exp \left(\sum_{m=1}^s G_m(z) \right) \quad (z = (z_1, \dots, z_k) \in \mathbf{C}^k),$$

where $\mathbf{Z}_+ = \{0, 1, 2, \dots\}$, $z^l = z_1^{l_1} \dots z_k^{l_k}$ for $l = (l_1, \dots, l_k) \in \mathbf{Z}_+^k$, and, for $m \in \{1, \dots, s\}$ and $z \in \mathbf{C}^k$,

$$G_m(z) = \frac{(-1)^{m+1}}{m} \sum_{j=1}^n [H_j(z)]^m, \quad H_j(z) = \sum_{r=1}^k p_{j,r} (z_r - 1) \quad (j \in \{1, \dots, n\}).$$

* Schwerpunkt Math. Statistik und Stoch. Prozesse, Fachbereich Mathematik, Universität Hamburg, Bundesstraße 55, D-20146 Hamburg, Germany; e-mail: roos@math.uni-hamburg.de

An approximation by $\mathcal{P}_s$ may be useful, since, for the probability generating function $\Psi_{S_n}(z)$ of P^{S_n} , the following relations hold:

$$\Psi_{S_n}(z) = \sum_{l \in \mathbf{Z}_+^k} \mathbf{P}\{S_n = l\} z^l = \prod_{j=1}^n (1 + H_j(z)) = \exp \left(\sum_{m=1}^{\infty} G_m(z) \right)$$

if $z \in \mathbf{C}^k$ with $\max_{1 \leq r \leq k} |z_r - 1| < 1$. Note that $\mathcal{P}_s(\mathbf{Z}_+^k) = \Psi_{\mathcal{P}_s}(1) = 1$.

In the univariate case $k = 1$, the first results concerning the above expansion are due to Kornya [9] and Presman [11]. Therefore we call this expansion a Kornya–Presman expansion. It should be mentioned that Presman considered the binomial case with $s = 2$. Further results came from Kruopis [10] and Barbour and Xia [2] (for $k = 1$ and $s = 2$), and Hipp [7] and Čekanavičius [3] (for $k = 1$ and arbitrary s). The multivariate Poisson case with $s = 1$ and arbitrary k was treated, for example, by Barbour [1], Deheuvels and Pfeifer [5], and Roos [13], [14]. For this case, see also the references in [13]. The method of this paper is originally due to Kerstan [8], who considered the case $k = s = 1$. Refinements of this method came from Daley and Vere-Jones [4, p. 297–299], Witte [15], and Roos [12, Chapter 8], [14].

In what follows, further notation is needed. Let

$$\begin{aligned} s \in \mathbf{N}, \quad p_0 &= \max_{1 \leq j \leq n} p_j, \quad \tilde{p}_0 = \sum_{r=1}^k \max_{1 \leq j \leq n} p_{j,r}, \\ \beta_s(x) &= \sum_{j=1}^n \left[\min \left\{ x \sum_{r=1}^k \frac{p_{j,r}^2}{\lambda_r}, p_j^2 \right\} \right]^{(s+1)/2} \quad (x \in [0, \infty)). \end{aligned}$$

Observe that

$$p_0 \leq \tilde{p}_0, \quad \beta_s(x) \leq [\beta_1(x)]^{(s+1)/2} \leq (x \tilde{p}_0)^{(s+1)/2}. \tag{1}$$

For $y \in \mathbf{C}$, let

$$U_s(y) = \exp \left(\sum_{m=1}^s \frac{y^m}{m} \right), \quad V_s(y) = [1 - (1 - y) U_s(y)] \frac{s+1}{y^{s+1}}.$$

Note that $(1 - y) U_s(y)$ is the prime function used by Weierstrass (see [6, p. 227]). For $x \in \mathbf{R}$, let $\lceil x \rceil$ denote the smallest integer $\geq x$. Let $A, B \in [0, 1]$ with $A + B = 1$. However, to get reasonable inequalities, it must often be assumed that $A, B \in (0, 1)$. In this paper, we frequently deal with power series of the type $W(z) = \sum_{l \in \mathbf{Z}_+^k} w_l z^l$ ($w_l \in \mathbf{R}$), which are absolutely convergent for all $z \in \mathbf{C}^k$. In particular, the order of summation may be chosen arbitrarily. We write $\|W(z)\| = \sum_{l \in \mathbf{Z}_+^k} |w_l|$ and use the easy fact that $\|W_1(z) W_2(z)\| \leq \|W_1(z)\| \|W_2(z)\|$ for two power series $W_1(z)$ and $W_2(z)$.

In the following theorem, we give an upper bound for the total variation distance $d_s = 2^{-1} \|\Psi_{S_n}(z) - \Psi_{\mathcal{P}_s}(z)\|$ between P^{S_n} and $\mathcal{P}_s$.

Theorem 1. *Let*

$$\begin{aligned} c_1(s) &= \begin{cases} (s+1) 2^{-5/2}, & \text{for odd } s, \\ (s+1) 2^{1/[2(s+1)]-5/2}, & \text{for even } s, \end{cases} \\ c_2(s, p_0) &= \frac{e 2^s \lceil s/2 - 1 \rceil!}{\sqrt{2\pi} (s+1)} V_s(2p_0), \\ c_3(s, p_0) &= \frac{e 2^{s+1}}{s+1} V_s(2p_0), \quad c_4(s, p_0) = 4e \sum_{m=2}^s \frac{(2p_0)^{m-2}}{m}, \end{aligned}$$

If $c_3(s, p_0) p_0^{s-1} \beta_1(2^{-3/2} B^{-1}) < 1$ and $c_4(s, p_0) \beta_1(2^{-3/2} A^{-1}) < 1$, then

$$d_s \leq \frac{c_2(s, p_0) \beta_s(c_1(s) B^{-1})}{[1 - c_3(s, p_0) p_0^{s-1} \beta_1(2^{-3/2} B^{-1})]^{\lceil s/2 \rceil} [1 - c_4(s, p_0) \beta_1(2^{-3/2} A^{-1})]}. \tag{2}$$

Remarks. (a) Considering the properties of the Weierstrass prime function (see [6, p. 227]), we obtain $V_s(y) = \sum_{i=0}^{\infty} v_{s,i} y^i$ ($y \in \mathbf{C}$), where $v_{s,0} = 1$ and $v_{s,i} \geq 0$ for all $i \geq 1$. Therefore $V_s(x)$ is increasing in $x \in [0, \infty)$. Further, for these x , we have $V_s(x) \leq U_s(x)$, since this is equivalent to

$$1 \leq U_s(x) \left(1 - x + \frac{x^{s+1}}{s+1} \right),$$

which follows from the observation that the term on the right-hand side is increasing in $x \in [0, \infty)$. Note also that $U_s(x) \leq (1-x)^{-1}$ for $x \in [0, 1)$. The inequalities given here can be used to obtain upper bounds for the constants $c_2(s, p_0)$ and $c_3(s, p_0)$.

(b) In the case $k = 1$ and $s = 2$, inequality (2) substantially coincides, up to constants, with the inequalities given by Presman [11, Assertion 1(a)] and Kruopis [10, Theorem 3], respectively. If k is arbitrary and $s = 1$, we can choose $A = 0$ and $B = 1$, since $c_4(1, p_0) = 0$; in this case, (2) is comparable with inequality (5) in [14].

(c) There exist positive constants c_5 and $c_6(s)$, such that, if $\tilde{p}_0 \leq c_5$, the inequality $d_s \leq c_6(s) \beta_s(1)$ is valid for all s . Note that c_5 does not depend on s . Indeed for $s = 1$, such an inequality with $c_6(1) = 8.8$ was proved in [14, formula (6)] without any restrictions on $\tilde{p}_0$. Further, letting $A = 0.745$ and using (1), (2), and Remark (a), it is easily shown that one can choose $c_5 = \frac{1}{4}$. Here it suffices to verify that an $\varepsilon \in (0, 1)$ exists such that, for all $s \in \{2, 3, \dots\}$ and $\tilde{p}_0 \leq \frac{1}{4}$,

$$c_3(s, p_0) p_0^{s-1} \beta_1(2^{-3/2} B^{-1}) < 1 - \varepsilon \quad \text{and} \quad c_4(s, p_0) \beta_1(2^{-3/2} A^{-1}) < 1 - \varepsilon.$$

The special value for A was taken to obtain a large c_5 . It should be mentioned that, in the general case $s \in \mathbf{N}$, it is not clear, whether the above condition $\tilde{p}_0 \leq c_5$ can be dropped.

For a successful approximation, we have to compute the values of $\mathcal{P}_s$. In the following proposition, we give a recursive formula for the counting density of $\mathcal{P}_s$. Observe that $\mathcal{P}_s(\{0\}) = \Psi_{\mathcal{P}_s}(0)$. For $l = (l_1, \dots, l_k) \in \mathbf{Z}_+^k$, we use the standard multi-index notation $|l| = l_1 + \dots + l_k$ and $l! = l_1! \dots l_k!$. Further, if additionally $t \in \mathbf{Z}_+^k$, we write $t \leq l$ in the case that $t_r \leq l_r$ for all r .

Proposition 1. Let $l \in \mathbf{Z}_+^k$ with $|l| \geq 1$, $M_{l,s} = \{t \in \mathbf{Z}_+^k \mid 1 \leq |t| \leq s; t \leq l\}$, and

$$b_l = \frac{(-1)^{|l|+1} |l|}{t!} \sum_{j=1}^n \left(\sum_{m=0}^{s-|l|} \frac{(m+|l|-1)!}{m!} p_j^m \right) \prod_{r=1}^k p_{j,r}^{t_r} \quad (t \in M_{l,s}).$$

Then

$$\mathcal{P}_s(\{l\}) = \frac{1}{|l|} \sum_{t \in M_{l,s}} \mathcal{P}_s(\{l-t\}) b_t.$$

2. Proofs. For the proof of Theorem 1, we need the following lemma.

Lemma 1. Let $x \in (0, \infty)$ and $j \in \{1, \dots, n\}$. Then

$$\|V_s(-H_j(z))\| = V_s(2p_j), \quad (3)$$

$$\left\| [H_j(z)]^{s+1} \exp(xG_1(z)) \right\| \leq \left[\min \left\{ \frac{4c_1(s)}{x} \sum_{r=1}^k \frac{p_{j,r}^2}{\lambda_r}, 4p_j^2 \right\} \right]^{(s+1)/2}. \quad (4)$$

If $c_4(s, p_0) \beta_1(2^{-3/2} A^{-1}) < 1$, then

$$T_1(A) := \left\| \exp \left(A G_1(z) + \sum_{m=2}^s G_m(z) \right) \right\| \leq \frac{1}{1 - c_4(s, p_0) \beta_1(2^{-3/2} A^{-1})}. \quad (5)$$

Proof. Let $V_s(y) = \sum_{i=0}^{\infty} v_{s,i} y^i$ ($y \in \mathbf{C}$) as in Remark (a) after Theorem 1. Using the polynomial theorem, we obtain

$$\begin{aligned} V_s(-H_j(z)) &= \sum_{i=0}^{\infty} v_{s,i} \left(p_j - \sum_{r=1}^k p_{j,r} z_r \right)^i = \sum_{i=0}^{\infty} v_{s,i} \sum_{l \in \mathbf{Z}_+^k: |l| \leq i} \frac{i! p_j^{i-|l|}}{l! (i-|l|)!} \prod_{r=1}^k (-p_{j,r} z_r)^{l_r} \\ &= \sum_{l \in \mathbf{Z}_+^k} \left[\sum_{i=|l|}^{\infty} \frac{v_{s,i} i! p_j^{i-|l|}}{l! (i-|l|)!} \prod_{r=1}^k p_{j,r}^{l_r} \right] (-z)^l, \end{aligned}$$

leading to (3): $\|V(-H_j(z))\| = V_s(-H_j((-1, \dots, -1))) = V_s(2p_j)$. Now we prove (4). For $s \in \{1, 2\}$, this inequality was proved in [14, formulas (19) and (26)]. For even s , we use

$$\left\| [H_j(z)]^{s+1} \exp(xG_1(z)) \right\| \leq \left\| H_j(z) \exp\left(\frac{xG_1(z)}{s+1}\right) \right\| \left\| [H_j(z)]^2 \exp\left(\frac{2xG_1(z)}{s+1}\right) \right\|^{s/2}$$

and the fact that (4) also holds for $s = 0$ with $c_1(0) = \frac{1}{4}$ (see [14, formula (18)]). For odd s , the proof of (4) is similar. For the proof of (5), we may assume that $s \geq 2$. Using (4) and Stirling's formula, we obtain

$$\begin{aligned} T_1(A) &= \left\| \exp\left(\sum_{m=2}^s \sum_{j=1}^n \frac{(-1)^{m+1}}{m} [H_j(z)]^m\right) \exp(A G_1(z)) \right\| \\ &= \left\| \exp(A G_1(z)) + \sum_{i=1}^\infty \frac{1}{i!} \left(\sum_{j=1}^n \sum_{m=2}^s \frac{(-1)^{m+1}}{m} [H_j(z)]^m \exp\left(\frac{A}{i} G_1(z)\right) \right)^i \right\| \\ &\leq 1 + \sum_{i=1}^\infty \frac{1}{i!} \left(\sum_{j=1}^n \sum_{m=2}^s \frac{\|H_j(z)\|^{m-2}}{m} \left\| [H_j(z)]^2 \exp\left(\frac{A}{i} G_1(z)\right) \right\|^i \right)^i \\ &\leq 1 + \sum_{i=1}^\infty \frac{i^i}{i!} \left[\frac{c_4(s, p_0)}{e} \beta_1(2^{-3/2} A^{-1}) \right]^i \leq \frac{1}{1 - c_4(s, p_0) \beta_1(2^{-3/2} A^{-1})}, \end{aligned}$$

if $c_4(s, p_0) \beta_1(2^{-3/2} A^{-1}) < 1$. The lemma is proved.

P r o o f o f T h e o r e m 1. Consider the following expansion of the difference of the generating functions of P^{S_n} and $\mathscr{P}_s$:

$$\begin{aligned} \Psi_{S_n}(z) - \Psi_{\mathscr{P}_s}(z) &= \left[\prod_{j=1}^n \left(1 - \frac{(-H_j(z))^{s+1}}{s+1} V_s(-H_j(z)) \right) - 1 \right] \Psi_{\mathscr{P}_s}(z) \\ &= \sum_{j=1}^n \sum_{1 \leq i(1) < \dots < i(j) \leq n} \prod_{\nu=1}^j \left[\frac{(-1)^s V_s(-H_{i(\nu)}(z))}{s+1} [H_{i(\nu)}(z)]^{s+1} \exp\left(\frac{B}{j} G_1(z)\right) \right] \\ &\quad \times \exp\left(A G_1(z) + \sum_{m=2}^s G_m(z)\right). \end{aligned}$$

By using the polynomial theorem, we obtain $2d_s \leq T_1(A) T_2(B)$, where $T_1(A)$ is defined as in Lemma 1 and

$$\begin{aligned} T_2(B) &:= \sum_{j=1}^n \frac{1}{j!} \left[\sum_{i=1}^n \frac{\|V_s(-H_i(z))\|}{s+1} \left\| [H_i(z)]^{s+1} \exp\left(\frac{B}{j} G_1(z)\right) \right\|^j \right] \\ &\leq \sum_{j=1}^n \left(\frac{1}{j!} \left[\frac{V_s(2p_0)}{s+1} (2p_0)^{s-1} \sum_{i=1}^n \left\| [H_i(z)]^2 \exp\left(\frac{B}{j} G_1(z)\right) \right\|^j \right]^{j-1} \right. \\ &\quad \times \frac{V_s(2p_0)}{s+1} \sum_{i=1}^n \left\| [H_i(z)]^{s+1} \exp\left(\frac{B}{j} G_1(z)\right) \right\| \Big) \\ &\leq \frac{2 c_2(s, p_0) \beta_s(c_1(s) B^{-1})}{[s/2 - 1]!} \sum_{j=1}^n j^{s/2-1} \left[c_3(s, p_0) p_0^{s-1} \beta_1(2^{-3/2} B^{-1}) \right]^{j-1}. \end{aligned}$$

For the latter inequality, we used (4) and Stirling's formula. In view of (5) and the relations

$$\sum_{j=1}^\infty j^\nu x^{j-1} \leq \frac{d^\nu}{dx^\nu} \sum_{j=1}^\infty x^{j+\nu-1} = \frac{d^\nu}{dx^\nu} \sum_{j=0}^\infty x^j = \frac{\nu!}{(1-x)^{\nu+1}}$$

for $x \in [0, 1)$ and $\nu \in \mathbf{Z}_+$, we see that (2) is valid. The proof is completed.

Proof of Proposition 1. Using the binomial and the polynomial theorems we obtain

$$\begin{aligned}
 [\Psi_{\mathcal{P}_s}(z)]^{-1} \sum_{l \in \mathbf{Z}_+^k} |l| \mathcal{P}_s(\{l\}) z^l &= [\Psi_{\mathcal{P}_s}(z)]^{-1} \sum_{r=1}^k z_r \frac{\partial}{\partial z_r} \Psi_{\mathcal{P}_s}(z) \\
 &= \sum_{r=1}^k z_r \sum_{m=1}^s \frac{(-1)^{m+1}}{m} \sum_{j=1}^n \frac{\partial}{\partial z_r} ([H_j(z)]^m) = \sum_{j=1}^n \left(\sum_{r=1}^k p_{j,r} z_r \right) \sum_{m=0}^{s-1} \left(p_j - \sum_{r=1}^k p_{j,r} z_r \right)^m \\
 &= - \sum_{j=1}^n \sum_{m=0}^{s-1} \sum_{i=0}^m \binom{m}{i} p_j^{m-i} \left(- \sum_{r=1}^k p_{j,r} z_r \right)^{i+1} \\
 &= - \sum_{j=1}^n \sum_{i=0}^{s-1} \sum_{l \in \mathbf{Z}_+^k: |l|=i+1} \sum_{m=i}^{s-1} \binom{m}{i} \frac{|l|!}{l!} p_j^{m-i} \left(\prod_{r=1}^k (-p_{j,r})^{l_r} \right) z^l \\
 &= \sum_{l \in \mathbf{Z}_+^k: 1 \leq |l| \leq s} \left[\frac{(-1)^{|l|+1} |l|!}{l!} \sum_{j=1}^n \left(\sum_{m=|l|-1}^{s-1} \binom{m}{|l|-1} p_j^{m-|l|+1} \right) \prod_{r=1}^k p_{j,r}^{l_r} \right] z^l \\
 &= \sum_{l \in \mathbf{Z}_+^k: 1 \leq |l| \leq s} b_l z^l.
 \end{aligned}$$

Therefore

$$\sum_{l \in \mathbf{Z}_+^k} |l| \mathcal{P}_s(\{l\}) z^l = \Psi_{\mathcal{P}_s}(z) \sum_{l \in \mathbf{Z}_+^k: 1 \leq |l| \leq s} b_l z^l = \sum_{l \in \mathbf{Z}_+^k: |l| \geq 1} \sum_{t \in M_{l,s}} \mathcal{P}_s(\{l-t\}) b_t z^l.$$

Comparing the power series, the assertion is shown.

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TEMIDO M. G.*, CANTO E CASTRO L.**

MAX-SEMISTABLE LAWS IN EXTREMES OF STATIONARY RANDOM SEQUENCES¹⁾

Рассматриваются стационарные последовательности, подчиняющиеся некоторому обобщению условия Лидбеттера $D(u_n)$. Для таких последовательностей доказывается, что если $\{k_n\}$ — неубывающая последовательность целых чисел и $\lim_{n \rightarrow +\infty} k_{n+1}/k_n = r \geq 1$, то предельным законом для максимума первых k_n величин является мах-полуустойчивый закон. Это утверждение обобщает соответствующий результат для последовательностей независимых одинаково распределенных случайных величин [3] и теорему Лидбеттера об экстремальных типах [4]. Мы доказываем также, что предельное поведение этого максимума можно вывести из предельного поведения соответствующего максимума ассоциированной независимой последовательности, и обобщаем хорошо известное понятие экстремального индекса.

Ключевые слова и фразы: максимум, слабая сходимость, стационарность, мах-полуустойчивые законы.

1. Introduction. Max-semistable laws arise in the theory of extremes as limit in distribution of the maximum, suitably normalized, of k_n independent and identically distributed (i.i.d.) random variables, where $\{k_n\}$ is a nondecreasing integer sequence satisfying

$$\lim_{n \rightarrow +\infty} \frac{k_{n+1}}{k_n} = r, \quad r \in [1, +\infty[. \quad (1)$$

Following [2] we will say that a distribution function (d.f.) G on $\mathbf{R}$ is ℓ -max-semistable if there are reals $r > 1$, $\gamma > 0$ and β such that

$$G(x) = G^r\left(\frac{x}{\gamma} + \beta\right), \quad x \in \mathbf{R}, \quad (2)$$

or equivalently, if there are a sequence of i.i.d. random variables with d.f. F and two real sequences $\{a_n > 0\}$ and $\{b_n\}$ for which

$$\lim_{n \rightarrow +\infty} F^{k_n}\left(\frac{x}{a_n} + b_n\right) = G(x), \quad (3)$$

for each continuity point of G , with $\{k_n\}$ satisfying (1) — where ℓ stands for *linear* normalization. The general definition of max-semistable law is given in [7].

* Departamento de Matemática, Universidade de Coimbra, Portugal; e-mail: mgtm@mat.uc.pt

** D.E.I.O., Universidade de Lisboa, Portugal.

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