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теоремы 1 и 2 в стационарном случае улучшают соответствующие результаты работы [5].

Доказательство теорем 1 и 2 аналогично доказательству в [5], но уже с применением других предельных теорем для гильбертовозначных с.в., аннотированных в [8].

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СПИСОК ЛИТЕРАТУРЫ

1. *Hoeffding W.* A non-parametric test of independence. — *Ann. Math. Statist.*, 1948, v. 19, p. 546–557.
2. *Dehling H., Denker M., Philipp W.* Invariance principles for von Mises and U -statistics. — *Z. Wahrscheinlichkeitstheor. verw. Geb.*, 1984, v. 67, p. 139–167.
3. *Dehling H.* The functional law of the iterated logarithm for von Mises functionals and multiple Wiener integrals. — *J. Multivariate Anal.*, 1989, v. 28, № 2, p. 177–189.
4. *Yoshihara K.* Weakly Dependent Stochastic Sequences and Their Applications. V. 2: Asymptotic Statistics Based on Weakly Dependent Data. Tokyo: Sanseido, 1993, 341 p.
5. *Kanagawa S., Yoshihara K.* The almost sure invariance principles of degenerate U -statistics of degree two for stationary random variables. — *Stochastic Process. Appl.*, 1994, v. 49, № 2, p. 347–356.
6. *Королюк В. С., Боровских Ю. В.* Асимптотический анализ распределений статистик. Киев: Наукова думка, 1984, 304 с.
7. *Dehling H.* Limit theorems for sums of weakly dependent Banach space valued random variables. — *Z. Wahrscheinlichkeitstheor. verw. Geb.*, 1983, v. 63, p. 393–432.
8. *Sharipov O. Sh.* Strong limit theorems for φ -mixing random variables with values in Hilbert space. — *The Seventh Vilnius conference on Probability Theory and Mathematical Statistics: Abstracts of communications.* Vilnius, 1998, p. 411.

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PENG Z.*, NADARAJAH S.**

ON THE JOINT LIMITING DISTRIBUTION OF SUMS AND MAXIMA OF STATIONARY NORMAL SEQUENCE

Пусть $X_1, X_2, \dots$ — стационарная последовательность стандартных нормально распределенных случайных величин и $\rho_n = \mathbf{E}(X_1 X_{n+1})$. В [5] найдено совместное предельное распределение для $\sum_{i=1}^n X_i$ и $\max_{1 \leq i \leq n} X_i$ в случае, когда $\rho_n \ln n \rightarrow \gamma \in [0, \infty)$. Мы распространяем этот результат на случай, когда ρ_n выпукла и $\rho_n = o(1)$, а $(\rho_n \ln n)^{-1}$ монотонна и $(\rho_n \ln n)^{-1} = o(1)$.

Ключевые слова и фразы: предельное распределение, максимумы, стационарная нормальная последовательность, сумма.

* Southwest Normal University, P. R. China.

** Department of Statistics and Applied Probability, University of California, Santa Barbara, California 93106-3110, U.S.A.; e-mail: snadar@pstat.ucsb.edu

1. Introduction. Since late 1970s there has been work to characterize the joint asymptotic distribution of sums and maxima of a sequence of random variables. Chow and Teugels [3] and Mori [10] studied the joint limiting behaviour of sums and maxima for independent stationary sequences. Anderson and Turkman [1] derived the joint limiting distribution of suitably normalized sums and maxima in a stationary strong mixing sequence. McCormick and Sun [8] studied the joint limiting behaviour of sums and maxima of stationary discrete-valued processes. Hsing [6] established asymptotic independence of sums and maxima of weakly dependent stationary random variables. Gomes and Oliveira [4] developed limit laws for sums and maxima of a sequence of independent exponentials.

In this paper we are concerned with a remarkable result by Ho and Hsing [5] that gives the joint limiting distribution of sums and maxima of stationary normal sequence. They showed that if the sequence is weakly dependent then the limit distribution is that of independent normal and extreme value variables, and if dependence is slightly stronger then the limit involves a mixture of extreme value distributions with normally varying location parameter. The aim of this paper is to consider the case when dependence is even stronger.

A good statistical motivation for the study of joint limiting distributions of sums and maxima comes from the analysis of environmental extremes — see the papers by Anderson and Turkman [1], [2] for details. Often environmental processes are stationary and strongly dependent, e.g. river flow data with long memory. Hence the need arises to establish the limiting distribution for the case of strong dependence.

Let $X_1, X_2, \dots$ be a stationary sequence of standard normal random variables. Let

$$\rho_n = \mathbf{E}(X_1X_{n+1}), \quad M_n = \max_{1 \leq i \leq n} X_i \quad \text{and} \quad S_n = \sum_{i=1}^n X_i.$$

A formal statement of Ho and Hsing’s result is as follows. If $\rho_n \ln n \rightarrow \gamma \in [0, \infty)$ as $n \rightarrow \infty$ then

$$\begin{aligned} \lim_{n \rightarrow \infty} \mathbf{P} \left\{ a_n(M_n - b_n) \leq y, \frac{S_n}{\sqrt{\mathbf{D}S_n}} \leq x \right\} \\ = \int_{-\infty}^x \exp \left(-\exp \left(-y - \gamma + \sqrt{2\gamma} z \right) \right) \varphi(z) dz \end{aligned} \tag{1}$$

for all $x \in \mathbf{R}, y \in \mathbf{R}$, where

$$a_n = \sqrt{2 \ln n}, \quad b_n = a_n - (2a_n)^{-1} (\ln \ln n + \ln 4\pi), \tag{2}$$

and φ is the standard normal density function. When $\gamma = 0$ the right-hand side of (1) factorizes and so M_n and S_n are asymptotically independent. In this paper we are concerned with the case where ρ_n is convex with $\rho_n = o(1)$ and $(\rho_n \ln n)^{-1}$ is monotone with $(\rho_n \ln n)^{-1} = o(1)$. Another technical motivation for considering this case comes from the following result due to Mittal and Ylvisaker [9]:

$$\lim_{n \rightarrow \infty} \mathbf{P} \left\{ \frac{M_n - (1 - \rho_n)^{1/2} b_n}{\rho_n^{1/2}} \leq y \right\} = \Phi(y) \tag{3}$$

for all $y \in \mathbf{R}$, where Φ is the standard normal distribution function. This result shows that the limiting distribution of the maximum, M_n , can be different from that of an extreme value variable under the conditions on ρ_n . The result that we provide below extends this result by considering the joint limiting distribution of M_n and S_n under the conditions.

2. Results.

Theorem 1. *Let $X_1, X_2, \dots$ be a stationary sequence of standard normal random variables. Let $\rho_n = \mathbf{E}(X_1X_{n+1})$, $M_n = \max_{1 \leq i \leq n} X_i$ and $S_n = \sum_{i=1}^n X_i$. Suppose that ρ_n is convex with $\rho_n = o(1)$ and that $(\rho_n \ln n)^{-1}$ is monotone with $(\rho_n \ln n)^{-1} = o(1)$. Then*

$$\lim_{n \rightarrow \infty} \mathbf{P} \left\{ \frac{M_n - (1 - \rho_n)^{1/2} b_n}{\rho_n^{1/2}} \leq y, \frac{S_n}{\sqrt{\mathbf{D}S_n}} \leq x \right\} = \Phi(\min(x, y)) \tag{4}$$

for all $x \in \mathbf{R}, y \in \mathbf{R}$, where b_n is as given by (2).

A proof of this theorem is sketched in the appendix.

Letting $x \rightarrow \infty$ in (4), we obtain Mittal and Ylvisaker's result in (3) as a special case.

Appendix: Proof of Theorem 1. We need two lemmas.

Lemma 1. *If ρ_n satisfies the conditions in Theorem 1 then there is a stationary sequence of standard normal random variables, $\{Y_i\}$, with*

$$\mathbf{E}(Y_1 Y_{i+1}) = \begin{cases} \frac{\rho_i - \rho_n}{1 - \rho_n}, & \text{for } i = 0, \dots, n-1, \\ 0, & \text{for } i \geq n. \end{cases} \quad (5)$$

Moreover, if $M'_n = \max_{1 \leq i \leq n} Y_i$, then for all $\varepsilon > 0$

$$\lim_{n \rightarrow \infty} \mathbf{P} \left\{ |M'_n - b_n| > \varepsilon \rho_n^{1/2} \right\} = 0,$$

where b_n is as given by (2).

P r o o f. Apply Polya's criterion to see (5). $\mathbf{P}\{M'_n \geq b_n - \varepsilon \rho_n^{1/2}\} \rightarrow 0$ follows by Lemma 6.6.2 in [7]. $\mathbf{P}\{M'_n \leq b_n - \varepsilon \rho_n^{1/2}\} \rightarrow 0$ follows by Theorem 2.4 in [9].

Lemma 2. *Let $X_1, X_2, \dots$ be a stationary sequence of standard normal random variables. Let $\rho_n = \mathbf{E}(X_1 X_{n+1})$ and $S_n = \sum_{i=1}^n X_i$. If ρ_n satisfies the conditions in Theorem 1 then $\mathbf{D}S_n \sim n^2 \rho_n$ as $n \rightarrow \infty$.*

P r o o f. By definition,

$$\mathbf{D}S_n = n + 2 \sum_{i=1}^n (n-i) \rho_i = 2 \sum_{i=1}^n i \rho_{n-i} - n. \quad (6)$$

Rewrite

$$\sum_{i=1}^n i \rho_{n-i} = \sum_{i=1}^{[n^{\varepsilon_n}]} i \rho_{n-i} + \sum_{i=[n^{\varepsilon_n}]+1}^{n-[n^{\varepsilon_n}]} i \rho_{n-i} + \sum_{i=n-[n^{\varepsilon_n}]+1}^n i \rho_{n-i} = I + II + III, \quad (7)$$

where $\varepsilon_n \in (0, 1)$ are chosen to satisfy $\varepsilon_n \rightarrow 1$ and $n^{\varepsilon_n-1} \ln n = O(1)$ as $n \rightarrow \infty$. Since $\rho_n \downarrow 0$ and $\rho_n \ln n \uparrow \infty$,

$$1 \leq \frac{\rho_i}{\rho_n} \leq \frac{1}{\varepsilon_n} \frac{\rho_i \ln i}{\rho_n \ln n} \leq \frac{1}{\varepsilon_n} \rightarrow 1, \quad \forall [n^{\varepsilon_n}] \leq i \leq n,$$

which implies uniform convergence of $\rho_i/\rho_n \rightarrow 1$, $\forall i \geq [n^{\varepsilon_n}]$. Using this result and the assumed properties of ρ_n and ε_n , we have $I = o(n^2 \rho_n)$, $III = o(n^2 \rho_n)$ and $II \sim \frac{1}{2} n^2 \rho_n$ as $n \rightarrow \infty$. Substituting into (6) and (7), we obtain $\mathbf{D}S_n \sim n^2 \rho_n$ as $n \rightarrow \infty$.

P r o o f o f T h e o r e m 1. Take $\{Y_i\}$ as defined in Lemma 1. Let Y be a standard normal random variable independent of $\{Y_i\}$. Then we can represent X_i by $(1 - \rho_n)^{1/2} Y_i + \rho_n^{1/2} Y$ for $i = 1, \dots, n$. Thus, if $M'_n = \max_{1 \leq i \leq n} Y_i$ and $S'_n = \sum_{i=1}^n Y_i$,

$$\begin{aligned} & \mathbf{P} \left\{ \frac{M_n - (1 - \rho_n)^{1/2} b_n}{\rho_n^{1/2}} \leq y, \frac{S_n}{\sqrt{\mathbf{D}S_n}} \leq x \right\} \\ &= \mathbf{P} \left\{ M'_n - b_n \leq \frac{\rho_n^{1/2} (y - Y)}{(1 - \rho_n)^{1/2}}, \frac{S'_n}{\sqrt{\mathbf{D}S_n}} \leq \frac{\sqrt{\mathbf{D}S_n} x - n \rho_n^{1/2} Y}{\sqrt{\mathbf{D}S_n - n^2 \rho_n}} \right\} \\ &= \int_{-\infty}^{\infty} \mathbf{P}(A_n(y, z) \cap B_n(x, z)) \varphi(z) dz, \end{aligned} \quad (8)$$

where $A_n(y, z) = \{M'_n - b_n \leq (1 - \rho_n)^{-1/2} \rho_n^{1/2} (y - z)\}$, $B_n(x, z) = \{S'_n / \sqrt{\mathbf{D}S'_n} \leq \sqrt{\mathbf{D}S_n} x - n \rho_n^{1/2} z / \sqrt{\mathbf{D}S_n - n^2 \rho_n}\}$. By Lemma 1 and since $\rho_n \downarrow 0$,

$$\lim_{n \rightarrow \infty} \mathbf{P}(A_n(y, z)) = \begin{cases} 1 & \text{if } y > z, \\ 0 & \text{if } y < z. \end{cases}$$

By Lemma 2,

$$\lim_{n \rightarrow \infty} \mathbf{P}(B_n(x, z)) = \begin{cases} 1 & \text{if } x > z, \\ 0 & \text{if } x < z. \end{cases}$$

Substituting into (8), we obtain the required limit as $n \rightarrow \infty$.

REFERENCES

1. *Anderson C. W., Turkman K. F.* The joint limiting distribution of sums and maxima of stationary sequences. — J. Appl. Probab., 1991, v. 28, p. 33–44.
2. *Андерсон К. У., Туркман К. Ф.* Предельные совместные распределения сумм и максимумов в статистическом контексте. — Теория вероятн. и ее примен., 1992, т. 37, в. 2, с. 352–355.
3. *Chow T. L., Teugels J. L.* The sum and the maximum of i.i.d. random variables. — Proceedings of the Second Prague Symposium on Asymptotic Statistics, 1978, 1979, p. 81–92.
4. *Gomes J., Oliveira O.* Limit laws for a sequence between the maximum and the sum of independent exponentials. — Statist. Probab. Lett., 1997, v. 35, № 1, p. 25–32.
5. *Ho H. C., Hsing T.* On the asymptotic joint distribution of the sum and maximum of stationary normal random variables. — J. Appl. Probab., 1996, v. 33, № 1, p. 138–145.
6. *Hsing T.* On the asymptotic independence of the sum and rare values of weakly dependent stationary random variables. — Stochastic Process. Appl., 1995, v. 60, № 1, p. 49–63.
7. *Leadbetter M. R., Lindgren G., Rootzén H.* Extremes and Related Properties of Random Sequences and Processes. New York: Springer-Verlag, 1983, 336 p.
8. *McCormick W. P., Sun J.* Sums and maxima of discrete stationary processes. — J. Appl. Probab., 1993, v. 20, № 4, p. 863–876.
9. *Mittal Y., Ylvisaker D.* Limit distributions for the maxima of stationary Gaussian processes. — Stochastic Process. Appl., 1975, v. 3, p. 1–18.
10. *Mori T.* The relation of sums and extremes of random variables. — Proceedings of the 43rd Session of the ISI, Buenos Aires, 1981, V. II. Bull. Inst. Internat. Statist., v. 49, № 2, p. 879–894.

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