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ON L_2 EFFICIENCY OF EMPIRIC DISTRIBUTION FOR ERGODIC DIFFUSION PROCESSES

Рассматривается задача асимптотически эффективного непараметрического оценивания инвариантного распределения диффузионного процесса. Предложен элементарный вывод минимаксной границы на риск интегрального типа произвольной оценки и затем показана асимптотическая эффективность эмпирической функции распределения.

Ключевые слова и фразы: эргодический диффузионный процесс, нижняя граница, непараметрическое оценивание, асимптотическая эффективность.

1. Introduction. We consider the problem of the estimation of the distribution function $F(\cdot)$, $x \in \mathbf{R}$, by the observation of a diffusion process $\{X_t: 0 \leq t \leq T\}$. We suppose that the process $\{X_t, t \geq 0\}$ possesses the ergodic property with invariant measure μ and $F(x) = \mu\{(-\infty, x]\}$. A natural estimator for $F(x)$ is the empirical distribution function $\hat{F}(x)$, $x \in \mathbf{R}$. It is known that this estimator is uniformly consistent by the Glivenko–Cantelli theorem, and asymptotically efficient in different metrics (see [5] and [9]). Here we will show that it is also asymptotically efficient in a sense that we are going to define. To do this we construct a local asymptotic minimax lower bound for the risk of any estimator of the invariant distribution function and we show that the empirical distribution function attains this bound. The risk considered here is the integrated mean square error. Our statement of the problem can be called semiparametric (see [1]). Indeed although the underlying model is a nonparametric one (due to the fact the trend coefficient is supposed to be an unknown function) and considering the integrated mean square error as risk we measure the difference between the estimator and the *true value* F in all the points, where they are defined, we have a $\sqrt{T}$ rate of convergence, which is typical for semiparametric model. For the model of independent and identically distributed random variables such kind of results have been established earlier by Levit [6] and Millar [8], using the theory of local asymptotic normality. Gill and Levit [3] obtained a similar result using a different approach based on a multidimensional version of the van Trees inequality. Further reference for the same kind of problem for other model can be found in [9]. The argument used here is essentially based on the reduction of our problem to a semiparametric one and then in applying the van Trees inequality to the obtained model. This technique has been applied also by Iacus [4] to the problem of state estimation of a diffusion process with small dispersion and can be applied in the same manner in the stationary density estimation problem. The assumptions we need here are less restrictive than the one we need to prove the efficiency of the empirical distribution function when we consider risk functions based on $\sup_{x \in \mathbf{R}} \sqrt{T} |\hat{F}_T(x) - F(x)|$ (see [9]) and this is essentially due to the structure of the integrated mean square error and to the fact that we don't need the weak convergence of the process in the space of real continuous function vanishing at infinity.

In the next section we present the statement of the problem and the assumptions. In Section 3 we present and prove the main result.

2. Preliminaries. Let us consider a one-dimensional strictly stationary diffusion process

$$dX_t = S(X_t)dt + \sigma(X_t)dW_t, \quad X_0, \quad 0 \leq t \leq T, \quad (1)$$

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where $\{W_t: t \geq 0\}$ is a standard Wiener process. The trend coefficient $S(\cdot)$ is unknown to the observer and the diffusion coefficient $\sigma^2(\cdot)$ is a known positive function. We suppose that the functions $S(\cdot)$ and $\sigma(\cdot)$ satisfy the Lipschitz condition

$$|S(x) - S(y)| + |\sigma(x) - \sigma(y)| \leq L|x - y|, \quad x, y \in \mathbf{R}, \quad (2)$$

so that (1) has a unique strong solution (see [7]). We suppose that

$$G(S) = \int_{-\infty}^{+\infty} \frac{1}{\sigma^2(x)} \exp \left\{ 2 \int_0^x \frac{S(v)}{\sigma^2(v)} dv \right\} dx < +\infty. \quad (3)$$

These conditions provide the existence of the invariant measure with stationary distribution function

$$F(x) = \frac{1}{G(S)} \int_{-\infty}^x \frac{1}{\sigma^2(y)} \exp \left\{ 2 \int_0^y \frac{S(v)}{\sigma^2(v)} dv \right\} dy.$$

Let us denote by $\mathcal{S}$ the class of Lipschitz functions $S(\cdot)$ satisfying condition (3). We suppose that the initial random variable X_0 has $F(\cdot)$ as distribution function, so the process $\{X_t, t \geq 0\}$ is stationary and possesses the ergodic property, that is, for every measurable function $g(\cdot)$ such that $\mathbf{E}|g(\xi)| < \infty$, where ξ is a stationary random variable with F as distribution function, we have

$$\mathbf{P} \left(\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T g(X_t) dt = \int_{\mathbf{R}} g(z) f(z) dz \right) = 1,$$

where $f(\cdot)$ is the probability density of stationary law (see [2]). We are interested in the estimation of $F(\cdot)$ by the observation $X^T = \{X_t: 0 \leq t \leq T\}$, solution of (1), when $S(\cdot)$ belongs to the class $\mathcal{S}$. By $\mathbf{P}_S^{(T)}$ we denote the law of the process X^T in the space $\mathcal{X} = \mathcal{C}(0, T)$ of continuous on $[0, T]$ functions. In the sequel if we want to emphasize the dependency of $f(\cdot)$, $F(\cdot)$ on S we shall put S as index; i.e., we write $f_S(\cdot)$ or $F_S(\cdot)$. Let $\bar{F}_T(x)$ be any estimator of $F_S(x)$, at the point $x \in \mathbf{R}$. We define the integrated mean square error as $\mathcal{R}_T(\bar{F}_T, F_S) = T \mathbf{E}_S (\bar{F}_T(x) - F_S(x))^2 \nu(dx)$, where ν is some finite measure on $\mathbf{R}$ and $\mathbf{E}_S$ is the mathematical expectation with respect to the measure $\mathbf{P}_S^{(T)}$. We will refer to it also as *global risk*.

A natural estimator of $F_S(x)$ is the empirical distribution function defined as follows $\hat{F}_T(x) = T^{-1} \int_0^T \chi_{\{X_t < x\}} dt$, where $\chi_{\{A\}}$ the indicator function of a set A . This estimator is uniformly consistent, asymptotically normal and asymptotically efficient in different metrics (see [5] and [9]).

3. Main result. We use the van Trees inequality to derive an asymptotic bound for an estimator in our nonparametric model; i.e., we consider the estimator of the stationary distribution function in its entirety and not pointwisely. We show that the empirical distribution function achieves a local asymptotic minimax lower bound for the integrated mean square error of an arbitrary estimator. The argument used here is essentially to reduce our model to a semiparametric one. As usual in such situations we consider a nonparametric vicinity of a fixed model and then we construct a parametric family of ergodic processes that belongs to this nonparametric vicinity. Then we apply the multidimensional van Trees inequality to this parametric model and we obtain our lower bound.

Let us fix a function $S_0 \in \mathcal{S}$ and, for some $\delta > 0$, introduce the set

$$V_\delta(S_0) = \left\{ S \in \mathcal{S}: \sup_{x \in \mathbf{R}} |S_0(x) - S(x)| < \delta \right\}.$$

We use below the following notation: $x \wedge y = \min(x, y)$ and $x \vee y = \max(x, y)$. Let us introduce the functions

$$r_S(x, y) = \frac{F_S(x \wedge y) - F_S(x) F_S(y)}{\sigma(y) f_S(y)}$$

and

$$G_{x,S}(y) = 2 \int_0^y \frac{F_S(v \wedge x) - F_S(v) F_S(x)}{\sigma^2(v) f_S(v)} dv.$$

We pose

$$\mathbf{E}_S r^2(x, \xi) = R_S(x, x) = 4\mathbf{E}_S \left(\frac{F_S(x \wedge \xi) [1 - F_S(x \vee \xi)]}{\sigma(\xi) f_S(\xi)} \right)^2$$

and we introduce the following conditions

$$\sup_{S \in V_\delta(S_0)} \int_{\mathbf{R}} R_S(x, x) \nu(dx) < +\infty; \quad (4)$$

$$\sup_{S \in V_\delta(S_0)} \int_{\mathbf{R}} \mathbf{E}_S (G_{x,S}(\xi))^2 \nu(dx) < +\infty. \quad (5)$$

Now we are able to state our main result.

Theorem. Let $S_0 \in \mathcal{S}$ and let $R_S(x, x)$ be continuous at the point S_0 with respect to the topology generated by the open sets V_δ . If conditions (4) and (5) are satisfied then

$$\lim_{\delta \rightarrow 0} \lim_{T \rightarrow \infty} \inf_{\bar{F}_T} \sup_{S \in V_\delta(S_0)} T \mathbf{E}_S \int_{\mathbf{R}} (\bar{F}_T(x) - F_S(x))^2 \nu(dx) = \int_{\mathbf{R}} R_{S_0}(x, x) \nu(dx), \quad (6)$$

where the inf is taken over all the estimators $\bar{F}_T(\cdot)$ of $F_S(\cdot)$.

P r o o f. Let us denote $\eta_T(x) = \sqrt{T}(\hat{F}_T(x) - F_S(x))$. The global risk for the empiric distribution function can be rewritten as

$$\mathcal{R}_T(\hat{F}_T, F_S) = T \mathbf{E}_S \int_{\mathbf{R}} (\bar{F}_T(x) - F_S(x))^2 \nu(dx) = \int_{\mathbf{R}} \mathbf{E}_S [\eta_T(x)]^2 \nu(dx).$$

Set $\rho_0 = \int_{\mathbf{R}} R_{S_0}(x, x) \nu(dx)$. First we prove that

$$\lim_{\delta \rightarrow 0} \lim_{T \rightarrow \infty} \sup_{S \in V_\delta(S_0)} \int_{\mathbf{R}} \mathbf{E}_S [\eta_T(x)]^2 \nu(dx) = \rho_0,$$

which will provide us the upper bound in (6). By the Itô formula we can write

$$\begin{aligned} \eta_T(x) &= \frac{1}{\sqrt{T}} \int_0^T (\chi_{\{X_t < x\}} - F_S(x)) dt \\ &= \frac{G_{x,S}(X_T) - G_{x,S}(X_0)}{\sqrt{T}} - \frac{2}{\sqrt{T}} \int_0^T r_S(x, X_t) dW_t. \end{aligned}$$

Taking the square and the expectation we obtain

$$\begin{aligned} T \mathbf{E}_S [\eta_T(x)]^2 &= \frac{1}{T} \mathbf{E}_S (G_{x,S}(X_T) - G_{x,S}(X_0))^2 \\ &\quad - \frac{2}{T} \mathbf{E}_S \left((G_{x,S}(X_T) - G_{x,S}(X_0)) \int_0^T r_S(x, X_t) dW_t \right) \\ &\quad + \frac{4}{T} \mathbf{E}_S \left(\int_0^T r_S(x, X_t) dW_t \right)^2. \end{aligned}$$

By condition (5) we have

$$\lim_{T \rightarrow +\infty} \sup_{S \in V_\delta(S_0)} \frac{1}{T} \int_{\mathbf{R}} \mathbf{E}_S (G_{x,S}(X_T) - G_{x,S}(X_0))^2 \nu(dx) = 0.$$

Applying the Cauchy-Schwarz inequality we can write

$$\begin{aligned} &\int_{\mathbf{R}} \mathbf{E}_S \left((G_{x,S}(X_T) - G_{x,S}(X_0)) \int_0^T r_S(x, X_t) dW_t \right) \nu(dx) \\ &\leq \left(\int_{\mathbf{R}} \mathbf{E}_S (G_{x,S}(X_T) - G_{x,S}(X_0))^2 \nu(dx) \right)^{1/2} \left(\int_{\mathbf{R}} T \mathbf{E}_S [r_S(x, \xi)]^2 \nu(dx) \right)^{1/2}. \end{aligned}$$

Therefore

$$\begin{aligned} &\lim_{T \rightarrow +\infty} \sup_{S \in V_\delta(S_0)} \frac{1}{T} \left(\int_{\mathbf{R}} \mathbf{E}_S (G_{x,S}(X_T) - G_{x,S}(X_0))^2 \nu(dx) \right)^{1/2} \\ &\quad \times \left(\int_{\mathbf{R}} T \mathbf{E}_S (r_S(x, \xi))^2 \nu(dx) \right)^{1/2} = 0. \end{aligned}$$

At last we have

$$\lim_{T \rightarrow +\infty} \sup_{S \in V_\delta(S_0)} \frac{1}{T} \int_{\mathbf{R}} \left(\int_0^T \mathbf{E}_S (r_S(x, X_t))^2 dt \right) \nu(dx) = \sup_{S \in V_\delta(S_0)} \int_{\mathbf{R}} R_S(x, x) \nu(dx).$$

All this allow us to conclude that

$$\lim_{T \rightarrow +\infty} \sup_{S \in V_\delta(S_0)} \mathcal{R}_T(\widehat{F}_T, F_S) = \sup_{S \in V_\delta(S_0)} \int_{\mathbf{R}} R_S(x, x) \nu(dx).$$

Further, by the continuity of $R_S(x, x)$ at the point S_0 , we have

$$\lim_{\delta \rightarrow 0} \lim_{T \rightarrow \infty} \sup_{S \in V_\delta(S_0)} \mathcal{R}_T(\widehat{F}_T, F_S) = \int_{\mathbf{R}} R_{S_0}(x, x) \nu(dx) = \rho_0.$$

So ρ_0 is certainly an upper bound to the left-hand side of (6). Now we will prove that

$$\lim_{\delta \rightarrow 0} \lim_{T \rightarrow \infty} \inf_{\widehat{F}_T} \sup_{S \in V_\delta(S_0)} \mathcal{R}_T(\widehat{F}_T, F_S) \geq \rho_0.$$

Our proof is motivated by the corresponding proof of the efficiency of the empiric distribution in i.i.d. case by Gill and Levit [3]. Introduce a parametric family of diffusion processes

$$dX_t = \left(S_0(X_t) + \sum_{i=1}^k \theta_i \psi_i(X_t) \sigma^2(X_t) \right) dt + \sigma(X_t) dW_t, \quad X_0^\theta, \quad 0 \leq t \leq T, \quad (7)$$

where $\psi_i(\cdot)$, $i = 1, \dots, k$, are continuously differentiable functions with compact support, parameter $\theta = (\theta_1, \dots, \theta_k) \in \Theta_\delta = [-\epsilon, \epsilon]^k$, $\epsilon = \epsilon(\delta)$, is such that $S(\theta, \cdot) = S_0(\cdot) + \sum_{i=1}^k \theta_i \psi_i(\cdot) \sigma^2(\cdot) \in V_\delta$. The stationary density function is

$$f(\theta, x) = \frac{1}{G(S(\theta)) \sigma^2(x)} \exp \left\{ 2 \int_0^x \frac{S_0(v)}{\sigma^2(v)} dv + 2 \sum_{i=1}^k \theta_i \int_0^x \psi_i(v) dv \right\},$$

and the initial value X_0^θ has this density function. Therefore the process $\{X_t: t \geq 0\}$ is ergodic. We denote by $\mathbf{P}_\theta^{(T)}$ and $\mathbf{E}_\theta^{(T)}$ the probability measure induced by the process (7) and expectation with respect to this measure respectively. Suppose that θ is a random vector with independent components and independent of the Wiener process $\{W_t: t \geq 0\}$ with independent components and its density function $\lambda(\theta) = \prod_{i=1}^k \lambda_i(\theta_i)$ is such that the Fisher information matrix $\Pi(\lambda)$ satisfy

$$\text{Tr } \Pi(\lambda) = \sum_{i=1}^k \Pi_{ii}(\lambda) = \sum_{i=1}^k \int_{-\epsilon}^{\epsilon} \left(\frac{\partial \lambda_i(\theta_i)}{\partial \theta_i} \right)^2 [\lambda_i(\theta_i)]^{-1} d\theta_i < \infty$$

and $\lambda_i(\pm\epsilon) = 0$, then for any absolutely continuous vector-function $\Psi(S(\theta)) = (\Psi_1(S(\theta)), \dots, \Psi_k(S(\theta)))$ and any estimator $\overline{\Psi}_T = (\overline{\Psi}_{T,1}, \dots, \overline{\Psi}_{T,k})$ of this function we have the (multidimensional, L^2 -norm type) van Trees inequality (see [3] in general case and [10] in the case of diffusion processes)

$$\mathbb{E} T \sum_{i=1}^k \left| \overline{\Psi}_{T,i} - \Psi_i(S(\theta)) \right|^2 \geq \frac{(\mathbb{E} \sum_{i=1}^k \partial \Psi_i(S(\theta)) / \partial \theta_i)^2}{\mathbb{E} \text{Tr } \Pi(\theta) + T^{-1} \text{Tr } \Pi(\lambda)}. \quad (8)$$

Here $\mathbb{E}$ is expectation with respect to measure $d\mathbb{P} = \lambda(\theta) d\theta \times d\mathbf{P}_\theta^{(T)}$ and the trace of the Fisher information matrix $\Pi(\theta)$ of our model (7) satisfy

$$\text{Tr } \Pi(\theta) = \sum_{i=1}^k \Pi_{ii}(\theta) = \sum_{i=1}^k \mathbf{E}_\theta^{(T)} \left(\psi_i(\xi) \sigma(\xi) \right)^2 (1 + o(1)) < \infty.$$

Let us introduce a complete orthonormal system of functions (base) $\{\varphi_i(\cdot), i = 1, 2, \dots\}$ in the space $L^2(\nu)$ and define the Fourier coefficients $\overline{\Phi}_T = (\overline{\Phi}_{T,1}, \dots, \overline{\Phi}_{T,k})$ and $\Phi(S) = (\Phi_1(S), \dots, \Phi_k(S))$ of the functions $\overline{F}_T(\cdot)$, $F_S(\cdot)$ as

$$\overline{\Phi}_{T,i} = \int_{\mathbf{R}} \overline{F}_T(x) \varphi_i(x) \nu(dx), \quad \Phi_i(S) = \int_{\mathbf{R}} F_S(x) \varphi_i(x) \nu(dx).$$

We have the obvious inequalities

$$\begin{aligned} \sup_{S(\cdot) \in V_\varepsilon} \mathcal{R}_T(\bar{F}_T, S) &\geq \sup_{\theta \in \Theta_\varepsilon} \mathcal{R}_T(\bar{F}_T, S(\theta)) \geq \int_{\Theta_\varepsilon} \mathcal{R}_T(\bar{F}_T, S(\theta)) \lambda(\theta) d\theta \\ &= T \mathbb{E} \sum_{i=1}^{\infty} \left| \bar{\Phi}_{T,i} - \Phi_i(S) \right|^2 \geq T \mathbb{E} \sum_{i=1}^k \left| \bar{\Phi}_{T,i} - \Phi_i(S) \right|^2, \end{aligned}$$

where we used the Parseval identity. We can now apply the van Trees inequality (8) to the last sum and obtain the estimate

$$T \mathbb{E} \sum_{i=1}^k \left| \bar{\Phi}_{T,i} - \Phi_i(S) \right|^2 \geq \frac{(\mathbb{E} \operatorname{div} \Phi(S(\theta)))^2}{\mathbb{E} \operatorname{Tr} \Pi(\theta) + T^{-1} \operatorname{Tr} \Pi(\lambda)}.$$

Direct calculation yields

$$\begin{aligned} \frac{\partial \Phi_i(S(\theta))}{\partial \theta_i} &= 2 \int_{\mathbf{R}} \int_{\mathbf{R}} \psi_i(y) \left((F_{S(\theta)}(x \vee y) - 1) F_{S(\theta)}(x \wedge y) \right) \varphi_i(x) \nu(dx) dy \\ &= 2 \int_{\mathbf{R}} \psi_i(y) \left(\int_{\mathbf{R}} \left((F_{S(\theta)}(x \vee y) - 1) F_{S(\theta)}(x \wedge y) \right) \varphi_i(x) \nu(dx) \right) dy. \end{aligned}$$

Choosing T sufficiently large and ε sufficiently small we can obtain the following estimate

$$\frac{(\mathbb{E} \operatorname{div} \Phi(S(\theta)))^2}{\mathbb{E} \operatorname{Tr} \Pi(\theta) + T^{-1} \operatorname{Tr} \Pi(\lambda)} = \frac{(\operatorname{div} \Phi(S_0))^2}{\sum_{i=1}^k \mathbb{E}_{S_0}(\psi_i(\xi) \sigma(\xi))^2} (1 + o(1)).$$

Let us introduce the functions

$$\psi_i^*(y) = \int_{\mathbf{R}} \frac{[F_{S_0}(x \vee y) - 1] F_{S_0}(x \wedge y)}{\sigma^2(y) f_{S_0}(y)} \varphi_i(x) \nu(dx), \quad i = 1, \dots, k.$$

We have $\mathbb{E}_{S_0}(\psi_i^*(\xi) \sigma(\xi))^2 \leq \int_{\mathbf{R}} R_{S_0}(x, x) \nu(dx) < \infty$. Therefore we can introduce a sequence $\psi_i^{(n)}(\cdot)$, $n = 1, 2, \dots$, $i = 1, \dots, k$, of continuously differentiable functions with compact supports and such that for all i

$$\lim_{n \rightarrow \infty} \mathbb{E}_{S_0} \left((\psi_i^{(n)}(\xi) - \psi_i^*(\xi)) \sigma(\xi) \right)^2 = 0.$$

Finally we put $\psi_i(\cdot) = \psi_i^{(n)}(\cdot)$; that is, the parametric family of diffusion processes (7) is constructed with such functions $\psi_i^{(n)}(\cdot)$. Then for the large values of n and k we have the desired estimate

$$\frac{(\operatorname{div} \Phi(S_0))^2}{\sum_{i=1}^k \mathbb{E}_{S_0}(\psi_i(\xi) \sigma(\xi))^2} = \int_{\mathbf{R}} R_{S_0}(x, x) \nu(dx) (1 + o(1)),$$

because by the Parseval identity $\mathbb{E}_{S_0} \sum_{i=1}^{\infty} (\psi_i(\xi) \sigma(\xi))^2 = \int_{\mathbf{R}} R_{S_0}(x, x) \nu(dx)$. And we have the result.

Example. Let us introduce the following class of functions satisfying (2)

$$\mathcal{S}(\gamma, M) = \left\{ S(\cdot): S(y) \operatorname{sign}(y) \leq -\gamma \text{ for } |y| > M \right\},$$

and suppose that the observed process is

$$dX_t = S(X_t) dt + \sigma(X_t) dW_t, \quad 0 \leq t \leq T, \quad X_0.$$

If the function $S(\cdot) \in \mathcal{S}(\gamma, M)$ and there exist two constants k_1 and k_2 such that $0 < k_1 < |\sigma(x)| < k_2 < +\infty$ for all $x \in \mathbf{R}$, then the conditions (4) and (5) are fulfilled and the continuity of $R_S(x, x)$ in the uniform topology can also be verified. So we have a class of processes for which the empirical distribution function is an efficient (in the sense of theorem) estimator for the invariant distribution function.

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ON COUPLING OF BROWNIAN BRIDGES

Пусть $\{B(t), 0 \leq t \leq 1\}$ — броуновский мост и $Y(t) = \int_0^t f(u) dB(u)$, где $f: [0, 1] \rightarrow \{+1, -1\}$, — неслучайная измеримая функция. Тогда задача: «Существует ли броуновский мост B^* такой, что $|Y(t)| \geq |B^*(t)|$ п.н., $0 \leq t \leq 1$?» имеет положительное решение. В статье доказывается, что в качестве B^* можно взять

$$B^*(t) = \begin{cases} Y(t), & 0 \leq t \leq \tau, \\ B(t), & \tau \leq t \leq 1, Y(\tau) = +B(\tau), \\ -B(t), & \tau \leq t \leq 1, Y(\tau) = -B(\tau), \end{cases}$$

где $\tau = \max\{t \geq 0: |Y(t)| = |B(t)|\}$.

Обсуждается также положительный ответ на вопрос о существовании броуновского моста B_* такого, что $\max_{0 \leq t \leq 1} |B_*(t)| = \max_{0 \leq t \leq 1} \{|X_+(t)| \vee |X_-(t)|\}$, где $X_+(t) = \int_0^t 1_{\{f=+1\}}(u) dB(u)$, $X_-(t) = \int_0^t 1_{\{f=-1\}}(u) dB(u)$, $0 \leq t \leq 1$.

В качестве следствия этих построений мы получаем строгое неравенство, сравнивающее распределения величин $\max_{0 \leq t \leq 1} |B(t)|$ и $\max_{0 \leq t \leq 1} |Y(t)|$.

Ключевые слова и фразы: броуновский мост, каплинг (coupling), перестановочные случайные величины.

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