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РОССИЙСКО-ЯПОНСКИЙ СИМПОЗИУМ
«СТОХАСТИЧЕСКИЙ АНАЛИЗ СЛОЖНЫХ
СТАТИСТИЧЕСКИХ МОДЕЛЕЙ»

С 15 по 17 сентября 2009 г. в Математическом институте им. В. А. Стеклова РАН работал Российско-японский симпозиум «Стохастический анализ сложных статистических моделей». Симпозиум был организован в рамках совместного Российско-японского проекта РФФИ и JSPS «Сложные стохастические модели: асимптотика и применения» (соруководители А. Н. Ширяев и Х. Вакаки). В течение трех дней участники Симпозиума заслушали 10 сорокаминутных докладов и 17 двадцатиминутных сообщений. Ниже публикуются тезисы докладов и сообщений участников симпозиума.

В. В. Ульянов, А. Н. Ширяев

Тезисы докладов японских участников

Fujita T. (Graduate School of Commerce and Management, Hitotsubashi University; *e-mail*: fujita@math.hit-u.ac.jp). **On restart options.**

Definition of restart option. Let us consider a path-dependent option with the maturity T . We denote by S_t the stock price at time t . Assume there exists a finite or infinite sequence of random times (usually stopping times) $T_1 < T_2 < \dots < T_i < \dots$. If T_i happens before T , then the payoff of the option is reset (kill the past) and restarts from T_i , we call this option *restart option*. We call T_i ($i = 1, 2, \dots$) *restart times*.

We see that «restart options» are more mild than existing exotic options like barrier options or lookback options.

Example 1 (Knock out barrier option with recovery). Take $S_0 = S > A$, $B > A$, $A < K < B$, $\tau_A = \inf\{t: S_t = A\}$, $\sigma_{A,B} = \inf\{t > \tau_A: S_t = B\}$. Option is knocked out if it reaches A before the maturity T . But after τ_A before the maturity T , if the stock price S_t reaches B , then option recovers and restarts. Finally, the option buyer gets the payoff $f(S_T)$ (for example, in the call case, $f(S_T) = \max(S_T - K, 0)$). In this case we note that $\sigma_{A,B} = T_1$ is the restart time.

Remarks. (i) This option belongs to the so-called *Edokko option* [1], which saves options from price manipulation.

(ii) We can consider «Knock out barrier option with infinite recovery» that recovery procedure continues until the maturity T .

The price of «Knock out barrier option with recovery» in the Black–Scholes model is the following.

We consider the following Black–Scholes model under the risk neutral measure Q :

$$dS_t = rS_t dt + \sigma S_t dW_t, \quad S_0 = S,$$

where S_t is the stock value at time t , r is the risk free rate, and σ is the volatility.

We get

$$S_t = S \exp((r - \frac{1}{2}\sigma^2)t + \sigma W_t).$$

Then the risk neutral valuation for derivative with payoff Y_T at maturity time T gives C , the present value of derivative Y :

$$C = \mathbf{E}(e^{-rT} Y_T) \quad (= C(T, S)).$$

In the case of «Knock out barrier option with recovery», $Y = f(S_T)(1_{\tau_A > T} + 1_{\tau_A < T} 1_{\sigma_{A,B} < T})$. We take the independent exponential random variable $\theta \sim \text{Exp}(\lambda)$ with the density $f_\theta(x) = 1_{x>0} \lambda e^{-\lambda x}$ (see [2] and [3]). We put $\mathbf{E} C(\theta, S) = \mathbf{E}(e^{-r\theta} Y_\theta) = \widehat{C}(\lambda, S)$ and $\widehat{F}(\lambda, S) = \mathbf{E}(e^{-r\theta} f(S_\theta))$. Let us remark that in the call case,

$$\widehat{F} = \frac{\lambda \sigma}{\sqrt{2\lambda'}((\sqrt{2\lambda'} - r/\sigma)^2 - \sigma^2/4)} S^{2\lambda'/\sigma - r/\sigma^2 + 1/2} K^{-2\lambda'/\sigma + r/\sigma^2 + 1/2},$$

where $\lambda' = \lambda + (r/\sigma + \sigma/2)^2/2$.

Then by the lack-of-memory property of exponential distribution and the strong Markov property of S_t , we get

$$\begin{aligned} \widehat{C}(\lambda, S) &= \mathbf{E}(e^{-r\theta} f(S_\theta)[1_{\tau_A > \theta} + 1_{\tau_A < \theta} 1_{\sigma_{A,B} < \theta}]) \\ &= \frac{\lambda}{r + \lambda} \mathbf{E}(f(S_{\theta'})[1_{\tau_A > \theta'} + 1_{\tau_A < \theta'} 1_{\sigma_{A,B} < \theta'}]) \\ &= \frac{\lambda}{r + \lambda} \left(\widehat{F}(r + \lambda, S) - \mathbf{E} e^{-(r+\lambda)\xi} \cdot \widehat{F}(r + \lambda, A) \right. \\ &\quad \left. - \mathbf{E} e^{-(r+\lambda)\xi} \cdot \mathbf{E} e^{-(r+\lambda)\xi'} \cdot \widehat{F}(r + \lambda, B) \right) \\ &= \frac{\lambda}{r + \lambda} \left(\widehat{F}(r + \lambda, S) \right. \\ &\quad - \exp \left\{ \frac{1}{\sigma} \left(r - \frac{\sigma^2}{2} \right) \frac{1}{\sigma} \ln \frac{A}{S} + \frac{1}{\sigma} \ln \frac{A}{S} \sqrt{2(r + \lambda) + \left(\frac{r - \sigma^2/2}{\sigma} \right)^2} \right\} \widehat{F}(r + \lambda, A) \\ &\quad + \exp \left\{ \frac{1}{\sigma} \left(r - \frac{\sigma^2}{2} \right) \frac{1}{\sigma} \ln \frac{A}{S} + \frac{1}{\sigma} \ln \frac{A}{S} \sqrt{2(r + \lambda) + \left(\frac{r - \sigma^2/2}{\sigma} \right)^2} \right\} \\ &\quad \times \exp \left\{ \frac{1}{\sigma} \left(r - \frac{\sigma^2}{2} \right) \frac{1}{\sigma} \ln \frac{B}{A} - \frac{1}{\sigma} \ln \frac{B}{A} \sqrt{2(r + \lambda) + \left(\frac{r - \sigma^2/2}{\sigma} \right)^2} \right\} \widehat{F}(r + \lambda, B) \Bigg), \end{aligned}$$

where $\theta' \sim \text{Exp}(r + \lambda)$ and $\xi = \inf\{t: W_t + t(r - \sigma^2/2)/\sigma = \sigma^{-1} \ln(A/S)\}$, $\xi' = \inf\{t: W_t + t(r - \sigma^2/2)/\sigma = \sigma^{-1} \ln(B/A)\}$.

We get the price by inverting the above with respect to λ .

Example 2 (*Meander lookback option*, [2]). The payoff of «meander lookback call option» equals $\max_{g_T^{(K)} \leq u \leq T} (S_u - K)^+$, where $x^+ = \max(x, 0)$ and S_t is a stock value process, $g_T^{(K)} = \sup\{t < T: S_t = K\}$.

The financial meaning of meander lookback option is the following: If we consider a usual lookback option (with payoff $\max_{0 \leq u \leq T} (S_u - K)^+$) the price of this option is sometimes extremely high. So partial lookback option (with payoff $\max_{u \in J} (S_u - K)^+$, where $J \subset [0, T]$) is considered and sometimes traded. Meander lookback option is one example of this partial lookback option.

Prices of derivatives might be unstable when the time is approaching to the maturity time T and option since if that option is traded, when the time approaches to maturity time T , this option is not affected by such instability of the market. For the price of «Meander lookback option» see [2]. We note that the path between $g_T^{(K)}$ and T is called *meander*.

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Iwata K. (Hiroshima University; *e-mail*: iwata@math.sci.hiroshima-u.ac.jp). **On a semigroup of optimal stopping times.** We discuss discrete time Markovian optimal stopping problems with finite horizon. Under a certain choice, optimal stopping times form a semigroup and there exists a natural associate linear semigroup.

Suppose that κ_n , where $n \geq 0$, is a probability kernel on a measurable space $(S, \mathcal{B})$, and $P_{n,x}$, where $n \geq 0$ and $x \in S$, describes the law of the associated time-inhomogeneous Markov chain: $P_{n,x}(X_{k+1} \in B | \sigma\{X_i\}_{i \leq k}) = \kappa_{n+k}(X_k, B)$ $P_{n,x}$ -a.s. for all $k \geq 0$ and $B \in \mathcal{B}$. Given a bounded measurable function $g: \mathbf{Z}_{\geq 0} \times S \rightarrow \mathbf{R}$ and $T \in \mathbf{Z}_{>0}$, consider the optimal stopping problem for $G_k := g(n+k, X_k)$ with the horizon T . Optimal stopping times are characterized as follows: We introduce $Kg(n, x) := \max\{g(n, x), \kappa g(n, x)\}$, where $\kappa f(n, x) := \int_S f(n+1, y) \kappa_n(x, dy)$, and

$$\zeta(T, g; n, \omega) := \min(\{k < T: \kappa K^{T-k-1}g(n+k, X_k) < K^{T-k}g(n+k, X_k)\} \cup \{T\}).$$

Then we have that

(a) $\tau \leq T$ is an optimal stopping time if and only if $\tau \leq \zeta(T, g; n, \cdot)$ $P_{n,x}$ -a.s. and $g(n+k, X_k) = K^{T-k}g(n+k, X_k)$ at $k = \tau$ $P_{n,x}$ -a.s.;

(b) $\eta(T, g; n, \cdot) := \min\{k \leq T: g(n+k, X_k) = K^{T-k}g(n+k, X_k)\}$ as well as $\zeta(T, g; n, \cdot)$ are optimal stopping times.

Given a sequence of stopping times $\tau(n, \cdot)$ and a measurable function $f: \mathbf{Z}_{\geq 0} \times S \rightarrow \mathbf{R}$ we write $(\kappa; \tau)f(n, x) := E_{n,x}[f(n+\tau(n, \cdot), X_{\tau(n, \cdot)})]$. Then (b) implies that $(\kappa; \eta(T, g))g = K^Tg$ and $(\kappa; \zeta(T, g))g = K^Tg$. Let $\sigma(n, \cdot)$ be another sequence of stopping times. We introduce a new sequence by

$$(\sigma \oplus \tau)(n, \omega) := \sigma(n, \omega) + \tau(n + \sigma(n, \omega), \omega(\sigma(n, \omega) + \cdot)).$$

It follows that $(\kappa; \sigma)(\kappa; \tau)f = (\kappa; \sigma \oplus \tau)f$ due to the strong Markov property. Two types of optimal stopping times enjoy semigroup property in the following sense:

$$\begin{aligned} \eta(s+t, g; n, \cdot) &= (\eta(s, K^t g) \oplus \eta(t, g))(n, \cdot) \quad P_{n,x}\text{-a.s.}, \\ \zeta(s+t, g; n, \cdot) &= (\zeta(s, K^t g) \oplus \zeta(t, g))(n, \cdot) \quad P_{n,x}\text{-a.s.} \end{aligned}$$

As a consequence we obtain time-inhomogeneous linear semigroups:

Theorem. *We have*

$$\begin{aligned} (\kappa; \eta(s+t, g))f &= (\kappa; \eta(s, K^t g))(\kappa; \eta(t, g))f, \\ (\kappa; \zeta(s+t, g))f &= (\kappa; \zeta(s, K^t g))(\kappa; \zeta(t, g))f. \end{aligned}$$

Kamakura T. (Chuo University; *e-mail*: kamakura@indsys.chuo-u.ac.jp). **Methods of testing for normality of observations.** Many statistical models are highly dependent on assumption of normal distribution. However, practitioners sometimes use the statistical model based on the normal assumption without any care of goodness of fitting observed data. In the field of the survival analysis positively skewed distributions such as gamma, lognormal, and Weibull distribution are frequently used to model lifetime data. In this paper we will investigate the several methods of testing for normality of observations such as skewness measure test like D'Agostino test (D'Agostino, 1973), Shapiro & Wilk test (Shapiro, Wilk and Chen, 1968), LM test (Jarque and Bera, 1987; Boman, 1973), and others based on simulation studies.