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K. Iwata, On a semigroup of optimal stopping times, *Теория вероятн. и ее примен.*, 2010, том 55, выпуск 3, 604

DOI: <https://doi.org/10.4213/tvp4252>

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26 апреля 2023 г., 17:37:47

2. Fujita T., Yor M. On the one-sided maximum of Brownian and random walk fragments and its applications to price of meander options. Preprint, 2009.
3. Yor M., Chesney M., Geman H., Jeanblanc-Picqué M. Some combinations of Asian, Parisian and Barrier options. — Mathematics of Derivative Securities. Ed. by M.H. Dempster and S.R. Pliska. Cambridge: Cambridge Univ. Press, 1997, p. 61–87.

Iwata K. (Hiroshima University; *e-mail*: iwata@math.sci.hiroshima-u.ac.jp). **On a semigroup of optimal stopping times.** We discuss discrete time Markovian optimal stopping problems with finite horizon. Under a certain choice, optimal stopping times form a semigroup and there exists a natural associate linear semigroup.

Suppose that κ_n , where $n \geq 0$, is a probability kernel on a measurable space $(S, \mathcal{B})$, and $P_{n,x}$, where $n \geq 0$ and $x \in S$, describes the law of the associated time-inhomogeneous Markov chain: $P_{n,x}(X_{k+1} \in B | \sigma\{X_i\}_{i \leq k}) = \kappa_{n+k}(X_k, B)$ $P_{n,x}$ -a.s. for all $k \geq 0$ and $B \in \mathcal{B}$. Given a bounded measurable function $g: \mathbf{Z}_{\geq 0} \times S \rightarrow \mathbf{R}$ and $T \in \mathbf{Z}_{>0}$, consider the optimal stopping problem for $G_k := g(n+k, X_k)$ with the horizon T . Optimal stopping times are characterized as follows: We introduce $Kg(n, x) := \max\{g(n, x), \kappa g(n, x)\}$, where $\kappa f(n, x) := \int_S f(n+1, y) \kappa_n(x, dy)$, and

$$\zeta(T, g; n, \omega) := \min(\{k < T: \kappa K^{T-k-1}g(n+k, X_k) < K^{T-k}g(n+k, X_k)\} \cup \{T\}).$$

Then we have that

(a) $\tau \leq T$ is an optimal stopping time if and only if $\tau \leq \zeta(T, g; n, \cdot)$ $P_{n,x}$ -a.s. and $g(n+k, X_k) = K^{T-k}g(n+k, X_k)$ at $k = \tau$ $P_{n,x}$ -a.s.;

(b) $\eta(T, g; n, \cdot) := \min\{k \leq T: g(n+k, X_k) = K^{T-k}g(n+k, X_k)\}$ as well as $\zeta(T, g; n, \cdot)$ are optimal stopping times.

Given a sequence of stopping times $\tau(n, \cdot)$ and a measurable function $f: \mathbf{Z}_{\geq 0} \times S \rightarrow \mathbf{R}$ we write $(\kappa; \tau)f(n, x) := E_{n,x}[f(n+\tau(n, \cdot), X_{\tau(n, \cdot)})]$. Then (b) implies that $(\kappa; \eta(T, g))g = K^Tg$ and $(\kappa; \zeta(T, g))g = K^Tg$. Let $\sigma(n, \cdot)$ be another sequence of stopping times. We introduce a new sequence by

$$(\sigma \oplus \tau)(n, \omega) := \sigma(n, \omega) + \tau(n + \sigma(n, \omega), \omega(\sigma(n, \omega) + \cdot)).$$

It follows that $(\kappa; \sigma)(\kappa; \tau)f = (\kappa; \sigma \oplus \tau)f$ due to the strong Markov property. Two types of optimal stopping times enjoy semigroup property in the following sense:

$$\begin{aligned} \eta(s+t, g; n, \cdot) &= (\eta(s, K^t g) \oplus \eta(t, g))(n, \cdot) \quad P_{n,x}\text{-a.s.}, \\ \zeta(s+t, g; n, \cdot) &= (\zeta(s, K^t g) \oplus \zeta(t, g))(n, \cdot) \quad P_{n,x}\text{-a.s.} \end{aligned}$$

As a consequence we obtain time-inhomogeneous linear semigroups:

Theorem. *We have*

$$\begin{aligned} (\kappa; \eta(s+t, g))f &= (\kappa; \eta(s, K^t g))(\kappa; \eta(t, g))f, \\ (\kappa; \zeta(s+t, g))f &= (\kappa; \zeta(s, K^t g))(\kappa; \zeta(t, g))f. \end{aligned}$$

Kamakura T. (Chuo University; *e-mail*: kamakura@indsys.chuo-u.ac.jp). **Methods of testing for normality of observations.** Many statistical models are highly dependent on assumption of normal distribution. However, practitioners sometimes use the statistical model based on the normal assumption without any care of goodness of fitting observed data. In the field of the survival analysis positively skewed distributions such as gamma, lognormal, and Weibull distribution are frequently used to model lifetime data. In this paper we will investigate the several methods of testing for normality of observations such as skewness measure test like D'Agostino test (D'Agostino, 1973), Shapiro & Wilk test (Shapiro, Wilk and Chen, 1968), LM test (Jarque and Bera, 1987; Boman, 1973), and others based on simulation studies.