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Iwata K. (Hiroshima University; *e-mail*: iwata@math.sci.hiroshima-u.ac.jp). **On a semigroup of optimal stopping times.** We discuss discrete time Markovian optimal stopping problems with finite horizon. Under a certain choice, optimal stopping times form a semigroup and there exists a natural associate linear semigroup.

Suppose that κ_n , where $n \geq 0$, is a probability kernel on a measurable space $(S, \mathcal{B})$, and $P_{n,x}$, where $n \geq 0$ and $x \in S$, describes the law of the associated time-inhomogeneous Markov chain: $P_{n,x}(X_{k+1} \in B | \sigma\{X_i\}_{i \leq k}) = \kappa_{n+k}(X_k, B)$ $P_{n,x}$ -a.s. for all $k \geq 0$ and $B \in \mathcal{B}$. Given a bounded measurable function $g: \mathbf{Z}_{\geq 0} \times S \rightarrow \mathbf{R}$ and $T \in \mathbf{Z}_{>0}$, consider the optimal stopping problem for $G_k := g(n+k, X_k)$ with the horizon T . Optimal stopping times are characterized as follows: We introduce $Kg(n, x) := \max\{g(n, x), \kappa g(n, x)\}$, where $\kappa f(n, x) := \int_S f(n+1, y) \kappa_n(x, dy)$, and

$$\zeta(T, g; n, \omega) := \min(\{k < T: \kappa K^{T-k-1}g(n+k, X_k) < K^{T-k}g(n+k, X_k)\} \cup \{T\}).$$

Then we have that

(a) $\tau \leq T$ is an optimal stopping time if and only if $\tau \leq \zeta(T, g; n, \cdot)$ $P_{n,x}$ -a.s. and $g(n+k, X_k) = K^{T-k}g(n+k, X_k)$ at $k = \tau$ $P_{n,x}$ -a.s.;

(b) $\eta(T, g; n, \cdot) := \min\{k \leq T: g(n+k, X_k) = K^{T-k}g(n+k, X_k)\}$ as well as $\zeta(T, g; n, \cdot)$ are optimal stopping times.

Given a sequence of stopping times $\tau(n, \cdot)$ and a measurable function $f: \mathbf{Z}_{\geq 0} \times S \rightarrow \mathbf{R}$ we write $(\kappa; \tau)f(n, x) := E_{n,x}[f(n+\tau(n, \cdot), X_{\tau(n, \cdot)})]$. Then (b) implies that $(\kappa; \eta(T, g))g = K^Tg$ and $(\kappa; \zeta(T, g))g = K^Tg$. Let $\sigma(n, \cdot)$ be another sequence of stopping times. We introduce a new sequence by

$$(\sigma \oplus \tau)(n, \omega) := \sigma(n, \omega) + \tau(n + \sigma(n, \omega), \omega(\sigma(n, \omega) + \cdot)).$$

It follows that $(\kappa; \sigma)(\kappa; \tau)f = (\kappa; \sigma \oplus \tau)f$ due to the strong Markov property. Two types of optimal stopping times enjoy semigroup property in the following sense:

$$\begin{aligned} \eta(s+t, g; n, \cdot) &= (\eta(s, K^t g) \oplus \eta(t, g))(n, \cdot) \quad P_{n,x}\text{-a.s.}, \\ \zeta(s+t, g; n, \cdot) &= (\zeta(s, K^t g) \oplus \zeta(t, g))(n, \cdot) \quad P_{n,x}\text{-a.s.} \end{aligned}$$

As a consequence we obtain time-inhomogeneous linear semigroups:

Theorem. *We have*

$$\begin{aligned} (\kappa; \eta(s+t, g))f &= (\kappa; \eta(s, K^t g))(\kappa; \eta(t, g))f, \\ (\kappa; \zeta(s+t, g))f &= (\kappa; \zeta(s, K^t g))(\kappa; \zeta(t, g))f. \end{aligned}$$

Kamakura T. (Chuo University; *e-mail*: kamakura@indsys.chuo-u.ac.jp). **Methods of testing for normality of observations.** Many statistical models are highly dependent on assumption of normal distribution. However, practitioners sometimes use the statistical model based on the normal assumption without any care of goodness of fitting observed data. In the field of the survival analysis positively skewed distributions such as gamma, lognormal, and Weibull distribution are frequently used to model lifetime data. In this paper we will investigate the several methods of testing for normality of observations such as skewness measure test like D'Agostino test (D'Agostino, 1973), Shapiro & Wilk test (Shapiro, Wilk and Chen, 1968), LM test (Jarque and Bera, 1987; Boman, 1973), and others based on simulation studies.

The LM test includes estimates of two parameter, skewness and kurtosis. For small samples this test may not have good property because it is based on the asymptotic distribution. First, we will present several estimators for these parameters. The standard estimates included in the computer program package S or R give rise to heavy biases in case of the small sample from our simulation studies. We can derive the less-biased statistics based on the k -statistics. For example,

$$\begin{aligned} k_4 &= \frac{-6s_1^4 + 12ns_1^2s_2 - 3n(n-1)s_2^2 - 4n(n+1)s_1s_3 + n^2(n+1)s_4}{n(n-1)(n-2)(n-3)} \\ &= \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^n (x_i - \bar{x})^4 - \frac{3(n-1)^2}{(n-2)(n-3)} \left\{ \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \right\}^2. \end{aligned}$$

The estimates of skewness and kurtosis are not correlated in case that the assumed distribution is normal distribution, but they are highly correlated for the skewed distributions.

We derived the covariance of both estimates up to the first order of sample size and improved the LM test for high performance of testing especially in case of positive skewed distributions.

Miyazaki E. (Chuo University). **Approximation for the distribution of the smallest latent root of Wishart matrix.** Let $\mathbf{S}$ be the sample covariance matrix based on random sample of size $N = n + 1$, from p -variate normal distribution with a mean vector $\boldsymbol{\mu}$ and a covariance matrix $\boldsymbol{\Sigma}$. Then $\mathbf{A} = n\mathbf{S}$ is distributed according to the following Wishart distribution $W_p(n, \boldsymbol{\Sigma})$:

$$f(\mathbf{A}) = C |\mathbf{A}|^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \text{tr } \boldsymbol{\Sigma}^{-1} \mathbf{A} \right\}.$$

Let $\lambda_1 \geq \dots \geq \lambda_p \geq 0$ be the population latent roots of $\boldsymbol{\Sigma}$ and let $\ell_1 > \dots > \ell_p > 0$ be the sample latent roots of $\mathbf{A}$ ($\mathbf{A} = \mathbf{H}' \mathbf{D}_\ell \mathbf{H}$, where $\mathbf{D}_\ell$ is a diagonal matrix with elements $\ell_1, \dots, \ell_p$ and $\mathbf{H}$ is $p \times p$ orthogonal matrix). Then the probability density function is given by

$$f(\mathbf{D}_\ell, \mathbf{H}) = C |\mathbf{D}_\ell|^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \text{tr } \boldsymbol{\Sigma}^{-1} \mathbf{H}' \mathbf{D}_\ell \mathbf{H} \right\} (d\mathbf{D}_\ell)(d\mathbf{H}), \quad (1)$$

where $(d\mathbf{D}_\ell) = d\ell_1 \cdots d\ell_p$ and $(d\mathbf{H})$ is Haar measure of the orthogonal group. Let $\mathbf{D}_u = \mathbf{D}_\ell - \ell_p \mathbf{I}$. The determinant $|\mathbf{D}_\ell|$ is written as follows:

$$|\mathbf{D}_u + \ell_p \mathbf{I}| = (u_1 + \ell_p) \cdots (u_{p-1} + \ell_p) \ell_p > u_1 \cdots u_{p-1} \ell_p.$$

Then we may obtain the following lower bound of the density function (1):

$$\begin{aligned} & C |\mathbf{D}_u^*|^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \text{tr } \boldsymbol{\Sigma}^{-1} \mathbf{H}' \mathbf{D}_u \mathbf{H} \right\} \\ & \times \prod_{1 \leq i < j \leq p-1} (u_i - u_j) \prod_{i=1}^{p-1} du_i (d\mathbf{H}) \ell_p^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \ell_p \text{tr } \boldsymbol{\Sigma}^{-1} \right\} d\ell_p, \end{aligned}$$

where $\mathbf{D}_u^* = \text{diag}(u_1, \dots, u_{p-1})$. Therefore, integrating out $\ell_1, \dots, \ell_{p-1}$ and $(d\mathbf{H})$, we obtain the following approximate χ^2 distribution of the smallest latent root ℓ_p :

$$f(\ell_p) = C' \ell_p^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \ell_p (\lambda_1^{-1} + \dots + \lambda_p^{-1}) \right\}. \quad (2)$$