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The LM test includes estimates of two parameter, skewness and kurtosis. For small samples this test may not have good property because it is based on the asymptotic distribution. First, we will present several estimators for these parameters. The standard estimates included in the computer program package S or R give rise to heavy biases in case of the small sample from our simulation studies. We can derive the less-biased statistics based on the k -statistics. For example,

$$\begin{aligned} k_4 &= \frac{-6s_1^4 + 12ns_1^2s_2 - 3n(n-1)s_2^2 - 4n(n+1)s_1s_3 + n^2(n+1)s_4}{n(n-1)(n-2)(n-3)} \\ &= \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^n (x_i - \bar{x})^4 - \frac{3(n-1)^2}{(n-2)(n-3)} \left\{ \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \right\}^2. \end{aligned}$$

The estimates of skewness and kurtosis are not correlated in case that the assumed distribution is normal distribution, but they are highly correlated for the skewed distributions.

We derived the covariance of both estimates up to the first order of sample size and improved the LM test for high performance of testing especially in case of positive skewed distributions.

Miyazaki E. (Chuo University). **Approximation for the distribution of the smallest latent root of Wishart matrix.** Let $\mathbf{S}$ be the sample covariance matrix based on random sample of size $N = n + 1$, from p -variate normal distribution with a mean vector $\boldsymbol{\mu}$ and a covariance matrix $\boldsymbol{\Sigma}$. Then $\mathbf{A} = n\mathbf{S}$ is distributed according to the following Wishart distribution $W_p(n, \boldsymbol{\Sigma})$:

$$f(\mathbf{A}) = C |\mathbf{A}|^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \text{tr } \boldsymbol{\Sigma}^{-1} \mathbf{A} \right\}.$$

Let $\lambda_1 \geq \dots \geq \lambda_p \geq 0$ be the population latent roots of $\boldsymbol{\Sigma}$ and let $\ell_1 > \dots > \ell_p > 0$ be the sample latent roots of $\mathbf{A}$ ($\mathbf{A} = \mathbf{H}' \mathbf{D}_\ell \mathbf{H}$, where $\mathbf{D}_\ell$ is a diagonal matrix with elements $\ell_1, \dots, \ell_p$ and $\mathbf{H}$ is $p \times p$ orthogonal matrix). Then the probability density function is given by

$$f(\mathbf{D}_\ell, \mathbf{H}) = C |\mathbf{D}_\ell|^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \text{tr } \boldsymbol{\Sigma}^{-1} \mathbf{H}' \mathbf{D}_\ell \mathbf{H} \right\} (d\mathbf{D}_\ell)(d\mathbf{H}), \quad (1)$$

where $(d\mathbf{D}_\ell) = d\ell_1 \cdots d\ell_p$ and $(d\mathbf{H})$ is Haar measure of the orthogonal group. Let $\mathbf{D}_u = \mathbf{D}_\ell - \ell_p \mathbf{I}$. The determinant $|\mathbf{D}_\ell|$ is written as follows:

$$|\mathbf{D}_u + \ell_p \mathbf{I}| = (u_1 + \ell_p) \cdots (u_{p-1} + \ell_p) \ell_p > u_1 \cdots u_{p-1} \ell_p.$$

Then we may obtain the following lower bound of the density function (1):

$$\begin{aligned} & C |\mathbf{D}_u^*|^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \text{tr } \boldsymbol{\Sigma}^{-1} \mathbf{H}' \mathbf{D}_u \mathbf{H} \right\} \\ & \times \prod_{1 \leq i < j \leq p-1} (u_i - u_j) \prod_{i=1}^{p-1} du_i (d\mathbf{H}) \ell_p^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \ell_p \text{tr } \boldsymbol{\Sigma}^{-1} \right\} d\ell_p, \end{aligned}$$

where $\mathbf{D}_u^* = \text{diag}(u_1, \dots, u_{p-1})$. Therefore, integrating out $\ell_1, \dots, \ell_{p-1}$ and $(d\mathbf{H})$, we obtain the following approximate χ^2 distribution of the smallest latent root ℓ_p :

$$f(\ell_p) = C' \ell_p^{(n-p-1)/2} \exp \left\{ -\frac{1}{2} \ell_p (\lambda_1^{-1} + \dots + \lambda_p^{-1}) \right\}. \quad (2)$$

Neglecting the term $\lambda_1^{-1} + \dots + \lambda_{p-1}^{-1}$ in (2), we obtain the following simplified approximate distribution of the smallest latent root ℓ_p , namely the χ^2 distribution of $n - p + 1$ degrees of freedom:

$$P(\ell_1 < z) = \gamma\left(\frac{n-p+1}{2}; \frac{nz}{2\lambda_p}\right) / \Gamma\left(\frac{n-p+1}{2}\right). \quad (3)$$

By the simulation of 1,000,000 times under the normality, we obtain the α percentile points of the smallest latent root (SLR) in the following cases (A to C) and the distribution function (3) on χ_{91}^2 approximation.

Case A: $\lambda_1 = 10^2, \lambda_2 = 9^2, \dots, \lambda_6 = 5^2, \lambda_7 = 4.5^2, \lambda_8 = 4^2, \lambda_9 = 3^2, \lambda_{10} = 1^2$,
 Case B: $\lambda_1 = 10^2, \lambda_2 = 9^2, \dots, \lambda_6 = 5^2, \lambda_7 = 4^2, \lambda_8 = 3^2, \lambda_9 = 2^2, \lambda_{10} = 1^2$,
 Case C: $\lambda_1 = 10^2, \lambda_2 = 9^2, \dots, \lambda_6 = 5^2, \lambda_7 = 4^2, \lambda_8 = 3^2, \lambda_9 = 1.5^2, \lambda_{10} = 1^2$.

	A		B		C	
α	SLR	χ_{91}^2	SLR	χ_{91}^2	SLR	χ_{91}^2
95	113.921	0.9476	113.505	0.9447	112.825	0.9397
90	108.318	0.8959	107.924	0.8912	107.289	0.8831
10	73.909	0.0957	73.675	0.0924	73.318	0.0874
05	69.731	0.0475	69.507	0.0456	69.185	0.0429

Nagai K. [*joint work with Y. Nishiyama and K. Hitomi*] (Yokohama National University; *e-mail*: knagai@ynu.ac.jp). **Asymptotics of unit root tests under sequential sampling via diffusion approximation.** We develop a general framework of sequential testing problem for some Markov chains converging to diffusion. Suppose that a Markov chain has a local parameter θ/N and converges as $N \rightarrow \infty$ to a one-dimensional diffusion with drift coefficient $a(x) + \theta b(x)$. We consider sequential testing problem $H_0: \theta \leq 0$ vs $H_1: \theta > 0$ for one-dimensional diffusion process in continuous time to solve the sequential testing problem for Markov processes in discrete time. The problem includes the traditional sequential testing problem of a drift parameter of Brownian motion described in [2] which corresponds to the diffusion approximation of independent and identically distributed observations with a local mean parameter. As an example, we consider a sequential test for a critical parameter $\mu = 1 + \theta/N$ of discrete-time branching process with immigration. The critical parameter corresponds to the expected number of offspring. The fixed accuracy estimation of the critical parameter was considered in [3]. We provide a sequential testing procedures using the stopping time based on the observed Fisher information and a sequential probability ratio test (SPRT). Combination of local hypothesis and the stopping time based on observed Fisher information enables us to apply change of time to the problem and use DDS Brownian motion (see [1]). The stopping time based on observed Fisher information and SPRT can be represented by DDS Brownian motion.

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Nishide K. [*joint work with M. Kijima and A. Ohyama*] (Yokohama National University; *e-mail*: knishide@ynu.ac.jp). **On the environmental Kuznets curve: A real options approach.** Previous researches have suggested that some pollutant levels first increase due to the economic growth and then start decreasing, the pattern being called the *environmental Kuznets curve*. Assuming that each policy maker optimally executes