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Neglecting the term $\lambda_1^{-1} + \dots + \lambda_{p-1}^{-1}$ in (2), we obtain the following simplified approximate distribution of the smallest latent root ℓ_p , namely the χ^2 distribution of $n - p + 1$ degrees of freedom:

$$P(\ell_1 < z) = \gamma\left(\frac{n-p+1}{2}; \frac{nz}{2\lambda_p}\right) / \Gamma\left(\frac{n-p+1}{2}\right). \quad (3)$$

By the simulation of 1,000,000 times under the normality, we obtain the α percentile points of the smallest latent root (SLR) in the following cases (A to C) and the distribution function (3) on χ_{91}^2 approximation.

Case A: $\lambda_1 = 10^2, \lambda_2 = 9^2, \dots, \lambda_6 = 5^2, \lambda_7 = 4.5^2, \lambda_8 = 4^2, \lambda_9 = 3^2, \lambda_{10} = 1^2$,
 Case B: $\lambda_1 = 10^2, \lambda_2 = 9^2, \dots, \lambda_6 = 5^2, \lambda_7 = 4^2, \lambda_8 = 3^2, \lambda_9 = 2^2, \lambda_{10} = 1^2$,
 Case C: $\lambda_1 = 10^2, \lambda_2 = 9^2, \dots, \lambda_6 = 5^2, \lambda_7 = 4^2, \lambda_8 = 3^2, \lambda_9 = 1.5^2, \lambda_{10} = 1^2$.

	A		B		C	
α	SLR	χ_{91}^2	SLR	χ_{91}^2	SLR	χ_{91}^2
95	113.921	0.9476	113.505	0.9447	112.825	0.9397
90	108.318	0.8959	107.924	0.8912	107.289	0.8831
10	73.909	0.0957	73.675	0.0924	73.318	0.0874
05	69.731	0.0475	69.507	0.0456	69.185	0.0429

Nagai K. [*joint work with Y. Nishiyama and K. Hitomi*] (Yokohama National University; *e-mail*: knagai@ynu.ac.jp). **Asymptotics of unit root tests under sequential sampling via diffusion approximation.** We develop a general framework of sequential testing problem for some Markov chains converging to diffusion. Suppose that a Markov chain has a local parameter θ/N and converges as $N \rightarrow \infty$ to a one-dimensional diffusion with drift coefficient $a(x) + \theta b(x)$. We consider sequential testing problem $H_0: \theta \leq 0$ vs $H_1: \theta > 0$ for one-dimensional diffusion process in continuous time to solve the sequential testing problem for Markov processes in discrete time. The problem includes the traditional sequential testing problem of a drift parameter of Brownian motion described in [2] which corresponds to the diffusion approximation of independent and identically distributed observations with a local mean parameter. As an example, we consider a sequential test for a critical parameter $\mu = 1 + \theta/N$ of discrete-time branching process with immigration. The critical parameter corresponds to the expected number of offspring. The fixed accuracy estimation of the critical parameter was considered in [3]. We provide a sequential testing procedures using the stopping time based on the observed Fisher information and a sequential probability ratio test (SPRT). Combination of local hypothesis and the stopping time based on observed Fisher information enables us to apply change of time to the problem and use DDS Brownian motion (see [1]). The stopping time based on observed Fisher information and SPRT can be represented by DDS Brownian motion.

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Nishide K. [*joint work with M. Kijima and A. Ohyama*] (Yokohama National University; *e-mail*: knishide@ynu.ac.jp). **On the environmental Kuznets curve: A real options approach.** Previous researches have suggested that some pollutant levels first increase due to the economic growth and then start decreasing, the pattern being called the *environmental Kuznets curve*. Assuming that each policy maker optimally executes

the switching options of regulation and deregulation for pollution, the switching dynamics of environmental policy is described by an alternating renewal process. It is shown that the double Laplace transform of transition density of a pollutant level can be obtained by a novel application of renewal theory. The expected level of overall pollutants is then calculated numerically and found to exhibit either a Λ -shaped or an N -shaped pattern in time. Our results present a simple explanation for the environmental Kuznets curve within a real options framework.

Sugiyama T. (Chuo University, Tokyo; *e-mail*: takakazu@math.chuo-u.ac.jp). **Distributions and test of each characteristic root in principal component analysis.** Principal components analysis is a data dimension reduction procedure that transforms the data representing a number of correlated variables into a new set of uncorrelated variables called principal components, and is to find the linear combinations with large variance. Let $\ell_1, \dots, \ell_p$ ($\ell_1 > \dots > \ell_p$) be the characteristic roots of the Wishart matrix $\mathbf{A} = n\mathbf{S}$, where $\mathbf{S}$ is the sample covariance matrix. Let $\lambda_1, \dots, \lambda_p$ be the characteristic roots of $\mathbf{\Sigma}$ ($\lambda_1 > \dots > \lambda_p > 0$). We apply principal component analysis for the covariance matrix $\mathbf{S}$, namely based on $\mathbf{S}$, we calculate the principal components and their variances. Each principal component accounts for a decreasing proportion of the entire variance inherent in the data; thus, the first principal component represents the largest variation (variance $n^{-1}\ell_1$), the second principal component represents the second largest variation (variance $n^{-1}\ell_2$), and so on. So, the distributions of each variance (characteristic root) are important.

The exact distribution of the largest characteristic root ℓ_1 is given in [1], [2]:

$$F(\ell_1 < nx) = \frac{|\mathbf{\Sigma}|^{n/2} \Gamma_p((p+1)/2)}{2^{np/2} \Gamma_p((n+p+1)/2)} \exp\left(-\frac{nx}{2} \text{tr } \mathbf{\Sigma}\right) \cdot {}_1F_1\left(\frac{p+1}{2}; \frac{n+p+1}{2}; \frac{nx}{2} \mathbf{\Sigma}^{-1}\right).$$

Now, the numerical computation of above distribution is possible until $p = 4$, $n = 50$. Sugiyama [3] also obtained the approximation $z_{1\alpha}$ of the upper 100α percentile point of the variance ℓ_1 , solving the following equation:

$$\prod_{i=1}^p \gamma\left(\frac{n-i+1}{2}; \frac{nz_{1\alpha}}{2\lambda_i}\right) / \Gamma\left(\frac{n-i+1}{2}\right) = \alpha,$$

where $\gamma(m/2; x/2)/\Gamma(m/2)$ is the characteristic distribution function of the chi-square distribution with m degrees of freedom. The above approximation is not so precise. So, Konishi and Sugiyama [4] obtained that $\ell_1^{1/3}$ is approximately distributed according to the normal distribution $\mathcal{N}(\lambda_1^{1/3}, 2\lambda_1^{2/3}/9)$.

We do not know the good mathematical results for the distributions of the second largest characteristic root ℓ_2 , the third ℓ_3 , and so on. Though we have these asymptotic distributions, it is not so simple. We wish to know the simple approximation of the distributions. When $\lambda_1 \gg \lambda_2 \gg \lambda_3 \gg \dots$ we propose the following χ^2 approximations:

$$\begin{aligned} \Pr(\ell_2 < z) &= \left[\gamma\left(\frac{n-1}{2}; \frac{nz}{2\lambda_2}\right) / \Gamma\left(\frac{n-1}{2}\right) \right], \\ \Pr(\ell_3 < z) &= \left[\gamma\left(\frac{n-2}{2}; \frac{nz}{2\lambda_3}\right) / \Gamma\left(\frac{n-2}{2}\right) \right], \quad \text{and so on.} \end{aligned} \tag{1}$$

To investigate the goodness of approximation, we consider the case $\lambda_1 = 100$, $\lambda_2 = 64$, $\lambda_3 = 25$, $\lambda_4 = \dots = \lambda_p = 1$, for the characteristic roots of $\mathbf{\Sigma}$, when $p = 10$.