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Table 1. The probability based on the χ^2 approximation (1)

n	Second largest ℓ_2		Third largest ℓ_3	
	exact prob. 0.90	exact prob. 0.95	exact prob. 0.90	exact prob. 0.95
100	0.8847	0.9381	0.8973	0.9528
200	0.8939	0.9500	0.9047	0.9530

We see that approximations (1) are valid for $n = 200$. Obtaining the upper or lower percentile points of each characteristic root ℓ_j , we may consider the test of the null hypothesis $H_0: \lambda_j = \lambda_{j0}$.

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Wakaki H. [*joint work with Shimizu H.*] (Graduate School of Science, Hiroshima University; *e-mail*: wakaki@math.sci.hiroshima-u.ac.jp). **An asymptotic expansion of the local power of a general test for testing a general covariance structure.** Let S be a $p \times p$ random matrix having a Wishart distribution $W_p(n, n^{-1}\Sigma)$. For testing a general covariance structure $\Sigma = \Sigma(\xi)$, we consider a class of test statistics $T_h = n \inf \rho_h(S, \Sigma(\xi))$, where $\rho_h(\Sigma_1, \Sigma_2) = \sum_{j=1}^p h(\lambda_j)$ is a divergence measure from Σ_1 to Σ_2 , λ_j 's are the eigenvalues of $\Sigma_1 \Sigma_2^{-1}$, and h is a given C^4 -class function on $(0, \infty)$ such that $h(1) = 0$, $h_1 = 0$, $h_2 = 1$, and $h(\lambda) > 0$ for any $\lambda \neq 1$, here h_r denotes the r -th derivative of h at $\lambda = 1$. It is assumed that all the elements of $\Sigma(\xi)$ are known C^4 -class functions on Ξ , an open subset of $\mathbf{R}^q$, and the Jacobian matrix of $\Sigma(\xi)$ is of full rank.

Wakaki, Eguchi, and Fujikoshi [1] derived an asymptotic expansion of the null distribution of T_h up to the order n^{-1} . Using the general asymptotic formula, they gave a condition for T_h to have a Bartlett adjustment factor.

We consider a sequence of local alternatives: $\Sigma = \Sigma_0 + n^{-1/2} \Sigma_0^{1/2} \Delta \Sigma_0^{1/2}$, where $\Sigma_0 = \Sigma(\xi_0)$, Δ is an arbitrarily fixed symmetric matrix such that $I_p + \Delta$ is positive definite. We derive an asymptotic expansion of the distribution function of T_h up to the order $n^{-1/2}$ under the local alternatives. Using the asymptotic formula, an asymptotic expansion of the power function of T_h is derived. Let β_h and β_{LR} be the power function of T_h and the likelihood ratio test, respectively. Then we find that

$$\begin{aligned} \beta_h = \beta_{LR} + \frac{1}{\sqrt{n}} \left(1 + \frac{h_3}{2} \right) & \left[\left\{ (p+1) \operatorname{tr}[D_\Delta] - \sum_{a,b=1}^q g^{ab} \operatorname{tr}[J_a J_b D_\Delta] \right\} g_{r+4}(\chi_\alpha^2; \tau) \right. \\ & \left. + \frac{1}{3} \operatorname{tr}[D_\Delta^3] g_{r+6}(\chi_\alpha^2; \tau) \right] + O(n^{-1}), \end{aligned}$$

where χ_α^2 is the upper α point of chi-square distribution with degree of freedom $r = p(p+1)/2 - q$, $g_s(x; \tau)$ is the noncentral chi-square density function with s degree of

freedom and noncentral parameter $\tau = (1/2) \operatorname{tr} [D_\Delta^2]$,

$$J_{ab\dots} = \Sigma_0^{1/2} \left[\left(\frac{\partial}{\partial \xi^a} \right) \left(\frac{\partial}{\partial \xi^b} \right) \dots \Sigma(\boldsymbol{\xi})^{-1} \right]_{\boldsymbol{\xi}=\boldsymbol{\xi}_0} \Sigma_0^{1/2},$$

$$G = (g_{ab}), \quad g_{ab} = (1/2) \operatorname{tr} [J_a J_b] \quad (a, b = 1, \dots, q),$$

$$G^{-1} = (g^{ab}), \quad D_\Delta = \Delta - (1/2) J_a g^{ab} \operatorname{tr} [J_b \Delta].$$

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Yanagawa T. (Biostatistics Center, Kurume University). **Observational chaos.**

The Lyapunov exponent is often used for detecting the evidence of chaos in physics, neurobiology, ecology, economics, and physiology. Conventional methods for estimating the Lyapunov exponent are reliable if the data are abundant, if measurement error is near 0, and if the data really come from a deterministic system. However, with limited data or a system subject to nonnegligible stochastic perturbations, it is well known that the estimates may be incorrect or ambiguous. Considering $\{X_t\}_{t=1,2,\dots,N}$ generated from the following nonlinear system with additive noise, we develop a method for estimating the Lyapunov exponent of $\{Y_t\}$ that will be defined below:

$$X_t = F(X_{t-\tau}, \dots, X_{t-d\tau}) + \varepsilon_t,$$

where d and τ are unknown positive integers and $F: \mathbf{R}^d \rightarrow \mathbf{R}$ is unknown nonlinear function such that $\{Y_t\}$ is ergodic, where $Y_t = F(Y_{t-1}, \dots, Y_{t-d})$. We assume that $\{X_t\}$ is a discrete-time strictly stationary time series with $\mathbf{E} X_t^2 < \infty$ and that $\mathbf{E} [\varepsilon_t | \mathbf{A}_1^{t-1}(X)] = 0$ a.s., $\mathbf{E} [\varepsilon_t^2 | \mathbf{A}_1^{t-1}(X)] = \sigma^2$ ($\sigma > 0$) a.s., for any positive integer t , where $\mathbf{A}_s^t(X)$ denotes the sigma-algebra generated by $(X_s, \dots, X_t)$.

We take similar approach as Pikovsky (1986), Landa and Rosenblum (1989), Cawley and Hsu (1992), and Sauer (1992), but use kernel type estimators for filtering out the noise.

Definition. $\{X_t\}$ is said to have the embedding dimension d_0 and the delay time τ_0 if and only if there exist nonnegative integers $d_0 < \infty$ and $\tau_0 < \infty$ such that $\mathbf{E}[X_t | X_{t-\tau}, X_{t-2\tau}, \dots, X_{t-d\tau}] \neq \mathbf{E}[X_t | X_{t-\tau_0}, X_{t-2\tau_0}, \dots, X_{t-d_0\tau_0}]$ a.e. for any $d < d_0$ and any $\tau > 0$, and $\mathbf{E}[X_t | X_{t-\tau}, X_{t-2\tau}, \dots, X_{t-d\tau}] = \mathbf{E}[X_t | X_{t-\tau_0}, X_{t-2\tau_0}, \dots, X_{t-d_0\tau_0}]$ a.e. for any $(d, \tau) \in \mathbf{B}(d_0, \tau_0)$, where $\mathbf{B}(d_0, \tau_0) = \{(d, \tau): \{\tau_0, 2\tau_0, \dots, d_0\tau_0\} \subset \{\tau, 2\tau, \dots, d\tau\}\}$.

First, an algorithm for the determination of d_0 and τ_0 is developed. Then, assuming that the embedding dimension and delay time are known, the Nadaraya–Watson kernel type estimator is introduced for estimating F . Finally, we generate $\{\hat{Y}_{N,t}\}$ from estimated F , and apply a modified Eckmann and Ruelle’s method for estimating the Lyapunov exponent. It is proved that $\hat{Y}_{N,t} \rightarrow Y_t$, in probability, as $N \rightarrow \infty$ and that the consistency of the proposed estimating procedure of the Lyapunov exponent.

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