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freedom and noncentral parameter $\tau = (1/2) \operatorname{tr} [D_\Delta^2]$,

$$J_{ab\dots} = \Sigma_0^{1/2} \left[\left(\frac{\partial}{\partial \xi^a} \right) \left(\frac{\partial}{\partial \xi^b} \right) \dots \Sigma(\boldsymbol{\xi})^{-1} \right]_{\boldsymbol{\xi}=\boldsymbol{\xi}_0} \Sigma_0^{1/2},$$

$$G = (g_{ab}), \quad g_{ab} = (1/2) \operatorname{tr} [J_a J_b] \quad (a, b = 1, \dots, q),$$

$$G^{-1} = (g^{ab}), \quad D_\Delta = \Delta - (1/2) J_a g^{ab} \operatorname{tr} [J_b \Delta].$$

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Yanagawa T. (Biostatistics Center, Kurume University). **Observational chaos.**

The Lyapunov exponent is often used for detecting the evidence of chaos in physics, neurobiology, ecology, economics, and physiology. Conventional methods for estimating the Lyapunov exponent are reliable if the data are abundant, if measurement error is near 0, and if the data really come from a deterministic system. However, with limited data or a system subject to nonnegligible stochastic perturbations, it is well known that the estimates may be incorrect or ambiguous. Considering $\{X_t\}_{t=1,2,\dots,N}$ generated from the following nonlinear system with additive noise, we develop a method for estimating the Lyapunov exponent of $\{Y_t\}$ that will be defined below:

$$X_t = F(X_{t-\tau}, \dots, X_{t-d\tau}) + \varepsilon_t,$$

where d and τ are unknown positive integers and $F: \mathbf{R}^d \rightarrow \mathbf{R}$ is unknown nonlinear function such that $\{Y_t\}$ is ergodic, where $Y_t = F(Y_{t-1}, \dots, Y_{t-d})$. We assume that $\{X_t\}$ is a discrete-time strictly stationary time series with $\mathbf{E} X_t^2 < \infty$ and that $\mathbf{E} [\varepsilon_t | \mathbf{A}_1^{t-1}(X)] = 0$ a.s., $\mathbf{E} [\varepsilon_t^2 | \mathbf{A}_1^{t-1}(X)] = \sigma^2$ ($\sigma > 0$) a.s., for any positive integer t , where $\mathbf{A}_s^t(X)$ denotes the sigma-algebra generated by $(X_s, \dots, X_t)$.

We take similar approach as Pikovsky (1986), Landa and Rosenblum (1989), Cawley and Hsu (1992), and Sauer (1992), but use kernel type estimators for filtering out the noise.

Definition. $\{X_t\}$ is said to have the embedding dimension d_0 and the delay time τ_0 if and only if there exist nonnegative integers $d_0 < \infty$ and $\tau_0 < \infty$ such that $\mathbf{E}[X_t | X_{t-\tau}, X_{t-2\tau}, \dots, X_{t-d\tau}] \neq \mathbf{E}[X_t | X_{t-\tau_0}, X_{t-2\tau_0}, \dots, X_{t-d_0\tau_0}]$ a.e. for any $d < d_0$ and any $\tau > 0$, and $\mathbf{E}[X_t | X_{t-\tau}, X_{t-2\tau}, \dots, X_{t-d\tau}] = \mathbf{E}[X_t | X_{t-\tau_0}, X_{t-2\tau_0}, \dots, X_{t-d_0\tau_0}]$ a.e. for any $(d, \tau) \in \mathbf{B}(d_0, \tau_0)$, where $\mathbf{B}(d_0, \tau_0) = \{(d, \tau): \{\tau_0, 2\tau_0, \dots, d_0\tau_0\} \subset \{\tau, 2\tau, \dots, d\tau\}\}$.

First, an algorithm for the determination of d_0 and τ_0 is developed. Then, assuming that the embedding dimension and delay time are known, the Nadaraya–Watson kernel type estimator is introduced for estimating F . Finally, we generate $\{\hat{Y}_{N,t}\}$ from estimated F , and apply a modified Eckmann and Ruelle’s method for estimating the Lyapunov exponent. It is proved that $\hat{Y}_{N,t} \rightarrow Y_t$, in probability, as $N \rightarrow \infty$ and that the consistency of the proposed estimating procedure of the Lyapunov exponent.

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