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A MAXIMAL INEQUALITY FOR REAL NUMBERS WITH APPLICATION TO EXCHANGEABLE RANDOM VARIABLES

Пусть $x = (x_1, \dots, x_n)$ — последовательность вещественных чисел такая, что $\sum_{i=1}^n x_i = 0$, и пусть $\Theta = \{\theta = (\theta_1, \dots, \theta_n): \theta_i = \pm 1\}$. Мы доказываем, что для любых $\theta \in \Theta$ и $t \geq 0$ справедливы неравенства

$$\frac{1}{2} \mathbf{P}\{|x_\pi| \geq 38t\} \leq \mathbf{P}\{|\theta \cdot x_\pi| \geq t\} \leq \mathbf{P}\left\{|x_\pi| \geq \frac{t}{2}\right\},$$

где $\mathbf{P}$ — равномерное распределение на группе $\{\pi\}$ всех перестановок чисел $\{1, \dots, n\}$, $x_\pi = (x_{\pi(1)}, \dots, x_{\pi(n)})$, $\theta \cdot x_\pi = (\theta_1 x_{\pi(1)}, \dots, \theta_n x_{\pi(n)})$ и $|y| = \max_{1 \leq k \leq n} \{|\sum_{i=1}^k y_i|\}$ для любого $y = (y_1, \dots, y_n) \in \mathbf{R}^n$.

Наше доказательство элементарно и автономно. В качестве следствия мы доказываем для случая вещественных чисел недавний результат

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Пруса [4]: если $X = (X_1, \dots, X_{2n})$ — перестановочная последовательность $2n$ вещественных случайных величин, то для любого $t > 0$

$$\mathbf{P}\left\{\max_{1 \leq j \leq 2n} \left| \sum_{i=1}^j X_i \right| > t\right\} \leq 16 \mathbf{P}\left\{\left| \sum_{i=1}^n X_i \right| > \frac{t}{3420}\right\}.$$

Ключевые слова и фразы: максимальное неравенство, перестановки, перестановочные случайные величины.

1. Introduction and main results. Let n be a positive integer, and let $\theta = (\theta_1, \dots, \theta_n)$ be a sequence of signs (i.e., $\theta_i = \pm 1$). Let $x = (x_1, \dots, x_n)$ be a sequence of real numbers such that $\sum_{i=1}^n x_i = 0$. In what follows we take as $\mathbf{P}$ the uniform probability on $\Pi = \{\pi\}$, the collection of all permutations of $\{1, \dots, n\}$. Chobanyan and Salehi [1] proved the following result

$$\frac{1}{10} \mathbf{P}\{|x_\pi| \geq 4t\} \leq \frac{1}{2^n} \sum_{\theta \in \Theta} \mathbf{P}\{|\theta \cdot x_\pi| \geq t\} \leq \mathbf{P}\left\{|x_\pi| \geq \frac{t}{2}\right\}, \quad (1)$$

where $\Theta = \{\theta\}$ is the collection of all possible n -dimensional sign vectors, and the following notation is used: $x_\pi = (x_{\pi(1)}, \dots, x_{\pi(n)})$, $\theta \cdot x_\pi = (\theta_1 x_{\pi(1)}, \dots, \theta_n x_{\pi(n)})$ and $|y| = \max_{1 \leq k \leq n} \{|\sum_{i=1}^k y_i|\}$ for every $y = (y_1, \dots, y_n) \in \mathbf{R}^n$.

In [3] we dealt with a special case. We proved that if $x_i = \pm 1$ then for every $\theta \in \Theta$ and $t > 0$

$$\mathbf{P}\{|x_\pi| \geq t\} \leq \mathbf{P}\{|\theta \cdot x_\pi| \geq t\} \leq \mathbf{P}\left\{|x_\pi| \geq \frac{t}{2}\right\}. \quad (2)$$

The result (2) can be considered as a refinement of result (1), in the case $x_i = \pm 1$, because in (1) there is averaging over Θ . Also (2) has better constants than (1). The proofs in [1] and in [3] are different.

In this paper we will drop the restriction $x_i = \pm 1$ and extend (2) to the case, where x_i are any real numbers.

Theorem. For every $\theta \in \Theta$ and $t \geq 0$ the following holds

$$\frac{1}{2} \mathbf{P}\{|x_\pi| \geq 38t\} \leq \mathbf{P}\{|\theta \cdot x_\pi| \geq t\} \leq \mathbf{P}\left\{|x_\pi| \geq \frac{t}{2}\right\}. \quad (3)$$

Our paper is self-contained and the method that we use in our proof is elementary. The proof of (3) will be presented in Section 2. It involves two different mappings on the class of the rearrangements of x , one for every side of (3). At the beginning of Section 2, the reader will find an explanation of those mappings.

While this paper was being typed (in Summer 1998) the author has learned about a result of Pruss [4]. This result, in the case of real numbers, follows from the left-hand side of our inequality (3). We will state it as a corollary to our main theorem.

Corollary. Let $X = (X_1, \dots, X_{2n})$ be an exchangeable sequence of $2n$ real valued random variables. For every $t > 0$ we have

$$\mathbf{P}\left\{\max_{1 \leq j \leq 2n} \left| \sum_{i=1}^j X_i \right| > t\right\} \leq 16 \mathbf{P}\left\{\left| \sum_{i=1}^n X_i \right| > \frac{t}{3420}\right\}. \quad (4)$$

The proof of the Corollary will appear in Section 3.

Remarks. 1. Chobanyan and Salehi [1] and Pruss [4] proved their respective results in Banach spaces. We deal here with real numbers.

2. It is not known if the constants in (3) are optimal.

3. One can also deduce (3) from (4), so the two are equivalent in the case of real numbers.

2. Proof of Inequality (3). The main effort here is to prove the left-hand side of (3). We believe that the technique that we are using here is original and simple. The idea in its informal shape is to find a (one-to-one) mapping, denoted by S , so that those rearrangements $\{x_\pi\}$ whose norm $|x_\pi|$ is large but $|\theta \cdot x_\pi|$ is small, are mapped into $S(x_\pi)$, where $|\theta \cdot S(x_\pi)|$ is large. Ideally we would like to have $|\theta \cdot S(x_\pi)| = |x_\pi|$. We cannot get that, but we try to come close to that. The mapping S is the identity map at the locations $\{\theta_i = 1\}$. The locations $\{\theta_i = -1\}$ are divided into subintervals and S reverses the order of the integers in each subinterval. The division of the locations $\{\theta_i = -1\}$ into subintervals is not universal, but depends to a certain extent, on the rearrangement itself. The basic idea appears already in Lemma 1. (See also the explanation at the beginning of the proof of Lemma 1). The proof of the left-hand side of (3) is presented in 5 steps that appear after Lemma 1. In particular, step 3 makes what I just explained into a concrete probabilistic inequality.

Now to the right-hand side of (3) whose proof appears at the end of the section. The idea of its (short) proof appears already in [1] and we bring it here for the sake of completeness. It involves a different (clever) mapping, denoted by T , of the rearrangements. This mapping is universal: All those integers at $\{\theta_i = 1\}$ are moved, in their natural order to the beginning of the permutation, and the integers at $\{\theta_i = -1\}$ come after that in reverse order.

Lemma 1. Let $y = (y_1, \dots, y_n) \in \mathbb{R}^n$. Assume $|\sum_{i=1}^n y_i| < t$, and $\max_{1 \leq i \leq n} \{|y_i|\} < t$. Then there is a permutation $\pi \in \Pi$ such that

$$|y + y_\pi| \leq |y| + t. \quad (5)$$

Furthermore, if z is a rearrangement of y , i.e., $z = y_\alpha$, for some $\alpha \in \Pi$, and $|z - y| < t$, then

$$|z + z_\pi| \leq |z| + 4t, \quad (6)$$

where π is the same permutation that was used in (5).

P r o o f. Before we present the formal proof we try to explain its simple idea. All we do is observing the locations, where the sequence $\sum_{i=1}^k y_i$ changes signs. Between each 2 such consecutive locations, the permutation π reverses the order of the integers and the result follows. The formal proof goes as follows. Define $k_0 = 0$, $k_{i+1} = \inf\{j > k_i : |\sum_{h=1}^j y_h| < t\}$, $i \geq 0$. By our assumption $k_1 = 1$, and there is m such that $n = k_m$. We define

$$\pi(j) = k_i + k_{i+1} - j + 1, \quad k_i < j \leq k_{i+1}, \quad 0 \leq i \leq m.$$

All that π does is to reverse the order of the integers in each interval $(k_i, k_{i+1}]$. To prove (5) we need to show

$$\left| \sum_{h=1}^j (y_h + y_{\pi(h)}) \right| < |y| + t, \quad 1 \leq j \leq n. \quad (7)$$

We deal with 3 cases: (I) $j \in \{k_i : 1 \leq i \leq m\}$, (II) $k_i < j < k_{i+1}$ and $0 < \sum_{h=1}^{k_i} y_h < t$, (III) $k_i < j < k_{i+1}$ and $-t < \sum_{h=1}^{k_i} y_h < 0$.

Case (I). We have from our construction

$$\sum_{h=1}^{k_i} y_h = \sum_{h=1}^{k_i} y_{\pi(h)}, \quad 0 < i \leq m, \quad (8)$$

so we get that $|\sum_{h=1}^{k_i} (y_h + y_{\pi(h)})| \leq |y| + t$, $0 < i \leq m$,

Case (II). In this case we have

$$t < \sum_{h=1}^j y_h, \quad \text{and} \quad -|y| < \sum_{h=k_i+1}^j y_{\pi(h)} < 0. \quad (9)$$

Obviously

$$\sum_{h=1}^j (y_h + y_{\pi(h)}) = \sum_{h=1}^j y_h + \sum_{h=1}^{k_i} y_{\pi(h)} + \sum_{h=k_i+1}^j y_{\pi(h)}, \quad (10)$$

so we get

$$\sum_{h=1}^j (y_h + y_{\pi(h)}) < \sum_{h=1}^j y_h + \sum_{h=1}^{k_i} y_{\pi(h)} < |y| + t, \quad (11)$$

and in the other direction

$$\sum_{h=1}^j (y_h + y_{\pi(h)}) > \sum_{h=1}^j y_h + \sum_{h=1}^{k_i} y_{\pi(h)} - |y| > -|y|. \quad (12)$$

From (11) and (12) we get (7).

Case (III). This is similar to Case (II) and we will omit the proof.

We have proved (5). We need to show (6). In Case (I) we get from (8) and the triangle inequality that

$$\left| \sum_{h=1}^{k_i} (z_h + z_{\pi(h)}) \right| \leq |z| + 2t, \quad 0 < i \leq m;$$

while in Case (II) we get the equivalent of (9) to z

$$0 < \sum_{h=1}^j z_h, \quad \text{and} \quad -|z| - 3t < \sum_{h=k_i+1}^j y_{\pi(h)} < 2t. \quad (13)$$

So the calculation will be similar to (11) and (12) and will lead to

$$\sum_{h=1}^j (z_h + z_{\pi(h)}) < |z| + 4t, \quad (14)$$

$$\sum_{h=1}^j (z_h + z_{\pi(h)}) > -|z| - 4t. \quad (15)$$

As before Case (III) is similar to Case (II). The proof of the Lemma is complete.

Proof of the left-hand side of (3). Fix $\theta \in \Theta$, $t > 0$. We need some notation. Let $\{+\} = \{1 \leq i \leq n: \theta_i = 1\}$, $\{-\} = \{1 \leq i \leq n: \theta_i = -1\}$. For any $y \in \mathbb{R}^n$, we define

$$y^+ = (y_1^+, \dots, y_n^+), \quad \text{where } y_i^+ = y_i 1_{\{+\}}(i), \quad 1 \leq i \leq n,$$

$$y^- = (y_1^-, \dots, y_n^-), \quad \text{where } y_i^- = y_i 1_{\{-\}}(i), \quad 1 \leq i \leq n.$$

We denote by A the set of all rearrangements of x , i.e., $A = \{x_\pi: \pi \in \Pi\}$. For every $y \in A$, we define

$$A_y = \{z \in A: z^+ = y^+\}, \quad \Lambda_y = \{z \in A_y: |\theta \cdot z| < t\}.$$

Obviously $\{A_y: y \in A\}$ is a decomposition of A .

Step 1. Assume $\max_{1 \leq i \leq n} \{|x_i|\} < t$. For every $y \in A$, there is a mapping $T: \Lambda_y \rightarrow A_y$ such that T is one-to-one and

$$|z^- + (Tz)^-| \leq |z^-| + 8t, \quad z \in \Lambda_y. \quad (16)$$

Proof. Without loss of generality we assume that $y \in \Lambda_y$. We start by observing that from $y = y^+ + y^-$ and $\theta \cdot y = y^+ - y^-$, it follows that $\sum_{i=1}^n y_i^+ + \sum_{i=1}^n y_i^- = 0$ and $|\sum_{i=1}^n y_i^+ - \sum_{i=1}^n y_i^-| < t$. We conclude that

$$\left| \sum_{i=1}^n y_i^- \right| < \frac{t}{2}. \quad (17)$$

Also, if $z \in \Lambda_y$ then $|\theta \cdot z| = |y^+ - z^-| < t$. Remembering that $|y^+ - y^-| < t$, we get that

$$|z^- - y^-| < 2t. \quad (18)$$

Now we use Lemma 1 (replacing t by $2t$), and we get that there is $\pi \in \Pi$ such that

$$|z^- + (z^-)_\pi| \leq |z^-| + 8t, \quad z \in \Lambda_y. \quad (19)$$

Observe that since $z^-(i) = 0$, $i \in \{+\}$, we can and will assume that π satisfy

$$\pi(i) = 1, \quad i \in \{+\}. \quad (20)$$

Finally we put

$$T(z) = z_\pi, \quad z \in \Lambda_y. \quad (21)$$

We get that the range of T is indeed a subset of A_y because of (20).

Step 2. Assume $\max_{1 \leq i \leq n} \{|x_i|\} < t$. Let $\Lambda = \{z \in A: |\theta \cdot z| < t\}$. There is a mapping $S: \Lambda \rightarrow A$ such that S is one-to-one and satisfy

$$|z^- + (Sz)^-| \leq |z^-| + 8t \quad \text{and} \quad (Sz)^+ = z^+, \quad z \in \Lambda. \quad (22)$$

Proof. Obviously we can find $\{y_i \in A, i \leq k\}$ such that $\Lambda = \cup_{i=1}^k \Lambda_{y_i}$, where $\Lambda_{y_i} \cap \Lambda_{y_j} = \emptyset$, $i \neq j$. By Step 1, we have for every $1 \leq i \leq k$, a mapping $T_i: \Lambda_{y_i} \rightarrow A_{y_i}$, that satisfy (16). Now we define

$$S(z) = T_i(z), \quad z \in \Lambda_{y_i}. \quad (23)$$

Step 3. Assume $\max_{1 \leq i \leq n} \{|x_i|\} < t$. Then $\mathbf{P}\{|x_\pi| \geq 19t, |\theta \cdot x_\pi| < t\} \leq \mathbf{P}\{|\theta \cdot x_\pi| \geq t\}$.

Proof. We need to show that if $z \in \Lambda$ satisfy $|z| \geq 19t$, then $|\theta \cdot S(z)| \geq t$. We calculate using Step 2:

$$\begin{aligned} |\theta \cdot S(z)| &= |z^+ - (Sz)^-| = |z - (z^- + (Sz)^-)| \geq |z| - |z^-| - 8t \\ &= |z| - \left| \frac{z - \theta \cdot z}{2} \right| - 8t \geq \left| \frac{z}{2} \right| - 8.5t \geq t. \end{aligned}$$

Step 4. Assume $\max_{1 \leq i \leq n} \{|x_i|\} < t$. Then

$$\mathbf{P}\{|x_\pi| \geq 19t\} \leq 2\mathbf{P}\{|\theta \cdot x_\pi| \geq t\}. \quad (24)$$

Proof. We will use Step 3:

$$\begin{aligned} \mathbf{P}\{|x_\pi| \geq 19t\} &= \mathbf{P}\{|x_\pi| \geq 19t, |\theta \cdot x_\pi| < t\} + \mathbf{P}\{|x_\pi| \geq 19t, |\theta \cdot x_\pi| \geq t\} \\ &\leq \mathbf{P}\{|\theta \cdot x_\pi| \geq t\} + \mathbf{P}\{|x_\pi| \geq 19t, |\theta \cdot x_\pi| \geq t\} \leq 2\mathbf{P}\{|\theta \cdot x_\pi| \geq t\}. \end{aligned}$$

Step 5. $\mathbf{P}\{|x_\pi| \geq 38t\} \leq 2\mathbf{P}\{|\theta \cdot x_\pi| \geq t\}$.

Proof. When $\max_{1 \leq i \leq n} \{|x_i|\} < 2t$, the result follows from Step 4 by replacing t by $2t$. If $\max_{1 \leq i \leq n} \{|x_i|\} \geq 2t$, then $\mathbf{P}\{|\theta \cdot x_\pi| \geq t\} = 1$.

This ends the proof of the left-hand side of (3).

The proof of the right-hand side of (3). We define a one-to-one (and onto) function $T: A \rightarrow A$. Let $z \in A$.

Put $\tilde{z} = (z_1^+, \dots, z_n^+, z_n^-, \dots, z_1^-)$. We take as $T(z)$ the vector $\tilde{z}$ after removing all zero components from it. A simple calculation shows that

$$|T(z)| = \max\{|z^+|, |z^-|\}. \quad (25)$$

The following inequality is obvious

$$|\theta \cdot z| \leq 2 \cdot \max\{|z^+|, |z^-|\}. \quad (26)$$

So we can combine (25) and (26) and get the result.

3. Application to exchangeable random variables. In this section we will show how to prove inequality (4) by using the left-hand side of our inequality (3). We will do it in 3 steps that show how to reduce the general problem to the setup that was used in this paper (Steps 2 and 3) and how to prove the result in our setup (Step 1).

Step 1. We will prove inequality (4) in our setup. We assume that $\sum_{i=1}^{2n} x_i = 0$ and that $\mathbf{P}$ stands for the uniform probability on $\{\pi\}$, the permutations of $\{1, \dots, 2n\}$. We define $X_i(\pi) = x_{\pi(i)}$, $1 \leq i \leq 2n$, and by that we have an exchangeable sequence of $2n$ random variables. Also, in the following calculation $\{\varepsilon_i, 1 \leq i \leq n\}$ will be independent identically distributed random variables with $\mathbf{P}\{\varepsilon_i = \pm 1\} = \frac{1}{2}$. The $\{\varepsilon_i\}$ will be independent of the random variables $\{X_i\}$. We will also denote $\theta_i = (-1)^{i+1}$, $1 \leq i \leq 2n$. We calculate (for the convenience of the reader we will drop the subscript notation of the permutation):

$$\begin{aligned} \mathbf{P}\left\{\left|\sum_{i=1}^n x_i\right| > t\right\} &= \mathbf{P}\left\{\left|\sum_{i=1}^n (x_i - x_{n+i})\right| > 2t\right\} = \mathbf{P}\left\{\left|\sum_{i=1}^n (x_{2i-1} - x_{2i})\right| > 2t\right\} \\ &= \mathbf{P}\left\{\left|\sum_{i=1}^n \varepsilon_i (x_{2i-1} - x_{2i})\right| > 2t\right\} \geq \frac{1}{2} \mathbf{P}\left\{\max_{1 \leq j \leq n} \left|\sum_{i=1}^j \varepsilon_i (x_{2i-1} - x_{2i})\right| > 2t\right\} \\ &= \frac{1}{2} \mathbf{P}\left\{\max_{1 \leq j \leq n} \left|\sum_{i=1}^j (x_{2i-1} - x_{2i})\right| > 2t\right\} \geq \frac{1}{4} \mathbf{P}\{|\theta \cdot x| > 6t\} \geq \frac{1}{8} \mathbf{P}\{|x| > 228t\}, \end{aligned}$$

where the first step follows from the assumption $\sum_{i=1}^{2n} x_i = 0$, the second and the third follow from exchangeability, the fourth step is Levy's inequality (more precisely, we condition on the x , apply Levy's inequality and then integrate over the x), the fifth step is again due to exchangeability. The seventh step is the left-hand side of (3) in our Theorem. We will show now the sixth step, namely

$$2\mathbf{P}\left\{\max_{1 \leq j \leq n} \left|\sum_{i=1}^j (x_{2i-1} - x_{2i})\right| > t\right\} \geq \mathbf{P}\{|\theta \cdot x| > 3t\}. \quad (27)$$

Equation (27) will follow from

$$\mathbf{P}\left\{|\theta \cdot x| > 3t, \max_{1 \leq j \leq n} \left|\sum_{i=1}^j (x_{2i-1} - x_{2i})\right| > t\right\} \leq \mathbf{P}\left\{\max_{1 \leq j \leq n} \left|\sum_{i=1}^j (x_{2i-1} - x_{2i})\right| > t\right\}. \quad (28)$$

Observe that if x belongs to the left-hand side event of (28), then $\max_{1 \leq i \leq n} \{x_{2i}\} \geq 2t$, and $|\sum_{i=1}^n x_{2i}| < t/2$ since $\sum_{i=1}^{2n} x_i = 0$. Let $x_{2k} = \max_{1 \leq i \leq n} \{x_{2i}\}$, $x_{2h} = \min_{1 \leq i \leq n} \{x_{2i}\}$ and assume without loss of generality that $h < k$, $x_{2k} = \max_{1 \leq i \leq n} \{x_{2i}\}$, and then necessarily $x_{2h} < 0$. To map the left-hand side to the right-hand side in (28) in a one-to-one way, we are exchanging $x_{2k} \leftrightarrow x_{2h}$. Now observe that $|\sum_{i=1}^h (x_{2i-1} - x_{2i})| < t$ and $x_{2k} - x_{2h} > 2t$ together imply that $|(\sum_{i=1}^{h-1} x_{2i-1} - x_{2i}) + (x_{2h-1} - x_{2k})| > t$, and we have proved (28).

Step 2. We will use the same setup as in Step 1 with one difference. Here $\sum_{i=1}^{2n} x_i = c$, where c is a constant not necessarily 0. We will show that the inequality can be easily reduced to the setup of Step 1.

Define

$$y_i = \begin{cases} x_i, & 1 \leq i \leq 2n-1, \\ x_{2n} - c, & i = 2n. \end{cases}$$

Without loss of generality we assume that $\max_{1 \leq i \leq 2n} \{|y_i|\} \geq |c|/2$. First we show

$$\mathbf{P}\left\{\left|\sum_{i=1}^n y_i\right| > t\right\} \leq 2\mathbf{P}\left\{\left|\sum_{i=1}^n x_i\right| > \frac{t}{3}\right\}. \quad (29)$$

To see (29) we need to observe that $\{|\sum_{i=1}^n y_i| > t\} \subset \{|\sum_{i=1}^n x_i| > t - |c|\}$, $\mathbf{P}\{|\sum_{i=1}^n x_i| \geq c/2\} \geq \frac{1}{2}$ as follows from $\sum_{i=1}^{2n} x_i = c$ and exchangeability, and that

$t/3 \leq \max\{|c|/2, t - |c|\}$. We now show

$$\mathbf{P}\{|x| > t\} \leq \mathbf{P}\left\{|y| > \frac{t}{5}\right\}. \quad (30)$$

To see (30) we observe that $\{|x| > t\} \subset \{|y| > t - |c|\}$, $\mathbf{P}\{|y| \geq c/4\} = 1$ as follows from $\max_{1 \leq i \leq 2n} \{|y_i|\} \geq |c|/2$, and that $t/5 \leq \max\{|c|/4, t - |c|\}$.

From Step 1, (29), (30) we get

$$\mathbf{P}\{|x| > t\} \leq 16\mathbf{P}\left\{\left|\sum_{i=1}^n x_i\right| > \frac{t}{3420}\right\}. \quad (31)$$

Step 3. Here we will deal with the general case. The conditional distribution of $X = (X_1, \dots, X_{2n})$ given the exchangeable σ -algebra, Δ , is exactly the setup of Step 2. So we get from (31)

$$\mathbf{P}\left\{\max_{1 \leq j \leq 2n} \left|\sum_{i=1}^j X_i\right| > t \mid \Delta\right\} \leq 16\mathbf{P}\left\{\left|\sum_{i=1}^n X_i\right| > \frac{t}{3420} \mid \Delta\right\} \quad \text{a.s.} \quad (32)$$

and when we integrate (32) we get the desired result.

Remark. To define the exchangeable σ -algebra Δ , we assume without loss of generality that the probability space is $\Omega = \mathbf{R}^{2n}$ and $X = (X_1, \dots, X_{2n})$ are the coordinate maps. We define $\Delta = \{A \in \beta(\mathbf{R}^{2n}) : \pi^{-1}(A) = A, \forall \pi \in \Pi\}$, where $\pi^{-1}(A) = \{x \in \mathbf{R}^{2n} : x_\pi \in A\}$ and Π denotes the collection of all the permutations of $\{1, \dots, 2n\}$. For more details the reader may consult [2, p. 174].

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ПИСЬМО В РЕДАКЦИЮ

With regard to my paper: «Representation of a class of semimartingales as stable integrals» (1998, v. 43, № 4, p. 808–818) a useless restriction appears in § 4 (Final considerations, p. 817). To remove it:

— on p. 817, lines 7 and 8 down, are to be replaced by the following ones: «there exists on $\mathbf{B}$ a predictable process H and an α -stable Lévy motion Z ($0 < \alpha < 2$) (respectively with a symmetric Lévy measure) such that»;

— on p. 817, line 14 up is to be replaced by the following one: «by means of a Z with a symmetric (for all α , $0 < \alpha < 2$) Lévy measure, X being decomposed, when $\alpha = 1$, as in Theorem 1b)».

Misprints: in formula (3.1), p. 813 the symbol H_s is to be replaced by H_t ; on p. 816, line 6 down, (2.3) is to be replaced by (2.1).

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