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Пусть $J_l \neq \emptyset$, $J_n \neq \emptyset$. Неравенства (7) равносильны неравенствам

$$\frac{(b_i - b_n) \cdot |J_n| \cdot x_n}{m_n} \neq b_i - a - \frac{(b_i - b_l)C_l}{d_l^{\gamma_l}} - \sum_{\substack{k=1, \\ k \neq l, n}}^r (b_i - b_k) \sum_{J_k} p_j. \quad (25)$$

Рассуждая, как и ранее, показываем, что неравенства (25) справедливы. Таким образом, построенная мартингальная мера P удовлетворяет ОУНБ.

Теорема В полностью доказана.

З а м е ч а н и е 4. Теорема В является далеко не окончательным результатом в описании (B,S)-рынков, на которых существуют мартингальные меры, удовлетворяющие ОУНБ. По мнению авторов настоящей работы, если $4 \leq r < \infty$, $b_1 < b_2 < \dots < b_r$, $b_1 < a < b_r$ и a не совпадает ни с одним b_k , то мартингальные меры, удовлетворяющие ОУНБ, всегда существуют. Однако в настоящее время даже для случая $r = 4$ авторы не имеют доказательства этого факта.

Авторы выражают глубокую благодарность академику РАН А.Н.Ширяеву за важные замечания, которые были использованы в данной статье.

СПИСОК ЛИТЕРАТУРЫ

1. Данежянц А. Г., Павлов И. В. Об ослабленном свойстве универсальной хааровской единственности. — Обозрение прикл. и промышл. матем., 2004, т. 11, в. 3, с. 506–508.
2. Данежянц А. Г. О специальных хааровских интерполяциях мартингалов. — Изв. вузов. Северо-Кавказский регион. Естественные науки, приложение, 2005, № 3, с. 3–20.
3. Павлов И. В., Цветкова И. В., Шамраева В. В. Некоторые результаты о мартингальных мерах одношаговых моделей финансовых рынков, связанные с условием несовпадения барицентров. — Вестник РГУПС, 2012, № 3, с. 177–181.

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ON AN OPTIMAL STOPPING PROBLEM OF AN INSIDER¹⁾

В работе рассматривается задача оптимальной остановки $v^{(\varepsilon)} := \sup_{\tau \in \mathcal{T}_{0,T}} \mathbf{E} B_{(\tau-\varepsilon)+}$, сформулированная А.Н.Ширяевым в докладе, сделанном на Международной конференции «Stochastic Optimization and Optimal Stopping», организованной Математическим институтом им. В. А. Стеклова в Москве в сентябре 2012 года. Пусть $T > 0$ — фиксированный временной горизонт, $(B_t)_{0 \leq t \leq T}$ — броуновское движение, $\varepsilon \in [0, T]$ — некоторая константа и $\mathcal{T}_{\varepsilon, T}$ — множество моментов остановки со значениями на $[\varepsilon, T]$. Решение

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этой задачи описывается обратными стохастическими дифференциальными уравнениями с отражением, коэффициенты которых зависят от траекторий, что влечет непрерывность $\varepsilon \rightarrow v^{(\varepsilon)}$. Для достаточно больших ε в работе получено явное выражение для $v^{(\varepsilon)}$, для малых ε получены верхняя и нижняя оценки. Основным результатом статьи является асимптотика $v^{(\varepsilon)}$ при $\varepsilon \searrow 0$. Помимо этого в статье получена теорема Леви о модуле непрерывности в L^1 -норме.

Ключевые слова и фразы: задача оптимальной остановки инсайдера, теорема Леви о модуле непрерывности для броуновского движения.

1. Introduction. In this paper we consider Shiryaev's optimal stopping problem

$$v^{(\varepsilon)} = \sup_{\tau \in \mathcal{T}_{0,T}} \mathbf{E} B_{(\tau-\varepsilon)+}, \quad (1)$$

where $T > 0$ is a fixed time horizon, $(B_t)_{0 \leq t \leq T}$ is the Brownian motion, $\varepsilon \in [0, T]$ is a constant, and $\mathcal{T}_{\varepsilon,T}$ is the set of stopping times taking values in $[\varepsilon, T]$. This can be thought of a problem of an insider in which she is allowed to peek ε into the future for the payoff before making her stopping decision.

We show that $v^{(\varepsilon)}$ is the solution of a corresponding path dependent reflected backward stochastic differential equation (RBSDEs). This is essentially an existence result, and it shows that an optimal stopping time exists. But the main advantage of using an RBSDE representation is that we can easily get the continuity of $v^{(\varepsilon)}$ with respect to ε from the stability of the RBSDEs. However, we want to compute the function as explicitly as possible, and the RBSDE representation of the problem does not help. This is because the problem is path dependent (one of the state variables would have been an entire path of length ε), and there is no numerical result available so far that can cover our case.

In fact, we will observe that $v^{(\varepsilon)} = \sqrt{2(T-\varepsilon)/\pi}$ if $\varepsilon \in [T/2, T]$, while as far as we know there is no explicit solution for $v^{(\varepsilon)}$ if $\varepsilon \in (0, T/2)$. But for smaller ε , there are only lower and upper bounds available. As the main result of this paper, we provide the asymptotic behavior of $v^{(\varepsilon)}$ as $\varepsilon \searrow 0$ (see Theorem A). As a byproduct, we also get Lévy's modulus of continuity theorem in the L^1 sense as opposed to the almost sure sense (compare Corollary 2 and, e.g., [2, Theorem 9.25, p. 114]).

2. First observations. Let $T > 0$ and let $\{B_t, t \in [0, T]\}$ be a Brownian motion defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and let $\mathbf{F} = \{\mathcal{F}_t, t \in [0, T]\}$ be the natural filtration augmented by the $\mathbf{P}$ -null sets of $\mathcal{F}$. We aim at the problem (1). But for the sake of generality, let us first look at the more general optimal stopping problem of an insider:

$$w = \sup_{\tau \in \mathcal{T}_{\varepsilon,T}} \mathbf{E} \left[\sum_{i=1}^n \varphi_{(\tau-\varepsilon^i)+}^i \right], \quad (2)$$

where $(\varphi_t^i)_{0 \leq t \leq T}$ is continuous and progressively measurable, $\varepsilon^i \in [0, T]$, $i = 1, \dots, n$, are given constants, and $\mathcal{T}_{\varepsilon,T}$ is the set of stopping times that lie between a constant $\varepsilon \in [0, T]$ and T . Observe that $\tau - \varepsilon^i$ is not a stopping time with respect to $\mathbf{F}$ for $\varepsilon^i > 0$. The solution to (2) is described by the following result.

Proposition 1. Assume $\mathbf{E} [\sup_{0 \leq t \leq T} (\xi_t^+)^2] < \infty$, where $\xi_t = \sum_{i=1}^n \varphi_{(t-\varepsilon^i)+}^i$, $0 \leq t \leq T$. Then the value defined in (2) can be calculated using a reflected backward stochastic differential equation (RBSDE). More precisely, $w = \mathbf{E} Y_\varepsilon$, for any $\varepsilon \in [0, T]$, where $(Y_t)_{0 \leq t \leq T}$

satisfies the RBSDE

$$\begin{aligned}\xi_t &\leq Y_t = \xi_T - \int_t^T Z_s dW_s + (K_T - K_t), \quad 0 \leq t \leq T, \\ \int_0^T (Y_t - \xi_t) dK_t &= 0.\end{aligned}\tag{3}$$

Moreover, there exists an optimal stopping time $\hat{\tau}$ described by

$$\hat{\tau} = \inf\{t \in [\varepsilon, T]: Y_t = \xi_t\}.$$

R e m a r k 1. One should note that the optimal stopping problem we are considering is path dependent (i.e., not of Markovian type) and therefore one would not be able to write down a classical free boundary problem corresponding to (1).

We prefer to use an RBSDE representation of the value function instead of directly using the representation directly from the classical optimal stopping theory because we want to use the stability result, which we will state in Corollary 3.6, associated with the former.

P r o o f o f P r o p o s i t i o n 1. For any $\tau \in \mathcal{T}_{\varepsilon, T}$,

$$\mathbf{E} \xi_\tau = \mathbf{E} [\mathbf{E} [\xi_\tau | \mathcal{F}_\varepsilon]] \leq \mathbf{E} \left[\operatorname{ess\,sup}_{\sigma \in \mathcal{T}_{\varepsilon, T}} \mathbf{E} [\xi_\sigma | \mathcal{F}_\varepsilon] \right].$$

Therefore,

$$w = \sup_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E} \xi_\tau \leq \mathbf{E} \left[\operatorname{ess\,sup}_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E} [\xi_\tau | \mathcal{F}_\varepsilon] \right].\tag{4}$$

By Theorem 5.2 in [1] there exists a unique solution (Y, Z, K) to the RBSDE in (3). Then by Proposition 2.3 (and its proof) in [1] we have

$$\sup_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E} \xi_\tau \geq \mathbf{E} \xi_{\hat{\tau}} = \mathbf{E} Y_{\hat{\tau}} = \mathbf{E} Y_\varepsilon = \mathbf{E} \left[\operatorname{ess\,sup}_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E} [\xi_\tau | \mathcal{F}_\varepsilon] \right].$$

Along with (4) the last inequality completes the proof.

Now let us get back to Shiryaev's problem (1). As a corollary of Proposition 1, we have the following result for $v^{(\varepsilon)}$, $\varepsilon \in [0, T]$.

Corollary 1. *The value defined in (1) can be calculated using an RBSDE. More precisely, $v^\varepsilon = Y_0$ almost surely, where $(Y_t)_{0 \leq t \leq T}$ satisfies the RBSDE (3) with ξ defined as $\xi_t = B_{(t-\varepsilon)+}$, $0 \leq t \leq T$. Moreover, there exists an optimal stopping time $\tilde{\tau}$ described by*

$$\tilde{\tau} = \inf\{t \geq 0: Y_t = B_{(t-\varepsilon)+}\} \geq \varepsilon 1_{\{\varepsilon < T\}} \quad \text{a.s.}\tag{5}$$

Furthermore, the function $\varepsilon \rightarrow v^{(\varepsilon)}$, $\varepsilon \in [0, T]$, is a continuous function.

P r o o f. By Proposition 1 $v^{(\varepsilon)} = Y_0$ a.s. and $\tilde{\tau}$ defined in (2.6) is optimal. Besides, the continuity of $\varepsilon \rightarrow v^{(\varepsilon)}$, $\varepsilon \in [0, T]$, is a direct consequence of the stability of RBSDEs indicated by Proposition 3.6 in [1]. Observe that for $\varepsilon \in (0, T)$ and $t \in [0, \varepsilon]$, $Y_t \geq \mathbf{E}[Y_\varepsilon | \mathcal{F}_t] > 0 = B_{(t-\varepsilon)+}$ a.s. Hence we have that $\tilde{\tau} \geq \varepsilon 1_{\{\varepsilon < T\}}$ a.s.

R e m a r k 2. In the above result, since for any $\delta \in [0, \varepsilon]$

$$v^{(\varepsilon)} = \sup_{\tau \in \mathcal{T}_{0, T}} \mathbf{E} B_{(\tau-\varepsilon)+} = \sup_{\tau \in \mathcal{T}_{\delta, T}} \mathbf{E} B_{(\tau-\varepsilon)+},$$

we can conclude from Proposition 1 that $v^{(\varepsilon)} = \mathbf{E} Y_\delta$, which implies that $(Y_t)_{t \in [0, \varepsilon]}$ is a martingale.

Further, we will make some observations about the magnitude of the function $\varepsilon \rightarrow v^{(\varepsilon)}$.

R e m a r k 3. Observe that for $\varepsilon \in (0, T)$, insider's value defined in (1) is strictly greater than 0 (and hence does strictly better than a stopper which does not possess the insider information):

$$v^{(\varepsilon)} \geq \mathbf{E} \left[\max_{0 \leq t \leq \varepsilon \wedge (T-\varepsilon)} B_t \right] = \sqrt{\frac{2}{\pi}} (\varepsilon \wedge (T-\varepsilon)) > v^{(0)} = 0,$$

which shows that there is an incentive for waiting. We also have an upper bound

$$v^{(\varepsilon)} \leq \mathbf{E} \left[\max_{0 \leq t \leq T} B_t \right] = \sqrt{\frac{2T}{\pi}}.$$

In fact, when $\varepsilon \in [T/2, T]$, $v^{(\varepsilon)}$ can be explicitly determined as

$$v^{(\varepsilon)} = \mathbf{E} \left[\max_{0 \leq t \leq T-\varepsilon} B_t \right] = \sqrt{\frac{2(T-\varepsilon)}{\pi}}, \quad \varepsilon \in \left[\frac{T}{2}, T \right],$$

and we have a strict lower bound for $\varepsilon \in [0, T/2)$

$$v^{(\varepsilon)} > \mathbf{E} \left[\max_{0 \leq t \leq \varepsilon} B_t \right] = \sqrt{\frac{2\varepsilon}{\pi}}, \quad \varepsilon \in \left[0, \frac{T}{2} \right).$$

3. Asymptotic behavior of $v^{(\varepsilon)}$ as $\varepsilon \searrow 0$. The following theorem states that the order of $v^{(\varepsilon)}$ defined in (1) is $\sqrt{2\varepsilon \ln \varepsilon^{-1}}$ as $\varepsilon \searrow 0$, which is the same as Levy's modulus for Brownian motion. Notice that

$$v^{(\varepsilon)} = \sup_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E} [B_{\tau-\varepsilon} - B_{\tau}].$$

Theorem A. *We have*

$$\lim_{\varepsilon \searrow 0} \frac{v^{(\varepsilon)}}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} = 1. \quad (6)$$

In order to prepare the proof of the theorem, we will need two lemmas.

Lemma 1. *We have*

$$\liminf_{\varepsilon \searrow 0} \frac{v^{(\varepsilon)}}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} \geq 1.$$

P r o o f. Let $d \in (0, 1)$ be a constant, and define $\tau^* \in \mathcal{T}_{\varepsilon, T}$

$$\tau^* := \inf \left\{ n\varepsilon: B_{(n-1)\varepsilon} - B_{n\varepsilon} \geq d\sqrt{2\varepsilon \ln \varepsilon^{-1}}, n = 1, \dots, \left\lceil \frac{T}{\varepsilon} \right\rceil - 1 \right\} \wedge T.$$

Then

$$\begin{aligned} \sup_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E} [B_{\tau-\varepsilon} - B_{\tau}] &\geq \mathbf{E} [B_{\tau^*-\varepsilon} - B_{\tau^*}] \\ &= \mathbf{E} [(B_{\tau^*-\varepsilon} - B_{\tau^*}) 1_{\{\tau^* \leq \varepsilon[T/\varepsilon]-\varepsilon\}}] + \mathbf{E} [(B_{\tau^*-\varepsilon} - B_{\tau^*}) 1_{\{\tau^* > \varepsilon[T/\varepsilon]-\varepsilon\}}] \\ &\geq d\sqrt{2\varepsilon \ln \varepsilon^{-1}} \mathbf{P} \left(\tau^* \leq \varepsilon \left\lceil \frac{T}{\varepsilon} \right\rceil - \varepsilon \right) + \mathbf{E} [(B_{T-\varepsilon} - B_T) 1_{\{\tau^* > \varepsilon[T/\varepsilon]-\varepsilon\}}] \\ &= d\sqrt{2\varepsilon \ln \varepsilon^{-1}} \mathbf{P} \left(\tau^* \leq \varepsilon \left\lceil \frac{T}{\varepsilon} \right\rceil - \varepsilon \right). \end{aligned}$$

We have

$$\begin{aligned} \mathbf{P} \left(\tau^* \leq \varepsilon \left\lceil \frac{T}{\varepsilon} \right\rceil - \varepsilon \right) &= 1 - \mathbf{P} \left(B_{(n-1)\varepsilon} - B_{n\varepsilon} < d\sqrt{2\varepsilon \ln \varepsilon^{-1}}, n = 1, \dots, \left\lceil \frac{T}{\varepsilon} \right\rceil - 1 \right) \\ &= 1 - \left[P \left(B_{\varepsilon} - B_0 < d\sqrt{2\varepsilon \ln \varepsilon^{-1}} \right) \right]^{[T/\varepsilon]-1} \\ &= 1 - \left[1 - \int_{d\sqrt{2\varepsilon \ln \varepsilon^{-1}}}^{\infty} \frac{1}{\sqrt{2\pi\varepsilon}} e^{-x^2/2\varepsilon} dx \right]^{[T/\varepsilon]-1} \\ &= 1 - (1 - \alpha)^{\frac{1}{\alpha}([T/\varepsilon]-1)\alpha}, \end{aligned} \quad (7)$$

where

$$\alpha := \int_{d\sqrt{2\varepsilon \ln \varepsilon^{-1}}}^{\infty} \frac{1}{\sqrt{2\pi\varepsilon}} e^{-x^2/2\varepsilon} dx = \frac{1}{2d\sqrt{\pi \ln \varepsilon^{-1}}} \varepsilon^{d^2} (1 + o(1)) \rightarrow 0,$$

by, e.g., [2, (9.20), p. 112]. Since $d \in (0, 1)$, $([T/\varepsilon] - 1)\alpha \rightarrow \infty$, and thus

$$\mathbf{P}\left(\tau^* \leq \varepsilon \left\lceil \frac{T}{\varepsilon} \right\rceil - \varepsilon\right) \rightarrow 1, \quad \varepsilon \searrow 0.$$

Therefore,

$$\liminf_{\varepsilon \searrow 0} \frac{v^{(\varepsilon)}}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} \geq \liminf_{\varepsilon \searrow 0} \left[dP\left(\tau^* \leq \varepsilon \left\lceil \frac{T}{\varepsilon} \right\rceil - \varepsilon\right) \right] = d.$$

Then (6) follows by letting $d \nearrow 1$.

Lemma 2. *The family*

$$\left\{ \frac{\sup_{\varepsilon \leq t \leq T} |B_{t-\varepsilon} - B_t|}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} : \varepsilon \in \left(0, \frac{T \wedge 1}{2}\right] \right\}$$

is uniformly integrable.

P r o o f. Since

$$\begin{aligned} \frac{\sup_{\varepsilon \leq t \leq T} |B_{t-\varepsilon} - B_t|}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} &\leq \frac{2 \max_{1 \leq n \leq [T/\varepsilon]+1} \sup_{(n-1)\varepsilon \leq t, t' \leq n\varepsilon} |B_t - B_{t'}|}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} \\ &\leq \frac{4 \max_{1 \leq n \leq [T/\varepsilon]+1} \sup_{(n-1)\varepsilon \leq t \leq n\varepsilon} |B_t - B_{(n-1)\varepsilon}|}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}}, \end{aligned}$$

it suffices to show that the family

$$\left\{ M_\varepsilon := \frac{\max_{1 \leq n \leq [T/\varepsilon]+1} \sup_{(n-1)\varepsilon \leq t \leq n\varepsilon} |B_t - B_{(n-1)\varepsilon}|}{\sqrt{\varepsilon \ln \varepsilon^{-1}}} : \varepsilon \in \left(0, \frac{T \wedge 1}{2}\right] \right\}$$

is uniformly integrable. For $a \geq 0$,

$$\mathbf{P}(M_\varepsilon \leq a) = \left[\mathbf{P}\left(\sup_{0 \leq t \leq \varepsilon} |B_t| \leq a \sqrt{\varepsilon \ln \varepsilon^{-1}}\right) \right]^{[T/\varepsilon]+1}.$$

Hence the density f_ε of M_ε , satisfies that for $a \geq 0$,

$$\begin{aligned} f_\varepsilon(a) &\leq ([T\varepsilon^{-1}] + 1) \left[\mathbf{P}\left(\sup_{0 \leq t \leq \varepsilon} |B_t| \leq a \sqrt{\varepsilon \ln \varepsilon^{-1}}\right) \right]^{[T/\varepsilon]} \sqrt{\frac{8}{\pi}} \sqrt{\ln \varepsilon^{-1}} e^{-\ln \varepsilon^{-1} a^2/2} \\ &\leq \frac{4T\sqrt{\ln \varepsilon^{-1}}}{\varepsilon} e^{-\ln \varepsilon^{-1} a^2/2}, \end{aligned}$$

where for the first inequality we use, e.g., [2, (8.3), p. 96], and the fact that the density of $\sup_{0 \leq t \leq \varepsilon} |B_t|$ is not greater than twice the density of $\sup_{0 \leq t \leq \varepsilon} B_t$. Then we have that for $N > 0$,

$$\begin{aligned} \mathbf{E}[M_\varepsilon 1_{\{M_\varepsilon > N\}}] &= \int_N^\infty x f_\varepsilon(x) dx \leq \frac{4T\sqrt{\ln \varepsilon^{-1}}}{\varepsilon} \int_N^\infty x e^{-\ln \varepsilon^{-1} x^2/2} dx \\ &= \frac{4T\varepsilon^{N^2/2-1}}{\sqrt{\ln \varepsilon^{-1}}} \leq \frac{T}{2^{N^2/2-3}\sqrt{\ln 2}}, \end{aligned}$$

i.e.,

$$\lim_{N \rightarrow \infty} \sup_{\varepsilon \in (0, T \wedge 1/2]} \mathbf{E}[M_\varepsilon 1_{\{M_\varepsilon > N\}}] = 0.$$

The lemma is proved.

Now let us turn to the proof of Theorem A.

Proof of Theorem A. We have

$$\begin{aligned} \limsup_{\varepsilon \searrow 0} \frac{\sup_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E}[B_{\tau-\varepsilon} - B_{\tau}]}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} &\leq \limsup_{\varepsilon \searrow 0} \mathbf{E} \left[\frac{\sup_{\varepsilon \leq t \leq T} |B_{t-\varepsilon} - B_t|}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} \right] \\ &\leq \mathbf{E} \left[\limsup_{\varepsilon \searrow 0} \frac{\sup_{\varepsilon \leq t \leq T} |B_{t-\varepsilon} - B_t|}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} \right] \leq 1, \end{aligned}$$

where we apply Lemma 2 for the second inequality, and use Levy's modulus for Brownian motion (see, e.g., [2, Theorem 9.25, p. 114]) for the third inequality. Together with (6), the conclusion follows. Theorem 1 is proved.

Using the above proof, we can actually show the following result, which is Lévy's modulus continuity result in the L^1 sense, as opposed to the almost sure sense (see, e.g., [2, Theorem 9.25, p. 114]).

Corollary 2. *We have*

$$\begin{aligned} \lim_{\varepsilon \searrow 0} \frac{\sup_{\tau \in \mathcal{T}_{\varepsilon, T}} \mathbf{E}[B_{\tau-\varepsilon} - B_{\tau}]}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} &= \lim_{\varepsilon \searrow 0} \mathbf{E} \left[\frac{\sup_{\varepsilon \leq t \leq T} (B_{t-\varepsilon} - B_t)}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} \right] \\ &= \lim_{\varepsilon \searrow 0} \mathbf{E} \left[\frac{\sup_{\varepsilon \leq t \leq T} |B_{t-\varepsilon} - B_t|}{\sqrt{2\varepsilon \ln \varepsilon^{-1}}} \right] = 1. \end{aligned}$$

REFERENCES

1. Karoui N. El., Kapoudjian C., Pardoux E., Peng S., Quenez M. C. Reflected solutions of backward SDE's, and related obstacle problems for PDE's. — Ann. Probab., 1997, v. 25, № 2, p. 702–737.
2. Karatzas I., Shreve S. E. Brownian Motion and Stochastic Calculus. New York: Springer-Verlag, 1988, 470 p.

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FIRST PASSAGE PROBLEMS OVER INCREASING BOUNDARIES FOR LÉVY PROCESSES WITH EXPONENTIALLY DECAYED LÉVY MEASURES

В этой заметке мы исследуем задачу о нахождении распределения времени первого выхода процесса Леви через одностороннюю нелинейную границу. Наша основная теорема утверждает, что хвост распределения времени пересечения через границу $f(t)$ имеет степенной порядок $-1/\alpha$, где $1 < \alpha \leq 2$, тогда и только тогда, когда функция $|f(t)|t^{-1/\alpha-1}$ является интегрируемой на интервале $[1, \infty)$. Этот результат обобщает результаты Новикова [10], [12].

Ключевые слова и фразы: задачу о нахождении распределения времени первого выхода; нелинейная граница, процессы Леви.

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