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NEUENSCHWANDER D.*

THE CLASS I_0 FOR RANDOM INCREASING UPPER SEMICONTINUOUS FUNCTIONS

Пусть C есть выпуклый конус $USC_*([0, 1], \mathbf{R}_+)$ возрастающих полунепрерывных сверху функций $g: [0, 1] \rightarrow \mathbf{R}_+$. Показано, что класс $I_0(C)$ распределений на C без неразложимого множителя строго вкладывается в класс безгранично-делимых распределений на C .

Ключевые слова и фразы: вероятностные меры без неразложимого множителя, полунепрерывные сверху функции, безгранично-делимые распределения.

1. Introduction. The class $I_0(\mathbf{R})$ is given as the set of those probability measures on $\mathbf{R}$ which have no indecomposable factor. It has been shown by Khintchine that every measure in I_0 is infinitely divisible (cf. [5, Theorem 5.4.2]). This inclusion is strict, cf. [7, 9.4]: A counterexample of an infinitely divisible probability distribution which is not in $I_0(\mathbf{R})$ is the compound Poisson distribution given by the characteristic function

$$\varphi(u) = \exp(\log(1 - a) - \log(1 - ae^{iu})) = \exp\left(\sum_{n=1}^{\infty} \frac{a^n}{n} (e^{iun} - 1)\right)$$

for some $0 < a < 1$. The same strict inclusion was proved in [6] in the context of q -addition of real-valued random variables. In this note, we will prove the corresponding statement for the convex cone $C = USC_*([0, 1], \mathbf{R}_+)$ of non-negative increasing upper semicontinuous functions on $[0, 1]$. Infinitely divisible laws on convex cones, in particular on $C = USC_*([0, 1], \mathbf{R}_+)$, have been considered in [3], [4]. Our methods will be based on a combination of these works and the classical proof of Theorem 5.4.2 in [5]. A major tool will be the theorem on infinite divisibility of limits of infinitesimal triangular arrays on C , which is of independent interest.

The topology on C will be one of [3, Section 6.3]. With this topology, C becomes a locally compact second countable convex cone.

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2. Limit distributions of infinitesimal triangular arrays. A well-known theorem in probability says that a distribution on $\mathbf{R}$ is infinitely divisible if and only if it is the limit of an infinitesimal triangular array Δ of distributions. The classical proof consists of showing that the row products of Δ can be approximated by compound Poisson laws, which then gives automatically the infinite divisibility of the limit. Another proof was suggested by Carnal [1] (see also [2, 6.6]), who showed directly that it is possible, for any $p \geq 1$, to split up the rows of Δ into p approximately equal convolution products. We will apply Carnal's method [1] to carry over this theorem to C .

Definition 1. A triangular array $\{\mu_{n,j}\}_{n \geq 1, 1 \leq j \leq k(n)}$ of probability measures on $C = USC_*([0, 1], \mathbf{R}_+)$ is called infinitesimal if for any Borel neighborhood U of 0 we have

$$\inf_{1 \leq j \leq k(n)} \mu_{n,j}(U) \longrightarrow 1 \quad (n \rightarrow \infty).$$

The convolution product $*$ on C is with respect to ordinary addition $+$. Let $M^1(C)$ be the topological monoid of probability measures of C with operation $*$ and the weak topology. A C -valued probability distribution μ is called infinitely divisible if, for any $p \geq 1$, there is a C -valued probability measure μ_p such that $\mu_p^{*p} = \mu$. A C -valued probability measure μ is called the limit of a triangular array $\{\mu_{n,j}\}_{n \geq 1, 1 \leq j \leq k(n)} \subset M^1(C)$ if it is the weak limit of row convolution products:

$$\mu_{n,1} * \mu_{n,2} * \cdots * \mu_{n,k(n)} \xrightarrow{w} \mu \quad (n \rightarrow \infty).$$

A character on C is a continuous function $\chi: C \rightarrow [0, 1]$ such that $\chi(x+y) = \chi(x)\chi(y)$ ($x, y \in C$) and $\chi(0) = 1$. Let X be the monoid of characters on C . A submonoid Y of X is called plain, if it is separating (i.e., for $x, y \in C$ with $x \neq y$ there is a $\chi \in Y$ such that $\chi(x) \neq \chi(y)$), $1 \in Y$, and for every $x \in C$ there is a $\chi \in Y$ such that $\chi(x) \neq 0$. The Laplace transform of $\mu \in M^1(C)$ is given by $\mathcal{L}\mu(\chi) = \int_C \chi(x) \mu(dx)$. The following continuity property holds (cf. [4, Lemma 3.1]):

Lemma 1. Let Y be a plain submonoid of X . Assume μ, μ_n ($n \geq 1$) are probability measures on C . Then $\mu_n \xrightarrow{w} \mu$ if and only if $\mathcal{L}\mu_n(\chi) \rightarrow \mathcal{L}\mu(\chi)$ for every $\chi \in Y$.

Let

$$\mathcal{F} := \left\{ \lambda(x): C \rightarrow \mathbf{R}_+ : \lambda(x) = \int_0^1 x \lambda(dx), \lambda \in M_{+, \text{cont}}^b \right\}$$

(where $M_{+, \text{cont}}^b$ denotes the class of bounded non-negative measures on $[0, 1]$ which are continuous on $[0, 1[$). By [4, 4.3], $e^{-\mathcal{F}}$ is a plain submonoid of X . In order to apply Carnal's method we must supply a countable plain submonoid of X . For $0 \leq a \leq b \leq 1$, let $U_{a,b}$ be the uniform probability measure on the interval $[a, b]$. Let L be the class of finite convex combinations of measures $U_{a,b}$ ($a, b \in \mathbf{Q}$, $0 \leq a \leq b \leq 1$). Then the following lemma can easily be verified:

Lemma 2. Let $\mathcal{F}_0 := \{\lambda(x): C \rightarrow \mathbf{R}_+ : \lambda(x) = \int_0^1 x \lambda(dx): \lambda \in L\}$. Then $Y_0 = e^{-\mathcal{F}_0}$ is a plain submonoid of X .

By looking at the Laplace transform it follows that, for any $p \geq 1$, the set of p -th convolution roots in $M^1(C)$ of some fixed $\mu \in M^1(C)$ is weakly compact.

Theorem 1. Every limit distribution of an infinitesimal triangular array $\{\mu_{n,j}\}_{n \geq 1, 1 \leq j \leq k(n)}$ of probability measures on $C = USC_*([0, 1], \mathbf{R}_+)$ is infinitely divisible on C .

Proof. Let $p \geq 1$ be fixed and write $F_0 = \{\lambda_q(x)\}_{q \geq 1}$. Let $\varepsilon > 0$ and $Q \geq 1$. As in the proof of Heyer, [2, Remark 6.5.3 and Theorem 6.6.4], there exists $n_0 = n_0(Q, \varepsilon)$ such that for every $n \geq n_0$ one can decompose the index set $\{1, 2, \dots, k(n)\}$ into p disjoint classes $\Pi_1^{(n)}, \Pi_2^{(n)}, \dots, \Pi_p^{(n)}$ with the property

$$\left| \prod_{j \in \Pi_r^{(n)}} \mathcal{L}\mu_{n,j}(e^{-\lambda_q(x)}) - \prod_{j \in \Pi_s^{(n)}} \mathcal{L}\mu_{n,j}(e^{-\lambda_q(x)}) \right| < \varepsilon \quad (1 \leq q \leq Q; 1 \leq r, s \leq p).$$

Now (as in the proof of Heyer, [2, Theorem 6.6.4]) putting $\varepsilon_Q := 1/Q$ and $n_Q := n_0(Q, \varepsilon_Q)$ and going over to a suitable subsequence one gets, by Lemmas 1 and 2, the assertion.

3. The class I_0 . Let $C = USC_*([0, 1], \mathbf{R}_+)$.

Definition 2. The class $I_0(C)$ on $C = USC_*([0, 1], \mathbf{R}_+)$ is defined as the set of those probability measures $\mu \in M^1(C)$ such that every non-degenerate convolution factor of μ is itself decomposable into a convolution product of two non-degenerate probability measures in $M^1(C)$.

For a probability measure $\mu \in M^1(C)$ for random elements $g \in C$, let $\mu_x \in M^1(\mathbf{R}_+)$ be the distribution of $g(x)$ ($x \in [0, 1]$).

The following proposition is maybe also of some independent interest.

Proposition 1. Suppose $\mu \in I_0(C)$ such that μ_1 is degenerate. Then μ is degenerate.

P r o o f. 1. Since the random elements $g \in C$ are non-negative and increasing, the supports of all the μ_x must be bounded under our assumptions. We suppose that μ is non-degenerate and show that μ and thus every μ_x ($x \in [0, 1]$) must also be infinitely divisible, which is not possible since the only infinitely divisible laws on $\mathbf{R}$ with bounded support are degenerate ones (this follows from the Lévy-Khintchine formula, cf. [5, 6.1]).

2. Denote, for $\ell \in \mathbf{N}$, by $\pi_\ell \in M^1(\mathbf{R}_+)$ the elementary Poisson measure with parameter $1/\ell$ on $\mathbf{N}_0$, and let $\bar{\pi}_\ell$ be the probability measure on C given as $(\bar{\pi}_\ell)_x = \delta_0 \in M^1(\mathbf{R}_+)$ for $0 \leq x < 1$ and $(\bar{\pi}_\ell)_1 = \pi_\ell \in M^1(\mathbf{R}_+)$. Then, under our assumption, evidently $\mu_\ell = \mu * \bar{\pi}_\ell$ is in $I_0(C)$ and $\mu_\ell \xrightarrow{w} \mu$ ($\ell \rightarrow \infty$), so μ remains infinitely divisible if every μ_ℓ is. So it suffices to verify, for $\ell \geq 1$, the infinite divisibility of μ_ℓ .

3. Let ℓ be fixed. If for two non-degenerate convolution factors $\alpha, \beta \in M^1(C)$ of μ_ℓ we would have, say, that α_1 is degenerate, then by the Raikov-Ottaviani Theorem on the factor closedness of the family of elementary Poisson distributions (cf. [5, Theorem 6.6.1]), β_1 has to be an elementary Poisson distribution ρ on $\mathbf{N}_0$. If β is the law of the random elements $g \in C$, let β_- be the distribution of the functions $[0, 1[\ni x \mapsto g(x)$. By the construction of μ_ℓ , we may assume that β_- and β_1 are independent. So we can decompose (using notation a little bit freely, but unambiguously)

$$\alpha * \beta = (\alpha * (\delta_0, \rho^{*(1/2)})) * (\beta * (\delta_0, \rho^{*(-1/2)})) \in M^1(C) \times M^1(C),$$

hence without loss of generality we may suppose that $\alpha_1, \beta_1 \in M^1(\mathbf{R}_+)$ are non-degenerate. So the first part of the proof of Theorem 5.4.2 in [5, p. 86] can be translated to our situation, when we look at the Khintchine functional of μ_1 etc.

4. The second part of the proof (again by looking at Khintchine functionals of μ_1 etc.) of Theorem 5.4.2 in [5] (bottom of p. 86 until middle of p. 87) can be directly translated as well. But now the fact that if the triangular array $\{(\mu_{n,j})_{1 \leq j \leq k(n)}\}_{n \geq 1} \subset M^1(\mathbf{R}_+)$ is infinitesimal, then (since all $g \in C$ are non-negative and increasing) also $\{\mu_{n,j}\}_{n \geq 1; 1 \leq j \leq k(n)} \subset M^1(C)$ is infinitesimal yields that μ has to be infinitely divisible, which (by 1. and Theorem 1) proves the lemma.

Now we are ready to state and prove our main theorem:

Theorem 2. For $C = USC_*([0, 1], \mathbf{R}_+)$, the class $I_0(C)$ is contained in the class of all infinitely divisible probability measures in $M^1(C)$.

P r o o f. By Proposition 1, non-degeneracy of a probability law for random elements g in C implies non-degeneracy of the law of $g(1)$. This allows once again to translate directly the first two parts of the proof of Theorem 5.4.2 in [5, p. 86–87] middle in the obvious way for the Khintchine functionals of the different occurring laws for random variables of the type $g(1)$ ($g \in C$). The same argument as in 4. of the proof of Proposition 1 shows that μ is the limit of an infinitesimal triangular array on B and thus infinitely divisible by Theorem 1.

R e m a r k. By looking at constant functions, the counterexample mentioned in the introduction gives also a counterexample in our situation to show that the inclusion of $I_0(C)$ in the class of infinitely divisible distributions on C is strict also here.

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THE INVARIANCE PRINCIPLE FOR SOME CLASS OF MARKOV CHAINS¹⁾

Работа посвящена принципу инвариантности для однородных марковских цепей с конечным пространством состояний. Доказательство основано на сравнении асимптотического поведения сумм случайных величин, построенных по марковским цепям, имеющим одну и ту же переходную матрицу (за один шаг) и разные начальные распределения.

Ключевые слова и фразы: принцип инвариантности, марковская цепь с конечным пространством состояний.

1. Introduction. Let $\{X_n, n \geq 1\}$ be a sequence of integer-valued Markov chain with stationary transition probabilities

$$p_{lj} = \mathbf{P}\{X_{k+1} = j \mid X_k = l\} \quad (1.1)$$

for all $k = 1, 2, \dots$, and $l, j \in S = \{-m, \dots, -1, 0, 1, \dots, m\}$, and the initial probabilities $\pi_j = \mathbf{P}\{X_1 = j\}$, $j \in S$. Denote $S_n = X_1 + \dots + X_n$. If the sequence is stationary with

$$p_j := \mathbf{P}\{X_k = j\} = \pi_j \quad (1.2)$$

for all $k = 1, 2, \dots$, then $\{X_n, n \geq 1\}$ is φ -mixing with mixing coefficients $C\rho^n$, $n = 1, 2, \dots$, for some $C > 0$ and $0 < \rho < 1$ (cf. [3]). Consequently, at this time, the weak invariance principle holds, in other words, putting

$$Y_n(t, \omega) = \frac{S_{[nt]}(\omega) - \mathbf{E}S_{[nt]}(\omega)}{\sqrt{\mathbf{D}S_n(\omega)}}, \quad 0 \leq t \leq 1, \quad (1.3)$$

we have

$$Y_n \Rightarrow W \quad \text{as } n \rightarrow \infty, \quad (1.4)$$

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