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AN EXAMPLE OF LARGE DEVIATIONS
FOR A STATIONARY PROCESS¹⁾†

В статье дается пример больших уклонений для некоторого семейства $(X_t^\varepsilon)_{t \geq 0, \varepsilon > 0}$ с $\dot{X}_t^\varepsilon = a(X_t^\varepsilon) + b(X_t^\varepsilon)\eta_{t/\varepsilon}$, где η_t — стационарный процесс, имеющий разложение Уолда $\eta_t = \int_{-\infty}^t h(t-s) dN_s$ относительно однородного процесса N_t с независимыми и интегрируемыми с квадратом приращениями.

Ключевые слова и фразы: большие уклонения, пространство Скорохода, разложение Уолда.

§ 1. Introduction and main result

1.1. It is well known (see [3]) that a family $X^\varepsilon = (X_t^\varepsilon)_{t \geq 0, \varepsilon > 0}$, of diffusion processes

$$dX_t^\varepsilon = a(X_t^\varepsilon) dt + \sqrt{\varepsilon} b(X_t^\varepsilon) dW_t \quad (1.1)$$

subject to a fixed X_0 , where W_t is a Wiener process and $a(x)$, $b(x)$ are Lipschitz continuous and $b^2(x) > 0$, obeys the large deviation principle (l.d.p.) in the space of continuous functions $\mathbf{C}_{[0,T]}$, and the corresponding rate function is given by the formula

$$I_T(\varphi) = \begin{cases} \int_0^T \left(\frac{\dot{\varphi}_t - a(\varphi_t)}{b(\varphi_t)} \right)^2 dt, & \varphi_0 = X_0, d\varphi \ll dt, \\ \infty, & \text{otherwise,} \end{cases} \quad (1.2)$$

where the notation $\ll \varphi_0 = X_0, d\varphi \ll dt \gg$ is used for designating $\varphi_t = X_0 + \int_0^t \dot{\varphi}_s ds$, $t \leq T$, and where $\dot{\varphi}_t$ is the Radon–Nikodym derivative of φ_t . Evidently, the l.d.p. for (X_t^ε) also holds in the metric space $(\mathbf{C}, \rho)$ with $\mathbf{C} = \mathbf{C}_{[0, \infty)}$,

$$\rho(X, Y) = \sum_{n \geq 1} 2^{-n} \frac{\sup_{t \leq n} |X_t - Y_t|}{1 + \sup_{t \leq n} |X_t - Y_t|},$$

and the rate function $I(\varphi) = \sup_T I_T(\varphi)$ (see, e.g., [9]).

By contrast to (1.1), in this paper we formulate the l.d.p. for a family of processes $(X_t^\varepsilon)_{t \geq 0, \varepsilon > 0}$, defined by an ordinary differential equation

$$\dot{X}_t^\varepsilon = a(X_t^\varepsilon) + b(X_t^\varepsilon) \eta_{t/\varepsilon}, \quad (1.3)$$

subject to a fixed X_0 , where $(\eta_t)_{t \in \mathbf{R}}$ is a strictly stationary and ergodic process with $\mathbf{E}\eta_0 = 0$.

Since the Markovian property for (η_t) is not to be assumed, the general approach consists in checking the l.d.p. for the occupation measures of (η_t) for the so called «third level» of the Donsker and Varadhan theory (see [2]) and applying the contraction principle

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† Три нижеследующие статьи (Gulinsky O.V., Liptser R.Sh., Barndorff-Nielsen O.E., Mordecki E.) готовились к публикации в пятом томе Трудов семинара «Статистика и управление случайными процессами». Об этом (не вышедшем в свет издании) детальную информацию можно найти в журнале «Обзорные прикладной и промышленной математики», 1998, т. 5, в. 2, с. 308. Там же опубликована часть предназначавшихся для этого тома статей.

of Varadhan (see [11]) to get the l.d.p. for X^ϵ . To avoid an application of the «third level» we restrict ourselves to the consideration of (η_t) obeying the Wold decomposition²⁾

$$\eta_t = \int_{-\infty}^t h(t-s) dN_s, \tag{1.4}$$

w.r.t. the right continuous having limits on the left homogeneous process with independent increments $N = (N_t)_{t \in \mathbf{R}}$ such that $N_0 = 0$ and $\mathbf{E} |N_t - N_s|^2 = k^2 |t - s|$, $\mathbf{E} (N_t - N_s) = 0$ and $\int_{\mathbf{R}} h^2(t) dt < \infty$ for any $s, t \in \mathbf{R}$ (so the integral in (1.4) is understood as an Itô's stochastic integral).

For such a process (η_t) a simple proof of the l.d.p. for X^ϵ is found, the explicit formula for the rate function takes place and furthermore, this model has useful applications.

1.2. Formally, letting

$$W_t^\epsilon = \frac{1}{\sqrt{\epsilon}} \int_0^t \eta_{s/\epsilon} ds, \tag{1.5}$$

we can rewrite equation (1.3) as $dX_t^\epsilon = a(X_t^\epsilon) dt + \sqrt{\epsilon} b(X_t^\epsilon) dW_t^\epsilon$, that is similar to (1.1). But the l.d.p. even for $(\sqrt{\epsilon} W_t^\epsilon)$ would be difficult to obtain by the method from [3]. Nevertheless, by virtue of (1.4) it is possible to obtain it by a different method. Two reasons are for $(\sqrt{\epsilon} W_t^\epsilon)$ to obey the l.d.p.:

- 1. By virtue of the Birkhoff–Khinchin theorem for any fixed $T > 0$

$$\limsup_{\epsilon \rightarrow 0} \sup_{t \leq T} |\sqrt{\epsilon} W_t^\epsilon| = 0 \quad (\mathbf{P}\text{-a.s.}).$$

- 2. Under the weak dependence assumptions

$$\int_0^\infty |h(t)| dt < \infty, \quad \int_0^\infty \sqrt{\int_t^\infty h^2(s) ds} dt < \infty \tag{1.6}$$

the functional central limit theorem for (W_t^ϵ) holds, i.e., $\mathcal{L}\text{-}\lim_{\epsilon \rightarrow 0} (W_t^\epsilon) = \Sigma(W_t)$, where (W_t) is the Wiener process, $\mathcal{L}$ means convergence in distribution, and

$$\Sigma = \int_0^\infty h(t) dt \tag{1.7}$$

(see [8, Chapter 9, Section 2, ex. 2]).

Along with (1.6) assume that

$$\int_0^\infty \left(\int_t^\infty h(s) ds \right)^2 dt < \infty. \tag{1.8}$$

Also the Cramér type conditions for the jumps $(\Delta N_t = N_t - \lim_{s \uparrow t} N_s)$ of N_t are required for getting the l.d.p. of $\sqrt{\epsilon} W_{t/\epsilon}^\epsilon$. The process N as any process with square integrable and independent increments, is characterized by its continuous Gaussian component $N^c = (N_t^c)_{t \in \mathbf{R}}$ with $\mathbf{E} (Y_t^c)^2 = \sigma^2 |t|$ and the Lévy measure $K(dx) dt$, $x \in \mathbf{R}^0 = \mathbf{R} \setminus \{0\}$, $t \in \mathbf{R}$, $\int_{\mathbf{R}^0} x^2 K(dx) < \infty$. Taking into account that for any $\lambda \in \mathbf{R}$

$$\begin{aligned} \mathbf{E} \sum_{t \leq 1} (e^{\lambda \Delta N_t} - 1 - \lambda \Delta N_t) &= \int_{\mathbf{R}^0} (e^{\lambda x} - 1 - \lambda x) K(dx), \\ \mathbf{E} \sum_{t \geq 0} \left(e^{\lambda h(t) \Delta N_t} - 1 - \lambda h(t) \Delta N_t \right) &= \int_0^\infty \int_{\mathbf{R}^0} \left(e^{\lambda h(t)x} - 1 - \lambda h(t)x \right) K(dx) dt, \end{aligned}$$

assume

$$\int_{\mathbf{R}^0} (e^{\lambda x} - 1 - \lambda x) K(dx) < \infty \quad \text{for any } \lambda \in \mathbf{R}, \tag{Cr.1}$$

$$\int_0^\infty \int_{\mathbf{R}^0} \left[e^{\lambda h(t)x} - 1 - \lambda h(t)x \right] K(dx) dt < \infty \quad \text{for any } \lambda \in \mathbf{R}. \tag{Cr.2}$$

²⁾ For discrete time case the corresponding result can be found in [4].

Under (Cr.1) the cumulant function is defined:

$$G(\lambda) = \frac{\lambda^2 \sigma^2}{2} + \int_{\mathbf{R}^0} (e^{\lambda x} - 1 - \lambda x) K(dx), \quad \lambda \in \mathbf{R}. \quad (1.9)$$

Due to (Cr.1) the function $G(\lambda)$ is twice continuous differentiable with

$$G'(\lambda) = \lambda \sigma^2 + \int_{\mathbf{R}^0} x (e^{\lambda x} - 1) K(dx), \quad G''(\lambda) = \sigma^2 + \int_{\mathbf{R}^0} x^2 e^{\lambda x} K(dx), \quad (1.10)$$

and so it is a non-negative (with $G(0) = 0$) and convex function. The Legendre–Fenchel transformation of $G(\lambda)$ is defined in the usual way (see [10]):

$$H_{\sigma^2}(y) = \sup_{\lambda \in \mathbf{R}} [\lambda y - G(\lambda)], \quad y \in \mathbf{R}. \quad (1.11)$$

As $G(\lambda)$ is continuous, the supremum in (1.11) may be taken over all rationals λ and so $H_{\sigma^2}(y)$ is a measurable function in y . In the notation $H_{\sigma^2}(y)$ the dependence on σ^2 is emphasized. In particular, the fact that $H_{\sigma^2}(y) \uparrow H_0(y)$ as $\sigma^2 \rightarrow 0$, is used in proving the l.d.p. below.

1.3. For any $\varphi \in \mathbf{C}$, letting $\frac{0}{0} = 0$, put

$$I(\varphi) = \begin{cases} \int_0^\infty H_{\sigma^2} \left(\frac{\dot{\varphi}_t}{\Sigma} \right) dt, & \varphi_0 = 0, \quad d\varphi \ll dt, \\ \infty, & \text{otherwise,} \end{cases} \quad (1.12)$$

where $\langle \varphi_0 = 0, d\varphi \ll dt \rangle$ is used for designating $\varphi_t = \int_0^t \dot{\varphi}_s ds$, $t \geq 0$, and where $\dot{\varphi}_t$ is the Radon–Nikodym derivative of φ_t . Evidently, if $\Sigma = 0$ then

$$I(\varphi) = \begin{cases} 0, & \varphi_t \equiv 0, \\ \infty, & \text{otherwise.} \end{cases}$$

Now, we are in the position to formulate the l.d.p. for the family $(\sqrt{\varepsilon} W_t^\varepsilon)$.

Theorem 1.1. 1) *Let assumptions (1.6), (1.8), and (Cr.1), (Cr.2) be fulfilled. Then the family $(\sqrt{\varepsilon} W_t^\varepsilon)$ obeys the l.d.p. in $(\mathbf{C}, \rho)$ with the rate function $I(\varphi)$ given in (1.12).*

The main result is implied by Theorem 1.1 and the contraction principle of Varadhan (see [11]).

Theorem 1.2. *Let the functions $a(x)$ and $b(x)$ be Lipschitz continuous and suppose there exist constants c and C such that $0 < c \leq |b(x)| \leq C$. Then under assumptions of Theorem 1.1 the family (X_t^ε) defined by (1.3) obeys the l.d.p. in $(\mathbf{C}, \rho)$ with the rate function $J(\varphi)$, $\varphi \in \mathbf{C}$, defined as*

$$J(\varphi) = \begin{cases} I(\psi), & \dot{\psi}_t = \frac{\dot{\varphi}_t - a(\varphi_t)}{b(\varphi_t)}, \quad \varphi_0 = X_0, \quad d\varphi \ll dt, \\ \infty, & \text{otherwise.} \end{cases}$$

These theorems are proved in Sections 4 and 5. In Section 3 we formulate the l.d.p. for $\varepsilon N_{t/\varepsilon}$. Section 2 contains an auxiliary result. In the last section an example having an independent interest is considered.

§ 2. Properties of $\int_0^t \eta_s ds$

Taking into account (1.4) and the well-known property of the stochastic Itô integral, we find

$$\mathbf{E}(\eta_t \mid \eta_s, s \leq 0) = \int_{-\infty}^0 h(t-s) dN_s \quad (\mathbf{P}\text{-a.s.}).$$

On the other hand, by (1.6), $\int_0^\infty \sqrt{\int_t^\infty h^2(s) ds} dt < \infty$. Both facts imply

$$\int_0^\infty \sqrt{\mathbf{E} \left(\mathbf{E}(\eta_t \mid \eta_s, s \leq 0) \right)^2} dt < \infty.$$

Then by [8, Lemma 9.2.1], there exists a semimartingale $(V_t)_{t \geq 0}$ w.r.t. the filtration $\mathbf{F} = (\mathcal{F}_t)_{t \in \mathbf{R}}$ generated by (η_t) satisfying the general conditions such that

$$V_t = V_0 + \int_0^t \eta_s ds - M_t, \quad (2.1)$$

where (M_t) is a square integrable martingale with strictly stationary ergodic increments and $(V_t, \eta_t)_{t \geq 0}$ forms a strictly stationary process. For η_t obeying the Wold decomposition (1.4) we have (with Σ given at (1.7))

$$M_t = \Sigma N_t, \quad V_t = \int_t^\infty \left(\int_{-\infty}^t h(u-s) dN_s \right) du \quad (2.2)$$

(for more details see [8, Chapter 9, Section 2, ex. 2]).

The aim of this section is to show an exponential integrability of $\sup_{t \leq T} |V_t|$.

Lemma 2.1. *Let (1.6), (1.8), and (Cr.1), (Cr.2) be fulfilled. Then for any $\lambda > 0$ and $T > 0$*

$$\mathbf{E} \exp \left\{ \lambda \sup_{t \leq T} |V_t| \right\} < \infty.$$

P r o o f. Assume there exists some positive martingale (L_t) w.r.t. $\mathbf{F}$ such that

$$|V_t| \leq L_t, \quad \forall t \leq T, \quad \mathbf{E} \exp\{\lambda L_T\} < \infty, \quad \forall \lambda > 0. \quad (2.3)$$

Then the result holds.

In fact, using Jensen's, Cauchy-Schwarz's, and Doob's inequalities, we get

$$\begin{aligned} \mathbf{E} \exp \left\{ \lambda \sup_{t \leq T} |V_t| \right\} &\leq \mathbf{E} \exp \left\{ \lambda \sup_{t \leq T} L_t \right\} = \mathbf{E} \sup_{t \leq T} \exp \left\{ \lambda L_t \right\} = \mathbf{E} \sup_{t \leq T} \exp \left\{ \mathbf{E}(\lambda L_T \mid \mathcal{F}_t) \right\} \\ &\leq \mathbf{E} \sup_{t \leq T} \mathbf{E} \left(\exp\{\lambda L_T\} \mid \mathcal{F}_t \right) \leq \sqrt{\mathbf{E} \sup_{t \leq T} \mathbf{E} \left(\exp\{\lambda L_T\} \mid \mathcal{F}_t \right)^2} \leq 2 \sqrt{\mathbf{E} \exp\{2\lambda L_T\}} < \infty. \end{aligned}$$

Thus, it remains to find an (L_t) satisfying (2.3). Two facts are used here: decomposition (2.1) and $\mathcal{F}_t$ -measurability of V_t . From (2.1) we find $V_t = V_T - \int_0^t \eta_s ds + \Sigma(N_T - N_t)$, $t \leq T$, and, what follows from the martingale property of (N_t) , $V_t = \mathbf{E}(V_T - \int_0^t \eta_s ds \mid \mathcal{F}_t)$. So $|V_t| \leq \mathbf{E}(|V_T| + \int_0^T |\eta_s| ds \mid \mathcal{F}_t)$, i.e., $L_t = \mathbf{E}(|V_T| + \int_0^T |\eta_s| ds \mid \mathcal{F}_t)$. Thereby, it remains to show only that

$$\mathbf{E} \exp \left\{ \lambda \left[|V_T| + \int_0^T |\eta_s| ds \right] \right\} < \infty. \quad (2.4)$$

We verify (2.4) by using Cauchy-Schwarz's and Jensen's inequalities:

$$\begin{aligned} \left(\mathbf{E} \exp \left\{ \lambda \left[|V_T| + \int_0^T |\eta_s| ds \right] \right\} \right)^2 &\leq \mathbf{E} \exp \left\{ 2\lambda |V_T| \right\} \mathbf{E} \exp \left\{ 2\lambda \int_0^T |\eta_s| ds \right\} \\ &= \mathbf{E} \exp \left\{ 2\lambda |V_0| \right\} \mathbf{E} \exp \left\{ \frac{1}{T} \int_0^T 2T\lambda |\eta_s| ds \right\} \\ &\leq \mathbf{E} \exp \left\{ 2\lambda |V_0| \right\} \mathbf{E} \frac{1}{T} \int_0^T \exp \left\{ 2T\lambda |\eta_s| \right\} ds = \mathbf{E} \exp \left\{ 2\lambda |V_0| \right\} \mathbf{E} \exp \left\{ 2T\lambda |\eta_0| \right\}. \end{aligned}$$

Thus (2.4) holds if $\mathbf{E} \exp\{\lambda |V_0|\} < \infty$ and $\mathbf{E} \exp\{\lambda |\eta_0|\} < \infty$ for any positive λ . The direct proof of the validity of these inequalities would be difficult. It is more convenient to use instead of V_0 and η_0 random variables $V'_0 = \int_0^\infty \int_s^\infty h(u) du dN_s$ (V'_0 is defined by virtue of (1.8)) and $\eta'_0 = \int_0^\infty h(s) dN_s$ which coincide with V_0 and η_0 in distribution and furthermore, using the inequality $e^{|x|} \leq e^x + e^{-x}$, to verify only that

$$\mathbf{E} \exp\{\lambda V'_0\} < \infty, \quad \mathbf{E} \exp\{\lambda \eta'_0\} < \infty, \quad \forall \lambda \in \mathbf{R}. \quad (2.5)$$

Denote by $H(t)$ any of the functions $h(t)$ or $\int_t^\infty h(u) du$. From the definition of V'_0 and η'_0 it follows that

$$\left. \begin{matrix} V'_0 \\ \eta'_0 \end{matrix} \right\} = \int_0^\infty H(t) dN_t \quad (\equiv Z_\infty).$$

Thereby, the validity of

$$\mathbf{E} \exp\{\lambda Z_\infty\} < \infty, \quad \forall \lambda \in \mathbf{R}, \quad (2.6)$$

has to be checked. To this end define a square integrable martingale $Z_t = \int_0^t H(s) dN_s$ having as the limit point $Z_\infty = \lim_{t \rightarrow \infty} Z_t$. By the Fatou lemma,

$$\mathbf{E} \exp\{\lambda Z_\infty\} \leq \limsup_{t \rightarrow \infty} \mathbf{E} \exp\{Z_t\}$$

and so it is sufficient to show that for any $\lambda \in \mathbf{R}$ there exists a constant $C(\lambda)$ depending only on λ such that

$$\mathbf{E} \exp\{Z_t\} \leq C(\lambda). \quad (2.7)$$

For finding $C(\lambda)$ we use the fact that (Z_t) is a process with independent increments. Namely, the Lévy measure $K(dx) dt$ is the compensator for the measure $\mu(dt, dx)$ of jumps of (N_t) w.r.t. the filtration $\mathbf{F}^N = (\mathcal{F}_t^N)_{t \geq 0}$ generated by (N_t) . Then the pure discontinuous part (N_t^d) of (N_t) can be represented as an Itô's integral w.r.t. the martingale measure $\mu(dt, dx) - K(dx) dt$:

$$N_t^d = \int_0^t \int_{\mathbf{R}^0} x [\mu(ds, dx) - K(dx) ds]$$

and so

$$N_t = N_t^c + \int_0^t \int_{\mathbf{R}^0} x [\mu(ds, dx) - K(dx) ds]. \quad (2.8)$$

Then

$$Z_t = \int_0^t H(s) dN_s = \int_0^t H(s) dN_s^c + \int_0^t \int_{\mathbf{R}^0} H(s) x [\mu(ds, dx) - K(dx) ds]$$

and, by [5, Chapter II], for any $\lambda \in \mathbf{R}$ we find

$$\mathbf{E} e^{\lambda Z_t} = \exp \left\{ \frac{\lambda^2 \sigma^2}{2} \int_0^t H^2(s) ds + \int_0^t \int_{\mathbf{R}^0} (e^{\lambda H(s)x} - 1 - \lambda H(s)x) K(dx) ds \right\}. \quad (2.9)$$

The right-hand side of this inequality grows as $t \rightarrow \infty$ to

$$C(\lambda) = \exp \left\{ \frac{\lambda^2 \sigma^2}{2} \int_0^\infty H^2(s) ds + \int_0^\infty \int_{\mathbf{R}^0} (e^{\lambda H(s)x} - 1 - \lambda H(s)x) K(dx) ds \right\} \quad (2.10)$$

and it remains only to show that this $C(\lambda)$ is finite. In case $H(t) \equiv h(t)$ it holds by (Cr.2). In case $H(t) \equiv \int_t^h(s) ds < \infty$ it is implied by (1.8) and (Cr.1), since $(\lambda_0 = |\lambda| \sup_{t \geq 0} |H(t)|)$

$$e^{\lambda H(t)x} - 1 - \lambda H(t)x \leq \lambda^2 H^2(t) e^{\lambda_0 |x|} \leq \lambda^2 H^2(t) (e^{\lambda_0 x} + e^{-\lambda_0 x})$$

and $\int_{\mathbf{R}^0} x^2 e^{\lambda x} K(dx) < \infty$ for any $\lambda \in \mathbf{R}$.

§ 3. The l.d.p. for $\varepsilon N_{t/\varepsilon}$

In this section the l.d.p. is established for the family $\varepsilon N_{t/\varepsilon}$ of homogeneous processes with independent increments (h.i.i.). Since N_t has paths in the Skorokhod space $\mathbf{D} = \mathbf{D}_{[0, \infty)}$ of the right continuous having limits on the left functions, we formulate the l.d.p. in the metric space $(\mathbf{D}, d)$ with the Skorokhod–Lindvall metric d (see [6] or, e.g., [8, Chapter 6]). The metric d is equivalent in some sense to the metric ρ (see Section 1) and is defined in the following way. For each $X = (X_t)_{t \geq 0}$ put $X_t^n = X_t \min(1, n - t)$, $0 \leq t \leq n$. If $X, Y \in \mathbf{D}$ and $X^n = (X_t^n)_{0 \leq t \leq n}$, $Y^n = (Y_t^n)_{0 \leq t \leq n}$ are defined as it was mentioned above, we have

$$d(X, Y) = \sum_{n \geq 1} 2^{-n} \frac{d_n(X^n, Y^n)}{1 + d_n(X^n, Y^n)},$$

where $d_n(X^n, Y^n)$ is the Skorokhod distance between X^n, Y^n (see [1]).

Theorem 3.1. *Let assumption (Cr.1) be fulfilled. The family $\varepsilon N_{t/\varepsilon}$ obeys the l.d.p. in the metric space $(\mathbf{D}, d)$ with the rate function*

$$I^{\text{hii}}(\varphi) = \begin{cases} \int_0^\infty H_{\sigma^2}(\dot{\varphi}_t) dt, & \varphi_0 = 0, \quad d\varphi \ll dt, \\ \infty, & \text{otherwise,} \end{cases}$$

with $H_{\sigma^2}(y)$ from (1.11).

Formally, Theorem 3.1 follows from a general result of the l.d.p. for Markovian processes (see [12]) or semimartingales (see [7]). We give here the direct proof. The reason for this is that the process N_t has a simpler structure than the Markovian processes or semimartingales considered in [12] and [7], so the suggested proof is simpler and could be of interest by itself.

Nevertheless, we use the methodology from [7] which is based on such notions as the exponential tightness, the partial l.d.p., and the Pukhalskii theorem (see [9]). Following it, only two sets of conditions have to be checked.

(C.1) **C-exponential tightness:** for any $T > 0$ and $\gamma > 0$

$$\lim_{c \rightarrow \infty} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon}| > c \right\} = -\infty,$$

$$\lim_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \sup \varepsilon \log \mathbf{P} \left\{ \sup_{0 \leq t \leq \delta} |\varepsilon(N_{(t+\tau)/\varepsilon} - N_{\tau/\varepsilon})| > \gamma \right\} = -\infty,$$

where sup is taken over all stopping times τ (w.r.t. $(\mathcal{F}_{t/\varepsilon}^N)_{t \geq 0}$) which are bounded by T .

(C.2) **C-local l.d.p.:** for any $T > 0$ and $\varphi \in \mathbf{C}$

$$\begin{aligned} \hat{I}_T(\varphi) &= - \limsup_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} \\ &= - \liminf_{\delta \rightarrow 0} \liminf_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\}. \end{aligned}$$

If (C.1) and (C.2) are fulfilled then the l.d.p. in $(\mathbf{D}, d)$ holds with the rate function

$$I^{\text{hii}}(\varphi) = \begin{cases} \sup_{T > 0} \hat{I}_T(\varphi), & \varphi \in \mathbf{C}, \\ \infty, & \varphi \in \mathbf{D} \setminus \mathbf{C}. \end{cases} \quad (3.1)$$

Conditions (C.1) and (C.2) are examined below in three lemmas.

Lemma 3.1. *Let assumption (Cr.1) be fulfilled. Then (C.1) holds.*

P r o o f. For checking the first condition in (C.1) exponential Chebyshev's, Jensen's, Cauchy-Schwarz's, and Doob's inequalities are applied:

$$\begin{aligned} \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon}| > c \right\} &\leq e^{-c/\varepsilon} \mathbf{E} \exp \left\{ \sup_{t \leq T} |N_{t/\varepsilon}| \right\} \\ &= e^{-c/\varepsilon} \mathbf{E} \sup_{t \leq T} \exp \left\{ |N_{t/\varepsilon}| \right\} = e^{-c/\varepsilon} \mathbf{E} \sup_{t \leq T} \exp \left\{ \left| \mathbf{E}(N_{T/\varepsilon} \mid \mathcal{F}_{t/\varepsilon}^N) \right| \right\} \\ &\leq e^{-c/\varepsilon} \mathbf{E} \sup_{t \leq T} \mathbf{E} \left(\exp \left\{ |N_{T/\varepsilon}| \right\} \mid \mathcal{F}_{t/\varepsilon}^N \right) \\ &\leq e^{-c/\varepsilon} \sqrt{\mathbf{E} \left[\sup_{t \leq T} \mathbf{E} \left(\exp \left\{ |N_{T/\varepsilon}| \right\} \mid \mathcal{F}_{t/\varepsilon}^N \right) \right]^2} \\ &\leq 2e^{-c/\varepsilon} \sqrt{\mathbf{E} \exp \left\{ 2|N_{T/\varepsilon}| \right\}} \leq 2e^{-c/\varepsilon} \sqrt{\mathbf{E} \left\{ e^{2N_{T/\varepsilon}} + e^{-2N_{T/\varepsilon}} \right\}}. \end{aligned} \quad (3.2)$$

Since (N_t) is a martingale with independent increments, by virtue of (2.8) and [5, Chapter II] we get

$$\mathbf{E} e^{\pm 2N_{T/\varepsilon}} = \exp \left\{ \frac{T}{\varepsilon} \left[2\sigma^2 + \int_{\mathbf{R}^0} (e^{\pm 2x} - 1 \mp 2x) K(dx) \right] \right\} \leq \exp \left\{ \frac{T\ell}{\varepsilon} \right\}, \quad (3.3)$$

where (see (Cr.1))

$$\ell = 2\sigma^2 + \int_{\mathbf{R}^0} (e^{2x} - 1 - 2x) K(dx) + \int_{\mathbf{R}^0} (e^{-2x} - 1 + 2x) K(dx) \quad (< \infty)$$

Then, as it follows from (3.2) and (3.3),

$$\lim_{c \rightarrow \infty} \limsup_{\varepsilon \rightarrow 0} \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon}| > c \right\} \leq \varepsilon \log 2 - c + \frac{\ell}{2} \rightarrow -\infty, \quad c \rightarrow \infty.$$

For checking the second condition in (C.1) note that by virtue of the strong Markovian property $(N_{(t+\tau)/\varepsilon} - N_{\tau/\varepsilon})_{t \geq 0}$ coincides in distribution with $(N_{t/\varepsilon})_{t \geq 0}$. So in the same way as (3.2) and (3.3) have been obtained, we find for any $\lambda > 0$

$$\begin{aligned} \mathbf{P} \left\{ \sup_{0 \leq t \leq \delta} |\varepsilon(N_{(t+\tau)/\varepsilon} - N_{\tau/\varepsilon})| > \gamma \right\} &\leq 2e^{-\lambda\gamma/\varepsilon} \sqrt{\mathbf{E} \left[e^{2\lambda N_{\delta/\varepsilon}} + e^{-2\lambda N_{\delta/\varepsilon}} \right]} \\ &\leq 2 \exp \left\{ -\frac{\lambda\gamma}{\varepsilon} + \frac{1}{2} \frac{\delta \ell(\lambda)}{\varepsilon} \right\}, \end{aligned}$$

where

$$\ell(\lambda) = 2\lambda^2 \sigma^2 + \int_{\mathbf{R}^0} (e^{2\lambda x} - 1 - 2\lambda x) K(dx) + \int_{\mathbf{R}^0} (e^{-2\lambda x} - 1 + 2\lambda x) K(dx).$$

Then

$$\limsup_{\varepsilon \rightarrow 0} \mathbf{P} \left\{ \sup_{0 \leq t \leq \delta} |\varepsilon(N_{(t+\tau)/\varepsilon} - N_{\tau/\varepsilon})| > \gamma \right\} \leq -\lambda\gamma + \frac{1}{2} \delta \ell(\lambda). \quad (3.4)$$

The function $\ell(\lambda)$, $\lambda \geq 0$, is non-negative, continuous, and increasing. If it is bounded then the result evidently holds. If it is increasing to ∞ then, taking λ_δ , such that $\ell(\lambda_\delta) = 2/\delta$, we arrive to the upper bound in (3.4): $-\lambda_\delta \gamma + 1$ which decreases to $-\infty$ as $\delta \rightarrow 0$.

Lemma 3.2. *Let assumption (Cr.1) be fulfilled. Then for any $T > 0$ and $\varphi \in \mathbf{C}$*

$$\begin{aligned} &\limsup_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} \\ &\leq - \begin{cases} \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt, & \varphi_0 = 0, \quad d\varphi \ll dt, \\ \infty, & \text{otherwise,} \end{cases} \end{aligned}$$

where $H_{\sigma^2}(\dot{\varphi}_t)$ is defined in (1.11).

P r o o f. Let $\nu(t)$ be a simple function of the form $\nu(t) = \sum_i \nu_i I_{[t_{i-1}, t_i]}(t)$. Put

$$Z_T^\varepsilon(\nu) = \exp \left\{ \int_0^T \nu(t) dN_{t/\varepsilon} - \frac{1}{\varepsilon} \int_0^T G(\nu(t)) dt \right\}, \quad (3.5)$$

where $G(\lambda)$ is the cumulant given at (1.9). By [5, Chapter II] we have

$$\mathbf{E} Z_T^\varepsilon(\nu) = 1 \quad (3.6)$$

which implies

$$1 \geq \mathbf{E} I \left(\sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right) Z_T^\varepsilon(\nu). \quad (3.7)$$

Inequality (3.7) is the general tool in proving the upper bound. It is natural to evaluate $Z_T^\varepsilon(\nu)$ from below on the set $\{\sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta\}$. Put by definition

$$\int_0^T \nu(t) d\varphi_t = \sum_i \nu_i [\varphi_{T \wedge t_i} - \varphi_{T \wedge t_{i-1}}].$$

Then

$$\begin{aligned} Z_T^\varepsilon(\nu) &\geq \exp \left\{ \frac{1}{\varepsilon} \left[\int_0^T \nu(t) d\varphi_t - \int_0^T G(\nu(t)) dt \right] \right\} \\ &\times \exp \left\{ - \left| \int_0^T \nu(t) \left[dN_{t/\varepsilon} - \frac{1}{\varepsilon} d\varphi_t \right] \right| \right\}. \end{aligned} \quad (3.8)$$

Taking into account that on the set $\{\sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta\}$ the following estimate holds:

$$\left| \int_0^T \nu(t) \left[dN_{t/\varepsilon} - \frac{1}{\varepsilon} d\varphi_t \right] \right| \leq \text{const} \frac{\delta}{\varepsilon},$$

where const depends on ν , only, we derive from (3.7) and (3.8) the following upper bound:

$$\limsup_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} \leq - \left[\int_0^T \nu(t) d\varphi_t - \int_0^T G(\nu(t)) dt \right].$$

The equality

$$\sup \left[\int_0^T \nu(t) d\varphi_t - \int_0^T G(\nu(t)) dt \right] = \begin{cases} \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt, & \varphi_0 = 0, \quad d\varphi \ll dt, \\ \infty, & \text{otherwise,} \end{cases}$$

where the supremum is taken over all simple functions $\nu(t)$, follows from [7, Lemma 6].

Lemma 3.3. *Let assumption (Cr.1) be fulfilled. Then for any $T > 0$ and $\varphi \in \mathbf{C}$*

$$\begin{aligned} & \limsup_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} \\ & \geq - \begin{cases} \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt, & \varphi_0 = 0, \quad d\varphi \ll dt, \\ \infty, & \text{otherwise,} \end{cases} \end{aligned}$$

where $H_{\sigma^2}(\dot{\varphi}_t)$ is defined in (1.11).

P r o o f. Since the announced lower bound coincides with the upper one, only the validity of the lower bound has to be checked, or it is sufficient to consider only the case « $\varphi_0 = 0, d\varphi \ll dt$ ».

Assume first that

$$\sigma^2 > 0, \quad |\dot{\varphi}_t| \leq N. \quad (3.9)$$

Under these assumptions there exists a measurable bounded function $\nu(t)$ satisfying for a.s. (w.r.t. the Lebesgue measure) $t \geq 0$ the equality $G'(\nu(t)) = 0$ (see (1.9)). Then we have

$$H_{\sigma^2}(\dot{\varphi}_t) = \nu(t) \dot{\varphi}_t - G(\nu(t)) \quad \text{a.s.} \quad (3.10)$$

For fixed $T > 0$ put $\nu_T(t) = I(T \geq t)\nu(t)$ and define the process

$$Z_t^\varepsilon(\nu_T) = \exp \left\{ \int_0^t \nu_T(s) dN_{s/\varepsilon} - \frac{1}{\varepsilon} \int_0^t G(\nu_T(t)) dt \right\}. \quad (3.11)$$

Since $G(0) = 0$, $\lim_{t \rightarrow \infty} Z_t^\varepsilon(\nu_T) = Z_T^\varepsilon(\nu_T)$ and

$$\mathbf{E} Z_T^\varepsilon(\nu_T) = 1. \quad (3.12)$$

Noticing that $Z_T^\varepsilon(\nu_T) > 0$ ($\mathbf{P}$ -a.s.) and letting $d\mathbf{P}_T^\varepsilon = Z_T^\varepsilon(\nu_T) d\mathbf{P}$ we get, due to (3.12), the probability measure $\mathbf{P}_T^\varepsilon$, which is equivalent to $\mathbf{P}$ and $d\mathbf{P}/d\mathbf{P}_T^\varepsilon = (Z_T^\varepsilon(\nu_T))^{-1}$ (the expectation w.r.t. $\mathbf{P}_T^\varepsilon$ is denoted by $\mathbf{E}_T^\varepsilon$). From the identity

$$\begin{aligned} \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} &= \mathbf{E} I \left(\sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right) Z_T^\varepsilon(\nu_T) \left(Z_T^\varepsilon(\nu_T) \right)^{-1} \\ &= \mathbf{E}_T^\varepsilon I \left(\sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right) \left(Z_T^\varepsilon(\nu_T) \right)^{-1} \end{aligned}$$

with

$$\left(Z_T^\varepsilon(\nu_T) \right)^{-1} = \exp \left\{ - \int_0^T \nu(t) \left[dN_{t/\varepsilon} - \frac{1}{\varepsilon} \dot{\varphi}_t dt \right] - \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt \right\},$$

in view of (3.10) and (3.11), we find

$$\begin{aligned} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} &\geq - \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt \\ &+ \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{E}_T^\varepsilon \left(I \left(\sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right) \exp \left\{ - \int_0^T \nu(t) \left[dN_{t/\varepsilon} - \frac{1}{\varepsilon} \dot{\varphi}_t dt \right] \right\} \right) \\ &\geq - \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt - \gamma \\ &+ \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P}_T^\varepsilon \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta, \left| \int_0^T \nu(t) \left[dN_{t/\varepsilon} - \frac{1}{\varepsilon} \dot{\varphi}_t dt \right] \right| \leq \frac{\gamma}{\varepsilon} \right\}. \end{aligned}$$

Thereby, by the arbitrariness of γ the desired lower bound holds if

$$\limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P}_T^\varepsilon \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta, \left| \int_0^T \nu(t) [\varepsilon dN_{t/\varepsilon} - \dot{\varphi}_t dt] \right| \leq \gamma \right\} = 0. \quad (3.13)$$

To this end we examine some properties of the process $Z_t^\varepsilon(\nu_T)$. Taking into account (2.8) and applying Itô's formula to the right-hand side of (3.11), we find that

$$\begin{aligned} dZ_t^\varepsilon(\nu_T) &= Z_{t-}^\varepsilon(\nu_T) dN_{t/\varepsilon} + Z_{t-}^\varepsilon(\nu_T) \\ &\times \int_{\mathbf{R}^0} \left(e^{\nu_T(t)x} - 1 - \nu_T(t)x \right) \left[\mu \left(d \left(\frac{t}{\varepsilon} \right), dx \right) - K(dx) d \left(\frac{t}{\varepsilon} \right) \right], \end{aligned}$$

so $Z_t^\varepsilon(\nu_T)$ is a martingale w.r.t. $(\mathbf{P}, (\mathcal{F}_{t/\varepsilon}^N)_{t \geq 0})$. Furthermore, it is a square integrable martingale since

$$\mathbf{E} \left(Z_t^\varepsilon(\nu_T) \right)^2 = \exp \left\{ \frac{1}{\varepsilon} \int_0^T \left[G(2\nu(t)) - 2G(\nu(t)) \right] dt \right\}.$$

Also note that the mutually quadratic variation of the pair of martingales $Z_t^\varepsilon(\nu_T)$ and $N_{t/\varepsilon}$ is defined as:

$$\left[Z^\varepsilon(\nu_T), N_{./\varepsilon} \right]_t = \frac{1}{\varepsilon} \int_0^t Z_s^\varepsilon(\nu_T) \sigma^2 \nu_T(s) ds + \int_0^t Z_s^\varepsilon(\nu_T) \int_{\mathbf{R}^0} x \left[e^{\nu_T(s)x} - 1 \right] \mu \left(d \left(\frac{s}{\varepsilon} \right), dx \right)$$

and the mutually predictable quadratic variation being the compensator of it is:

$$\left\langle Z^\varepsilon(\nu_T), N_{./\varepsilon} \right\rangle_t = \frac{1}{\varepsilon} \left[\int_0^t Z_s^\varepsilon(\nu_T) \sigma^2 \nu_T(s) ds + \int_0^t Z_s^\varepsilon(\nu_T) \int_{\mathbf{R}^0} x \left[e^{\nu_T(s)x} - 1 \right] K(dx) ds \right].$$

Define a new process $N_t^{T,\varepsilon} = N_{t/\varepsilon} - (1/\varepsilon) \varphi_{t \wedge T}$. It is easy to check that

$$N_t^{T,\varepsilon} = N_{t/\varepsilon} - \frac{1}{\varepsilon} \int_0^t \left(Z_s^\varepsilon(\nu_T) \right)^{-1} \left\langle Z^\varepsilon(\nu_T), N_{./\varepsilon} \right\rangle_s.$$

Then by [8, Chapter 4] the process $N_t^{T,\varepsilon}$ is a martingale w.r.t. $(\mathbf{P}_T^\varepsilon, (\mathcal{F}_{t/\varepsilon}^N)_{t \geq 0})$ with a Gaussian continuous component $N_t^{T,\varepsilon,c}$, $\mathbf{E}_T^\varepsilon (N_t^{T,\varepsilon,c})^2 = \sigma^2 t/\varepsilon$, and its Lévy measure is $K_T^\varepsilon(dt, dx) = e^{\nu_T(t)x} K(dx) d(t/\varepsilon)$. So it is a square integrable martingale whose predictable quadratic variation is given by the formula:

$$\left\langle N^{T,\varepsilon,c} \right\rangle_t = \frac{1}{\varepsilon} \left[\sigma^2 t + \int_0^t \int_{\mathbf{R}^0} x^2 e^{\nu_T(t)x} K(dx) dt \right].$$

Consequently, by Doob's inequality (see, e.g., [8, Chapter I]) we have

$$\mathbf{E}_T^\varepsilon \left(\sup_{t \leq T} \left| \varepsilon N_t^{T,\varepsilon,c} \right|^2 \right) \leq 4\varepsilon \left[\sigma^2 T + \int_0^T \int_{\mathbf{R}^0} x^2 e^{\nu_T(t)x} K(dx) dt \right] = \varepsilon \text{ const.} \quad (3.14)$$

Analogously we get

$$\mathbf{E}_T^\varepsilon \left(\sup_{t \leq T} \left| \int_0^t \nu_T(s) d(\varepsilon N_s^{T, \varepsilon, c}) \right|^2 \right) = \varepsilon \text{ const.} \quad (3.15)$$

Since for $t \leq T$ we have $\varepsilon N_t^{T, \varepsilon} = \varepsilon N_{t/\varepsilon} - \varphi_t$, (3.13) is implied by (3.14) and (3.15). In the next step we show the validity of the lower bound under weaker conditions than (3.9). Namely, taking into account that for $d\varphi \ll dt$ we have $\int_0^t |\dot{\varphi}_s| ds < \infty \forall t > 0$, and so only

$$\sigma^2 > 0 \quad (3.16)$$

has to be assumed.

For $N \geq 1$, taking $\dot{\varphi}_t^N = \dot{\varphi}_t I(|\dot{\varphi}_t| \leq N)$, put $\varphi_t^N = \int_0^t \dot{\varphi}_s^N ds$. Evidently, φ_t^N satisfies (3.9) and so, by the result obtained above, for any $\delta > 0$ we have

$$\limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t^N| \leq \delta \right\} \geq - \int_0^T H_{\sigma^2}(\dot{\varphi}_t^N) dt.$$

On the other hand, $H_{\sigma^2}(0) = 0$ and so $H_{\sigma^2}(\dot{\varphi}_t^N) = H_{\sigma^2}(\dot{\varphi}_t) I(|\dot{\varphi}_t| \leq N)$ what implies the inequality $\int_0^T H_{\sigma^2}(\dot{\varphi}_t^N) dt \leq \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt$.

Then

$$\begin{aligned} & \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta - \sup_{t \leq T} |\varphi_t - \varphi_t^N| \right\} \\ & \geq \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t^N| \leq \delta \right\} \geq - \int_0^T H_{\sigma^2}(\dot{\varphi}_t) dt. \end{aligned}$$

The desired lower bound evidently holds, since $|\varphi_t - \varphi_t^N| \leq \int_0^T |\dot{\varphi}_t| I(|\dot{\varphi}_t| > N) dt \rightarrow 0$ as $N \rightarrow \infty$.

Thus, it remains to check the validity of the lower bound under

$$\sigma^2 = 0. \quad (3.17)$$

Due to $\sigma^2 = 0$ the Gaussian component of N_t equals zero. Put $N_t^\gamma = N_t + \gamma W_t$, where W_t is a Wiener process which is to be assumed independent of N_t and $\gamma > 0$. By the result obtained above we get

$$\limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon}^\gamma - \varphi_t| \leq \delta \right\} \geq - \int_0^T H_{\gamma^2}(\dot{\varphi}_t) dt.$$

As it was mentioned in Section 1, function $H_{\sigma^2}(y)$ is increasing to $H_0(y)$ as $\sigma^2 \rightarrow 0$ and so $\int_0^T H_{\gamma^2}(\dot{\varphi}_t) dt \leq \int_0^T H_0(\dot{\varphi}_t) dt$, i.e.,

$$\limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon}^\gamma - \varphi_t| \leq \delta \right\} \geq - \int_0^T H_0(\dot{\varphi}_t) dt. \quad (3.18)$$

Now we use the following chain of the lower estimates:

$$\begin{aligned} & 2 \max \left\{ \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq 2\delta \right\}, \mathbf{P} \left\{ \gamma \sup_{t \leq T} |\varepsilon W_{t/\varepsilon}| > \delta \right\} \right\} \\ & \geq \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq 2\delta \right\} + \mathbf{P} \left\{ \gamma \sup_{t \leq T} |\varepsilon W_{t/\varepsilon}| > \delta \right\} \\ & \geq \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta + \gamma \sup_{t \leq T} |\varepsilon W_{t/\varepsilon}| \right\} \geq \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon}^\gamma - \varphi_t| \leq \delta \right\}. \end{aligned} \quad (3.19)$$

From (3.19) and (3.18) it follows that for any $\delta > 0, \gamma > 0$

$$\max \left\{ \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon N_{t/\varepsilon} - \varphi_t| \leq 2\delta \right\}, \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon W_{t/\varepsilon}| > \frac{2\delta}{\gamma} \right\} \right\} \geq - \int_0^T H_0(\dot{\varphi}_t) dt$$

and what follows from Lemma 3.1

$$\lim_{\gamma \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon W_{t/\varepsilon}| > \frac{2\delta}{\gamma} \right\} = -\infty.$$

Thus, the desired lower bound holds.

§ 4. Proof of Theorem 1.1

Decomposition (2.1) with $M_t = \Sigma N_t$ (see (2.2)) implies

$$\sqrt{\varepsilon} W_t^\varepsilon = \varepsilon(V_{t/\varepsilon} - V_0) + \Sigma \varepsilon N_{t/\varepsilon}. \quad (4.1)$$

The first step consists in showing the exponential negligibility of the process $\varepsilon(V_{t/\varepsilon} - V_0)$ in a sense given below.

Lemma 4.1. *Let Cramér's conditions (Cr.1) and (Cr.2) be fulfilled. Then for any $T > 0$, $\gamma > 0$*

$$\lim_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon(V_{t/\varepsilon} - V_0)| \geq \gamma \right\} = -\infty.$$

Proof. Evidently, only

$$\lim_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\varepsilon V_{t/\varepsilon}| \geq \gamma \right\} = -\infty$$

has to be checked. Let $\alpha_k = \sup_{(k-1) \leq t \leq Tk} |V_t|$, $k \geq 1$. Noticing that $(\alpha_k)_{k \geq 1}$ forms a strictly stationary sequence and using the exponential Chebyshev inequality (with parameter $\lambda > 0$), we find

$$\begin{aligned} \mathbf{P} \left\{ \varepsilon \sup_{t \leq T} |V_{t/\varepsilon}| \geq \gamma \right\} &= \mathbf{P} \left\{ \sup_{t \leq T/\varepsilon} |V_t| \geq \frac{\gamma}{\varepsilon} \right\} \leq \mathbf{P} \left\{ \max_{k \leq 1/\varepsilon} \alpha_k \geq \frac{\gamma}{\varepsilon} \right\} \leq \sum_{k \leq 1/\varepsilon} \mathbf{P} \left\{ \alpha_k \geq \frac{\gamma}{\varepsilon} \right\} \\ &\leq \frac{1}{\varepsilon} \mathbf{P} \left\{ \alpha_1 \geq \frac{\gamma}{\varepsilon} \right\} \leq \frac{1}{\varepsilon e^{\lambda \gamma / \varepsilon}} \mathbf{E} e^{\lambda \alpha_1} = \frac{1}{\varepsilon e^{\lambda \gamma / \varepsilon}} \mathbf{E} e^{\lambda \sup_{t \leq T} |V_t|}. \end{aligned}$$

Then

$$\varepsilon \log \mathbf{P} \left\{ \varepsilon \sup_{t \leq T} |V_{t/\varepsilon}| \geq \gamma \right\} \leq -\varepsilon \log \varepsilon - \lambda \gamma + \varepsilon \log \mathbf{E} e^{\lambda \sup_{t \leq T} |V_t|}.$$

By virtue of Lemma 2.1 the right-hand side of the last inequality goes to $-\infty$ as « $\lim_{\lambda \rightarrow \infty} \limsup_{\varepsilon \rightarrow 0}$ » is taken.

In the second step, the fact that the family $\varepsilon N_{t/\varepsilon}$, as well as $\Sigma \varepsilon N_{t/\varepsilon}$, satisfies conditions (C.1) and (C.2) and Lemma 4.1 is used. According to this, for any $\varphi \in \mathbf{C}$ and $T > 0$ we get

$$\begin{aligned} &\limsup_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\Sigma \varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} \\ &= \limsup_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\sqrt{\varepsilon} W_{t/\varepsilon} - \varphi_t| \leq \delta \right\} = I_T(\varphi) \end{aligned}$$

and

$$\begin{aligned} &\liminf_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\Sigma \varepsilon N_{t/\varepsilon} - \varphi_t| \leq \delta \right\} \\ &= \liminf_{\delta \rightarrow 0} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} |\sqrt{\varepsilon} W_{t/\varepsilon} - \varphi_t| \leq \delta \right\} = I_T(\varphi), \end{aligned}$$

where, letting $\frac{0}{0} = 0$,

$$I_T(\varphi) = \begin{cases} \int_0^T H_{\sigma^2} \left(\frac{\dot{\varphi}_t}{\Sigma} \right) dt, & \varphi_0 = 0, d\varphi \ll dt, \\ \infty, & \text{otherwise.} \end{cases} \quad (4.2)$$

Thus, by the method used in the proof of Theorem 3.1, we get the l.d.p. for the family $\sqrt{\varepsilon} W_t^\varepsilon$ in the metric space $(\mathbf{D}, d)$ with the rate function $I(\varphi) = \sup_T I_T(\varphi)$ which coincides with (1.12).

On the other hand, since for each ε the paths of the process $\sqrt{\varepsilon} W_t^\varepsilon$ are continuous, the l.d.p. for this family holds also in the metric space $(\mathbf{C}, \rho)$ with the same rate function.

§ 5. Proof of Theorem 1.2

5.1. To apply the contraction principle of Varadhan (see [11]) a continuous mapping $(\sqrt{\varepsilon} W_s^\varepsilon)_{t \geq 0} \Rightarrow (X_t^\varepsilon)_{t \geq 0}$ has to be constructed.

Due to (1.3) consider a differential equation

$$\dot{X}_t = a(X_t) + b(X_t) \dot{Y}_t, \quad (5.1)$$

subject to X_0 , where $\dot{Y}_t$ is the Radon–Nikodym derivative of some absolutely continuous function Y_t from $\mathbf{C}$.

We show that the mapping $(Y_t)_{t \geq 0} \Rightarrow (X_t)_{t \geq 0}$ defined by (5.1) can be extended for any function Y_t from $\mathbf{C}$.

Lemma 5.1. *Let functions $a(x)$ and $b(x)$ be Lipschitz continuous and suppose there exist constants c and C such that $0 < c \leq |b(x)| \leq C$. Then for fixed X_0 there exists a continuous (w.r.t. the metric ρ) mapping $(Y_t)_{t \geq 0} \Rightarrow (X_t)_{t \geq 0}$, $(X_t), (Y_t) \in \mathbf{C}$, such that for any absolutely continuous function (Y_t) this mapping is defined by the differential equation (5.1).*

P r o o f. Let

$$F(x) = \int_0^x \frac{dy}{b(y)}. \quad (5.2)$$

By virtue of the assumptions of the lemma, the function $F(x)$ is continuous differentiable and has an inverse $F^{-1}(x)$ which is continuous differentiable too and both $F(x)$ and $F^{-1}(x)$ satisfy the linear growth condition: there exists a positive constant ℓ such that

$$\left. \begin{array}{l} |F(x)| \\ |F^{-1}(x)| \end{array} \right\} \leq \ell(1 + |x|).$$

Put

$$g(x) = \frac{a(F^{-1}(x))}{b(F^{-1}(x))}. \quad (5.3)$$

Let Y_t be an absolutely continuous function and X_t be a solution of (5.1). Put $\theta_t = F(X_t)$. It is easy to check that θ_t is a solution of a differential equation $\dot{\theta}_t = g(\theta_t) + \dot{Y}_t$ subject to $\theta_0 = F(X_0)$ which is unique by virtue of the local Lipschitzianity of $g(x)$. Then we arrive to a mapping defined by

$$X_t = F^{-1}(\theta_t), \quad \theta_t = F(X_0) + \int_0^t g(\theta_s) ds + [Y_t - Y_0]. \quad (5.4)$$

Evidently, this mapping can be defined not only for absolutely continuous but also for continuous functions Y_t . In fact, $g(x)$ in the integral equation from (5.4), satisfies the linear growth and the local Lipschitz conditions, so this integral equation has the unique solution also in the case of a continuous function Y_t .

Thus it remains to show that this mapping is continuous in the metric ρ . Note that the continuity in the metric ρ is equivalent to the following implication: for any $T > 0$

$$\left\{ \begin{array}{l} \limsup_n \sup_{t \leq T} |Y_t^n - Y_t| = 0, \\ (Y_t^n), (Y_t) \in \mathbf{C}, \quad n \geq 1, \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \limsup_n \sup_{t \leq T} |X_t^n - X_t| = 0, \\ (X_t^n), (X_t) \in \mathbf{C}, \quad n \geq 1, \end{array} \right. \quad (5.5)$$

where X_t^n , $n \geq 1$, and X_t are the solutions of (5.4) corresponding to Y_t^n , $n \geq 1$, and Y_t , respectively.

The validity of (5.5) also is implied by the local Lipschitz condition for the function $g(x)$.

5.2. Taking $Y_t \equiv \sqrt{\varepsilon} W_t^\varepsilon$, $X_t \equiv X_t^\varepsilon$, we get the desired continuous mapping.

Then, due to the contraction principle of Varadhan (see [11]), the family X_t^ε obeys the l.d.p. in the metric space $(\mathbf{C}, \rho)$ with the rate function

$$J(\varphi) = \begin{cases} \inf I(\psi), & \varphi_0 = X_0, d\psi \ll dt, \\ \infty, & \text{otherwise,} \end{cases}$$

where $I(\psi)$ is the rate function corresponding to the l.d.p. for $(\sqrt{\varepsilon} W_t^\varepsilon)$ and inf is taken over all absolutely continuous functions from $\mathbf{C}$ with $\varphi_0 = 0$ such that (see (5.1)) $\dot{\varphi}_t = a(\varphi_t) + b(X_t) \dot{\psi}_t$. Since this equation has a unique solution, the infimum is attained at

$$\psi_t = \int_0^t \frac{\dot{\varphi}_s - a(\varphi_s)}{b(\varphi_s)} ds.$$

§ 6. Application

From the point of view of applications, a more realistic model than (1.4) for the process η_t , involved in (1.3), is

$$\tilde{\eta}_t = \int_0^t h(t-s) dN_s, \quad (6.1)$$

where N_t and $h(t)$ are the same as in (1.4) (note that $\tilde{\eta}_t$ is not a stationary process). Below we consider such kind of processes and show that nevertheless X^ε obeys the l.d.p. and the rate function remains the same.

Let $\xi_t = (\xi_t^1, \xi_t^2, \dots, \xi_t^n)$ be a vector-column Itô's process

$$d\xi_t = A\xi_t dt + B dN_t, \quad t \in \mathbf{R}, \quad (6.2)$$

where A and B are matrices of dimensions $n \times n$ and $n \times 1$ respectively and also the eigenvalues of A belong to the left half of plane. Under the assumption made above

$$\xi_t = \int_{-\infty}^t e^{A(t-s)} dN_s = e^{At} \xi_0 + \int_0^t e^{A(t-s)} dN_s = e^{At} \xi_0 + \tilde{\xi}_t. \quad (6.3)$$

By the process $\tilde{\eta}_t$ take $\tilde{\xi}_t^1$ ($\tilde{\eta}_t \equiv \tilde{\xi}_t^1$). Following (6.3) we have for $\tilde{\eta}_t$ a decomposition of the type (6.1) with

$$h(t) = \sum_{j=1}^n \{e^{At}\}_{1,j} B_j, \quad (6.4)$$

where $\{e^{At}\}_{1,j}$, $1 \leq j \leq n$, are the elements of the first row of the matrix e^{At} and B_j , $1 \leq j \leq n$, are the elements of B .

Put

$$\tilde{W}_t^\varepsilon = \frac{1}{\sqrt{\varepsilon}} \int_0^t \tilde{\eta}_{s/\varepsilon} ds \quad (6.5)$$

(comp. (1.5)). We show that the family $\sqrt{\varepsilon} \tilde{W}_t^\varepsilon$ obeys the l.d.p. in the metric space $(\mathbf{C}, \rho)$ with the rate function $I(\varphi)$ given in (1.12) with $\Sigma = \int_0^\infty \sum_{j=1}^n \{e^{As}\}_{1,j} B_j$. To this end introduce

$$W_t^\varepsilon = \frac{1}{\sqrt{\varepsilon}} \int_0^t \xi_s^1 ds. \quad (6.6)$$

Due to (6.3), $\xi_s^1 = \int_{-\infty}^s h(s-u) dN_u$ with $h(t)$ from (6.4). Under the assumptions made above there exist positive constants c_1 and c_2 such that $|h(t)| \leq c_1 e^{c_2 t}$, so Theorem 1.1 can be applied, i.e., $\sqrt{\varepsilon} W_t^\varepsilon$ obeys the l.d.p. with the rate function mentioned above.

Thus, the desired result for $\sqrt{\varepsilon} \tilde{W}_t^\varepsilon$ holds if for any $T > 0$ and $\gamma > 0$

$$\lim_{\varepsilon \rightarrow 0} \log \mathbf{P} \left\{ \sup_{t \leq T} |\sqrt{\varepsilon} (W_t^\varepsilon - \tilde{W}_t^\varepsilon)| > \gamma \right\} = -\infty. \quad (6.7)$$

To check (6.7) note that under our assumptions $\int_0^\infty \sum_{j=1}^n |\{e^{As}\}_{1,j}| ds = c < \infty$. Then

$$\begin{aligned} |\sqrt{\varepsilon} (W_t^\varepsilon - \tilde{W}_t^\varepsilon)| &= \varepsilon \left| \int_0^{t/\varepsilon} (\xi_s^1 - \tilde{\xi}_s^1) ds \right| \leq \varepsilon \int_0^\infty |\xi_s^1 - \tilde{\xi}_s^1| ds \\ &= \varepsilon \int_0^\infty \sum_{j=1}^n |\{e^{As}\}_{1,j} \xi_0^j| ds \leq c\varepsilon \sum_{j=1}^n |\xi_0^j|. \end{aligned} \quad (6.8)$$

Then, using the exponential Chebyshev inequality, we find for any $\lambda > 0$:

$$\begin{aligned} \mathbf{P} \left\{ \sup_{t \leq T} \left| \sqrt{\varepsilon} (W_t^\varepsilon - \widetilde{W}_t^\varepsilon) \right| > \gamma \right\} &\leq \mathbf{P} \left\{ \sum_{j=1}^n |\xi_0^j| < \frac{\gamma}{c\varepsilon} \right\} \leq \sum_{j=1}^n \mathbf{P} \left\{ |\xi_0^j| < \frac{\gamma}{nc\varepsilon} \right\} \\ &\leq n \max_{1 \leq j \leq N} \left\{ \mathbf{P} \left\{ |\xi_0^j| < \frac{\gamma}{nc\varepsilon} \right\} \right\} \leq n \max_{1 \leq j \leq N} \{ e^{-\lambda\gamma/(nc\varepsilon)} \mathbf{E} e^{\lambda|\xi_0^j|} \}. \end{aligned}$$

In accordance to the last inequality the following upper estimate holds:

$$\begin{aligned} \varepsilon \log \mathbf{P} \left\{ \sup_{t \leq T} \left| \sqrt{\varepsilon} (W_t^\varepsilon - \widetilde{W}_t^\varepsilon) \right| > \gamma \right\} &\leq \varepsilon \log n - \frac{\lambda\gamma}{nc} + \max_{1 \leq j \leq n} \{ \varepsilon \log \mathbf{E} e^{\lambda|\xi_0^j|} \} \\ &\leq \varepsilon \log n - \frac{\lambda\gamma}{nc} + \sum_{j=1}^n \varepsilon \log \mathbf{E} e^{\lambda|\xi_0^j|}. \end{aligned} \quad (6.9)$$

Assume that

$$\mathbf{E} e^{\lambda\xi_0^j} < \infty, \quad \forall \lambda \in \mathbf{R}, \quad j = 1, 2, \dots, n. \quad (6.10)$$

Then, due to the inequality $e^{|x|} \leq e^x + e^{-x}$, we have $\mathbf{E} e^{\lambda|\xi_0^j|} < \infty$ for each $j = 1, 2, \dots, n$ and for all $\lambda > 0$. Consequently under assumption (6.10) the right-hand side of (6.9) goes to $-\infty$ if $\llim_{\lambda \rightarrow \infty} \lim_{\varepsilon \rightarrow 0}$ is taken and so the desired property (6.7) takes place.

Therefore, only (6.10) has to be checked. From (6.3) it follows that $\xi_0 = \int_{-\infty}^0 e^{-As} \times B dN_s$ and by the homogeneity of increments of N_t the random vector ξ_0 coincides in distribution with

$$\xi_0' = \int_0^\infty e^{As} B dN_s. \quad (6.11)$$

Let $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$ be a vector-row with $\lambda_j \in \mathbf{R}$, $j = 1, 2, \dots, n$. Putting $H(s) = \Lambda e^{As} B$ as in the proof of (2.6) we get

$$\mathbf{E} e^{\Lambda \xi_0} = \mathbf{E} e^{\Lambda \xi_0'} = \mathbf{E} \exp \left\{ \int_0^\infty H(s) dN_s \right\} < \infty,$$

i.e., (6.10) holds.

Thus, $\sqrt{\varepsilon} \widetilde{W}_t^\varepsilon$ obeys the l.d.p. The l.d.p. for $\widetilde{X}_t^\varepsilon$ follows from this result due to the contraction principle of Varadhan (see [11]) and Lemma 5.1.

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BARNDORFF-NIELSEN O. E.*
L-NONFORMATION, L-ANCILLARITY,
AND L-SUFFICIENCY

Определяется понятие отсутствия информации об интересующем параметре *L*-неинформативности. Из него вытекают понятия *L*-подчиненности и *L*-достаточности (последнее у нее было изучено ранее). В статье *L* означает правдоподобие. В предлагаемой работе обсуждаются свойства этих понятий.

Ключевые слова и фразы: *L*-неинформативность (*L*-nonformation), *L*-достаточность (*L*-sufficiency), *L*-подчиненность (*L*-ancillarity), правдоподобие, параметризация, информация.

1. Introduction. Rémon [6] introduced a concept of generalized sufficiency. He called it *L*-sufficiency, but for reasons of consistency with other notation that term is now being used in a restricted sense (cf. [2], [5]). Here *L* as a prefix is short for likelihood.

Inherent in the concept of *L*-sufficiency is a notion of «no information about a parameter of interest». In the present paper this notion is extracted and termed *L*-nonformation. In addition to *L*-sufficiency, it leads to a concept of generalized ancillarity, to be termed *L*-ancillarity.

L-nonformation is defined and discussed in Section 2 and its relation to *L*-sufficiency is considered in Section 3. Section 4 introduces the concept of *L*-ancillarity and derives some properties of this, particularly in the context of exponential models. The rest of the present section sets out a few preliminaries.

Let $p(x; \omega)$ be the model function of a parametric model M and let (ψ, χ) be a partition of the parameter ω into an interest parameter ψ and a nuisance parameter χ . We assume that ψ and χ are variation independent (i.e., $\Omega = \Psi \times X$, where Ω , Ψ , and X are the domains of variation of ω , ψ , and Ξ , respectively). Reparametrizations $(\psi, \chi) \rightarrow (\varphi, \lambda)$ of M of the form

$$\varphi = \varphi(\psi), \quad \lambda = \lambda(\psi, \chi)$$

are called *interest respecting reparametrizations*.

The maximum likelihood estimate for χ when ψ is considered as known is denoted by $\hat{\chi}_\psi$ and for any combinant $q = q(x; \omega)$ we use the convention that the symbols « $\hat{}$ » and « $\tilde{}$ » placed above q indicate evaluation at the full and partial maximum likelihood estimate, respectively. That is, $\hat{q} = q(x; \hat{\omega})$ and $\tilde{q} = q(x; \psi, \hat{\chi}_\psi)$. The evaluation is taken to be the last operation in the calculation of the quantity concerned, in order to avoid possible ambiguities.

We denote the profile likelihood for the interest parameter ψ by $L_P(\psi)$ or by $\tilde{L}(\psi)$, i. e., $L_P(\psi) = L(\psi, \hat{\chi}_\psi) = \tilde{L}(\psi)$. Furthermore, the normed version of the profile likelihood for ψ will be denoted by $\bar{L}_P(\psi)$, i. e.,

$$\bar{L}_P(\psi) = \frac{\tilde{L}(\psi)}{\tilde{L}(\hat{\psi})}.$$

Let u and v be statistics. For given value of ψ we write $\hat{\chi}_\psi(\tilde{v} \mid \tilde{u})$ for the maximum likelihood estimate of χ based on the conditional distribution of v given u , and we denote

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