KOLMOGOROV’S STRONG LAW OF LARGE NUMBERS HOLDS FOR PAIRWISE UNCORRELATED RANDOM VARIABLES\textsuperscript{1)}

1. Introduction and results. In publication [3] from 1981, N. Etemadi weakened the requirements of Kolmogorov’s first Strong Law of Large Numbers (SLLN) for identically distributed random variables and provided an elementary proof of a more general SLLN. We will first state the main theorem of [3] after introducing some notation that we will use.

Notation. Throughout the paper, \((\Omega, \mathcal{A}, P)\) is a probability space. All random variables (usually denoted by \(X\) or \(X_n\)) are assumed to be measurable functions from \(\Omega\) to the set of real numbers \(\mathbb{R}\). If \(X\) is a random variable, then \(E X\) stands for its expected value and \(D X\) stands for its variance.

Etemadi’s Theorem [3]. Let \((X_n)_{n \in \mathbb{N}}\) be a sequence of pairwise independent and identically distributed random variables with \(E |X_1| < \infty\). Then

\[
\lim_{n \to \infty} \frac{X_1 + \ldots + X_n}{n} = E X_1 \quad \text{almost surely (a.s.)}.
\] (1)

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