1. Introduction. Kotlarski [6] (cf. [10, Theorem 2.1.1]) proved the following result.

**Theorem 1.1.** Let $X_1$, $X_2$, and $X_3$ be three independent real-valued random variables. Let $Z_1 = X_1 - X_3$ and $Z_2 = X_2 - X_3$. If the characteristic function of the random vector $(Z_1, Z_2)$ does not vanish, then the joint distribution of the random vector $(Z_1, Z_2)$ determines the distributions of $X_1$, $X_2$, and $X_3$ up to a change of location.

Miller [8] generalized Theorem 1.1 to random vectors. Theorem 1.1 has been extended to independent random elements taking values in a Hilbert space in [5] and for random elements taking values in a locally compact abelian group in [9]. It is possible to relax the condition of nonvanishing characteristic function in Theorem 1.1 under some additional condition of analyticity of the characteristic functions. For details, see [14] (cf. [10, p. 15]).

Rao [13] (cf. [10, Theorem 2.1.4]) proved the following result.

**Theorem 1.2.** Suppose $X_1$, $X_2$, and $X_3$ are three independent real-valued random variables. Consider two linear forms

$$Z_1 = a_1 X_1 + a_2 X_2 + a_3 X_3$$

and

$$Z_2 = b_1 X_1 + b_2 X_2 + b_3 X_3$$

such that $a_i : b_i \neq a_j : b_j$ for $i \neq j$. If the characteristic function of $(Z_1, Z_2)$ does not vanish, then the distribution of $(Z_1, Z_2)$ determines the distribution of $X_1$, $X_2$ and $X_3$ up to a change of location.

Kagan and Székely [4] in their paper introduced the notion of $Q$-independence and $Q$-identically distributed random variables and proved that the classical characterization properties of normal distribution due to Cramér [1],

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