The replica method in quantum mechanics

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Quantum entropic characterization of Gaussian optical transformations using the replica method C. N. Gagatsos, A. I. Karanikas, G. Kordas, N. J. Cerf quant-ph arXiv:1408.5062 (2014)

$$S(\hat{\rho}) = -Tr(\hat{\rho} \ln \hat{\rho})$$

$$S(\hat{\rho}) = -\sum \lambda_k \ln \lambda_k$$

The Tsallis entropy of order n

C. Tsallis, Lecture Notes in Physics, Springer, Berlin (2001)

$$S^{(n)}{}_T(\hat{\rho}) = -\frac{1 - Tr\,\hat{\rho}^n}{n-1}$$

$$\lim_{n\to1} S^{(n)}{}_T(\hat{\rho}) = -Tr(\hat{\rho}\ln\hat{\rho}) = S(\hat{\rho})$$

$$\lim_{n\to1} S^{(n)}{}_T(\hat{\rho}) = -\frac{\partial}{\partial n} Tr\,\hat{\rho}^n \Big|_{n\to1}$$

$$S(\hat{\rho}) = -\lim_{n\to1}\frac{\partial}{\partial n} Tr\,\hat{\rho}^n$$

$$Tr\,\hat{\rho}^{\,n} = \int dx_1 \int dx_2 .. \int dx_n \, \rho(x_1, x_2)\rho(x_2, x_3)..\rho(x_n, x_1)$$

$$\rho(x_1, x_2) = \langle x_1 | \hat{\rho} | x_2 \rangle$$

$$\hat{\rho}_{th} = \frac{1}{N+1} \sum_{n=0}^{\infty} \left(\frac{N}{N+1} \right)^{n} |n\rangle\langle n|$$

$$S(\hat{\rho}_{th}) = (N+1)\ln(N+1) - (N)\ln(N)$$

$$\hat{\rho}_{th} = \frac{1}{\pi N} \int d^{2}\alpha e^{-N|\alpha|^{2}} |\alpha\rangle\langle\alpha|$$

$$Tr\,\widehat{\rho_{th}}^{\,n} = \frac{1}{(1+N)^{n} - N^{n}}$$

N. A. Ansari and V. I. Man'ko, Phys. Rev. A, 50, 1942 (1994)

V. V. Dodonov, V. I. Man'ko, and D. E. Nikonov, Phys. Rev. A, 51, 3328 (1995)

$$\hat{\rho} = \frac{1}{1+a}\left(\left|\alpha\right\rangle\left\langle\alpha\right| + a\left|-\alpha\right\rangle\left\langle-\alpha\right|\right)$$

$$\alpha = |\alpha| e^{i\varphi}$$

$$\left\langle x | \alpha \right\rangle = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{x^2}{2} + \sqrt{2}\alpha x - \frac{|\alpha|^2}{2} - \frac{\alpha^2}{2}}$$

$$\hat{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle$$

$$Tr\hat{\rho}^{n}=\frac{1}{(1+a)^{n}}\left(1+ae^{-8|\alpha|^{2}}\varphi\left[a^{(1)}\right]\right)\left(1+ae^{-8|\alpha|^{2}}\varphi\left[a^{(2)}\right]\right)..\left(1+ae^{-8|\alpha|^{2}}\varphi\left[a^{(n)}\right]\right)$$

$$a^{(1)}=\left\{1,1,0,..0\right\} \qquad a^{(2)}=\left\{0,1,1,..0\right\} \qquad a^{(n)}=\left\{1,0,0,..1\right\}$$

$$\varphi\left[a^{(1)}\right]=e^{2|\alpha|^{2}\sum_{j=1}^{n}\left(a_{j}{}^{(1)}\right)^{2}}$$

$$\varphi\left[a^{(1)}\right]\varphi\left[a^{(2)}\right]=e^{2|\alpha|^{2}\sum_{j=1}^{n}\left(a_{j}{}^{(1)}\right)^{2}+\left(a_{j}{}^{(2)}\right)^{2}}$$

$$\varphi\left[a^{(1)}\right]\varphi\left[a^{(2)}\right]\varphi\left[a^{(3)}\right]=e^{2|\alpha|^{2}\sum_{j=1}^{n}\left(a_{j}{}^{(1)}\right)^{2}+\left(a_{j}{}^{(2)}\right)^{2}+\left(a_{j}{}^{(3)}\right)^{2}}$$

$$\mu = Tr\hat{\rho}^2 = \frac{1}{(1+a)^2}\left(1 + 2ae^{-4|\alpha|^2} + a^2\right)$$

$$Tr\hat{\rho}^3 = \frac{1}{(1+a)^3}\left(1 + 3ae^{-4|\alpha|^2} + 3a^2e^{-4|\alpha|^2} + a^3\right)$$

$$Tr\hat{\rho}^4 = \frac{1}{(1+a)^4}\left(1 + 4ae^{-4|\alpha|^2} + 4a^2e^{-4|\alpha|^2} + 2a^2e^{-8|\alpha|^2} + 4a^3e^{-4|\alpha|^2} + a^4\right)$$

Purity inequality for bipartite system

Deformed Subadditivity Condition for Qudit States and Hybrid Positive Maps Margarita A. Man'ko, Vladimir I. Man'ko in Journal of Russian Laser Research (2014)

$\rho(1,2)$ Is the density matrix of a bipartite system

$\rho(1)$ $\rho(2)$

$$\rho(1) = Tr_2\,\rho(1,2) \qquad \rho(2) = Tr_1\,\rho(1,2)$$

$$S^{(n)}{}_T\left(1,2\right) \leq S^{(n)}{}_T\left(1\right) + S^{(n)}{}_T\left(2\right)$$

$n = 2$ $\qquad 1 + \mu_{12} \geq \mu_1 + \mu_2$

$$\mu_{12} = Tr\,\rho^2(1,2)$$

$\mu_1 = Tr\,\rho^2(1)$ $\qquad \mu_2 = Tr\,\rho^2(2)$

N. A. Ansari and V. I. Man'ko, Phys. Rev. A, 50, 1942 (1994)

V. V. Dodonov, V. I. Man'ko, and D. E. Nikonov, Phys. Rev. A, 51, 3328 (1995)

$$\widehat{\rho} = \frac{1}{2}\left|\alpha_1\alpha_2\right\rangle\left\langle\alpha_1\alpha_2\right| + \frac{1}{2}\left|-\alpha_1\alpha_2\right\rangle\left\langle-\alpha_1\alpha_2\right|$$

$$\left|\alpha_1\alpha_2\right\rangle = \left|\alpha_1\right\rangle\left|\alpha_2\right\rangle$$

$$\widehat{a}_i\left|\alpha_i\right\rangle = \alpha_i\left|\alpha_i\right\rangle$$

$$\mu_{12} = \frac{1}{2}\left(1+e^{-4|\alpha_1|^2-4|\alpha_2|^2}\right)$$

$$\mu_1 = \frac{1}{2}\left(1+e^{-4|\alpha_1|^2}\right) \qquad \mu_2 = \frac{1}{2}\left(1+e^{-4|\alpha_2|^2}\right)$$

$$1+\mu_{12}-\mu_1-\mu_2 = \frac{1}{2}\left(1-e^{-4|\alpha_1|^2}\right)\left(1-e^{-4|\alpha_2|^2}\right) \geq 0$$