

Nikolskii-type inequalities for algebra polynomials in regions with cusps

F. Abdullayev, N. Özkartepe
Mersin University, Turkey

Let $G \subset \mathbb{C}$ be a finite Jordan region, with $0 \in G$, $L := \partial G$; $P_n(z)$, $\deg P_n \leq n$, $n \in \mathbb{N}$, be an arbitrary algebraic polynomials and let $h(z)$ be a weight function. For $p > 0$ we denote by $A_p(h, G)$ the class of analytic in G functions f such that

$$\iint_G h(z)|f(z)|^p dx dy < \infty, \quad z = x + iy;$$

and, when L is rectifiable, by $\mathcal{L}_p(h, L)$, $p > 0$, the class of measurable on L functions f such that

$$\int_L h(z)|f(z)|^p |dz| < \infty.$$

In this work, we study the Nikol'skii-type inequalities for algebraic polynomials $P_n(z)$ and pointwise estimations for these polynomials in various regions of the complex plane through their $A_p(h, G)$ and $\mathcal{L}_p(h, L)$ -norms, depending on the geometrical properties of regions and generalized Jacobi weight function $h(z)$ for some Jordan regions of complex plane.

References

- [1] F. G. Abdullayev, "On the some properties of the orthogonal polynomials over the region of the complex plane (Part III)", *Ukr. Math. J.*, **53**:12 (2001), 1934–1948.
- [2] E. Hille, G. Szegő, J. D. Tamarkin, "On some generalization of a theorem of A. Markoff", *Duke Math.*, **3** (1937), 729–739.
- [3] N. Stylianopoulos, "Fine asymptotics for Bergman orthogonal polynomials over domains with corners", CMFT 2009 (Ankara, June 2009).
- [4] J. L. Walsh, *Interpolation and Approximation by Rational Functions in the Complex Domain*, AMS, 1960.