Recent progress in the study of the boundedness of classical operators of real analysis in general Morrey-type spaces

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Let $0 < p, \theta \leqslant \infty$ and let w be a non-negative measurable function on $(0, \infty)$. We denote by $LM_{p\theta,w}$, $GM_{p\theta,w}$, the local Morrey-type spaces, the global Morrey-type spaces respectively, which are the spaces of all functions $f \in L_p^{loc}(\mathbb{R}^n)$ with finite quasi-norms

$$\big\| w(r) \|f\|_{L_p(B_r)} \big\|_{L_\theta(0,\infty)}, \qquad \sup_{x \in \mathbb{R}^n} \|f(x + \cdot)\|_{LM_{p\theta,w}}$$

respectively. (Here B_r is the ball of radius r centered at the origin.) For $w(r) = r^{-\frac{\lambda}{p}}$ with $0 < \lambda < n$ the spaces $GM_{p\theta,w}$ were introduced by C. Morrey in 1938 and appeared to be quite useful in various problems in the theory of partial differential equations.

A survey will be given of recent results in which, for a certain range of the numerical parameters p_1, θ_1, p_2, θ_2, necessary and sufficient conditions on the functions w_1 and w_2 are established ensuring the boundedness of the maximal operator, fractional maximal operator, Riesz potential, genuine singular integrals, the Hardy operator as operators from one local Morrey-type space $LM_{p_1\theta_1,w_1}$ to another one $LM_{p_2\theta_2,w_2}$.

Under discussion there will also be interpolation theorems for general local Morrey-type spaces $LM_{p\theta,w}$.

References

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