# Uniform boundness of Steklov's operator in variable exponent Morrey space

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Let $p(\,\cdot\,)$ be a continuous function on $I_0 = [0, 1]$ with values in $[1, \infty)$. We suppose that

$$1 \leqslant p_- \leqslant p(x) \leqslant p_+ < \infty, \tag{1}$$

where $p_- := \operatorname{ess\,inf}_{x \in I_0} p(x) \geqslant 1$, $p_+ := \operatorname{ess\,sup}_{x \in I_0} p(x) < \infty$, and also suppose the $p(\,\cdot\,)$ satisfy the log-condition i.e.

$$|p(x) - p(y)| \leqslant \frac{A}{-\ln|x - y|}, \qquad |x - y| \leqslant \frac{1}{2}, \quad x, y \in I_0. \tag{2}$$

Let $\lambda(\,\cdot\,)$ be a measurable function on $I_0$ with values in $[0, 1]$. We define the variable exponent Morrey space $M^{p(\,\cdot\,),\lambda(\,\cdot\,)}(I_0)$ as the set of integrable functions $f$ on $I_0$ such that

$$I_{p(\,\cdot\,),\lambda(\,\cdot\,)}(f) := \sup_{\substack{x \in I_0 \\ 0 < r < 2\pi}} r^{-\lambda(x)} \int_{\widetilde{I}(x,r)} |f|^{p(y)}\, dy < \infty.$$

The norm of space $M^{p(\,\cdot\,),\lambda(\,\cdot\,)}(I_0)$ may be defined in two forms,

$$\|f\|_1 := \inf\left\{\eta > 0 : I_{p(\,\cdot\,),\lambda(\,\cdot\,)}\left(\frac{f}{\eta}\right) < 1\right\},$$

and

$$\|f\|_2 := \sup_{\substack{x \in I_0 \\ 0 < r < 2\pi}} r^{-\frac{\lambda(x)}{p(x)}} \|f \chi_{\widetilde{I}(x,r)}\|_{L^{p(\,\cdot\,)}(I_0)}.$$

Since two norms coincide, we put

$$\|f\|_{M^{p(\,\cdot\,),\lambda(\,\cdot\,)}(I_0)} := \|f\|_1 = \|f\|_2.$$

The Steklov operator is defined as

$$s_h(f)(x) := \frac{1}{h} \int_0^h f(x + t)\, dt.$$

Our main result is following.

Theorem. *Let $f \in M^{p(\,\cdot\,),\lambda(\,\cdot\,)}(I_0)$, $\lambda_+ := \operatorname{ess\,sup}_{x \in I_0} \lambda(x)$, $0 \leqslant \lambda(x) \leqslant \lambda_+ < 1$, and $p(\,\cdot\,)$ satisfy conditions (1) and (2), then the family of operators $s_h(f)$, $0 < h \leqslant 1$, is uniformly bounded in $M^{p(\,\cdot\,),\lambda(\,\cdot\,)}(I_0)$.*

This contribution is based on recent joint work with Professor Vagif Guliyev.

## References

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