Necessary and sufficient conditions for the boundedness of the fractional integral operators in the local Morrey-type spaces on Carnot groups

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Let $\mathbb{G}$ be a Carnot group (nilpotent stratified Lie group), ρ its homogeneous norm and Q its homogeneous dimension. The fractional integral $I_\alpha f$ on Carnot group $\mathbb{G}$ is defined by

$$I_\alpha f(x) = \int_{\mathbb{G}} \rho(y^{-1}x)^{\alpha-Q} f(y)\, dy, \qquad 0 < \alpha < Q.$$

Let $0 < p, \theta \leqslant \infty$ and let w be a non-negative measurable function on $(0, \infty)$. We denote by $LM_{p\theta,w}(\mathbb{G})$, $GM_{p\theta,w}(\mathbb{G})$, the local Morrey-type spaces, the global Morrey-type spaces respectively, which are the spaces of all functions $f \in L_p^{\mathrm{loc}}(\mathbb{G})$ with finite quasi-norms

$$\|f\|_{LM_{p\theta,w}(\mathbb{G})} = \left(\int_0^\infty w(r)^\theta \left(\int_{\{x\in\mathbb{G}:\rho(x)<r\}} |f(x)|^p\, dx \right)^{\theta/p} dr \right)^{1/\theta},$$

$$\|f\|_{GM_{p\theta,w}(\mathbb{G})} = \sup_{x\in\mathbb{G}} \left(\int_0^\infty w(r)^\theta \left(\int_{\{y\in\mathbb{G}:\rho(y^{-1}\cdot x)<r\}} |f(y)|^p\, dy \right)^{\theta/p} dr \right)^{1/\theta}$$

respectively. For $\theta = \infty$ and $w(r) = r^{-\frac{\lambda}{p}}$ with $0 < \lambda < Q$ the space $M_{p,\lambda}(\mathbb{G}) \equiv GM_{p\infty,r^{-\lambda/p}}(\mathbb{G})$ is the Morrey space, for $\theta = \infty$ the space $M_{p,w}(\mathbb{G}) \equiv GM_{p\infty,w}(\mathbb{G})$ is the generalized Morrey space on Carnot group $\mathbb{G}$.

A survey will be given of recent results in which, for certain ranges of the numerical parameters n, p_1, θ_1, p_2, θ_2 necessary and sufficient conditions on the functions w_1 and w_2 are established ensuring the boundedness of the fractional integral operators from one local Morrey-type space $LM_{p_1\theta_1,w_1}(\mathbb{G})$ to another one $LM_{p_2\theta_2,w_2}(\mathbb{G})$.

It is shown that from the above result the Sobolev-Morrey embeddings for Carnot groups follow easily. A priori estimates for the sub-Laplacian in corresponding Besov-Morrey spaces are also proved.

Note that, the local Morrey-type spaces $LM_{p\theta,w}(\mathbb{G})$ defined on homogeneous Lie groups $\mathbb{G}$ were introduced in doctoral thesis [1] by Guliyev (see also [2]) and the global Morrey-type spaces $GM_{p\theta,w}(\mathbb{R}^n)$ defined on n-dimensional Euclidian space $\mathbb{R}^n$ were introduced in [3] by Burenkov and Guliyev (see also [4], [5]). The main purpose of [1] (also of [2]) is to give some sufficient conditions for the boundedness of fractional integral operators and singular integral operators defined on homogeneous Lie groups in the local Morrey-type space $LM_{p\theta,w}(\mathbb{G})$. In a series of papers by Burenkov, H. Guliyev and V. Guliyev, etc. (see [3], [4], [5], [6]), some necessary and sufficient conditions for the boundedness of fractional maximal operators, fractional integral operators and singular integral operators in local Morrey-type spaces $LM_{p\theta,w}(\mathbb{R}^n)$ were given.

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References

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