Norm of union for dual Morrey spaces with applications to nonlinear elliptic PDEs

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For Banach spaces, the union as a general construction is a nonsence – it is not even a linear set. For a purposes of analysis, the construction of a sum of spaces is sufficient in (almost) all situations. For nolinear PDEs it is not so. Even minor simpleness of a construction $\inf_j \|f\|_{X_j}$ in comparison with $\inf_{\sum_j f_j = f} \sum_j \|f_j\|_{X_j}$ can be crucial.

For $p \in (1, \infty)$, $a \in (0, n(p-1))$ we let define spaces $L_{p,a} = L_{p,a}(R^n)$ by the (quasi)norm

$$\|f\|_{p,a} = \inf_{\sigma} \|f; L_p(R^n; \omega)\|,$$

$L_p(R^n; \omega)$ – weighted Lebesgue spaces with

$$\omega(x) = \left(\int_{R_+^{n+1}} r^{a/(1-p)} 1_{\{|x-y|<r\}} \, d\sigma(y,r) \right)^{1-p}$$

inf is taken over nonnegative Borel measures σ on $R_+^{n+1} = \{(y,r) : y \in R^n, r > 0\}$ with normalization $\sigma(R_+^{n+1}) = 1$.

N.B. The dual for $L_{p,a}$ with $a > 0$ are classical Morrey spaces.

We consider nonlinear elliptic equations of the form

$$\operatorname{div}^m A(x, D^m u) = f(x)$$

in R^n with the natural energetic space W_p^m, $p \in (1, \infty)$, and standard structure conditions (e.g. m, p-Laplacian).

We establish the existence of very weak solution $u \in W_{p,a}^m$ for some range of $a \in (0, a^*)$ where $a^* > 0$ depends on n, m, p and a modulus of ellipticity of equation.

Key difference from spaces W_q^m with $q \neq p$ (a priori estimates in $W_{p-\varepsilon}^m$ are known since 1993) is that weighted spaces $W_{p,\omega}^m$ allow to establish pseudomonotonicity of our nonlinear operator.