Квантовые операции добавления и удаления фотонов и дифференциатор волновой функции на их основе

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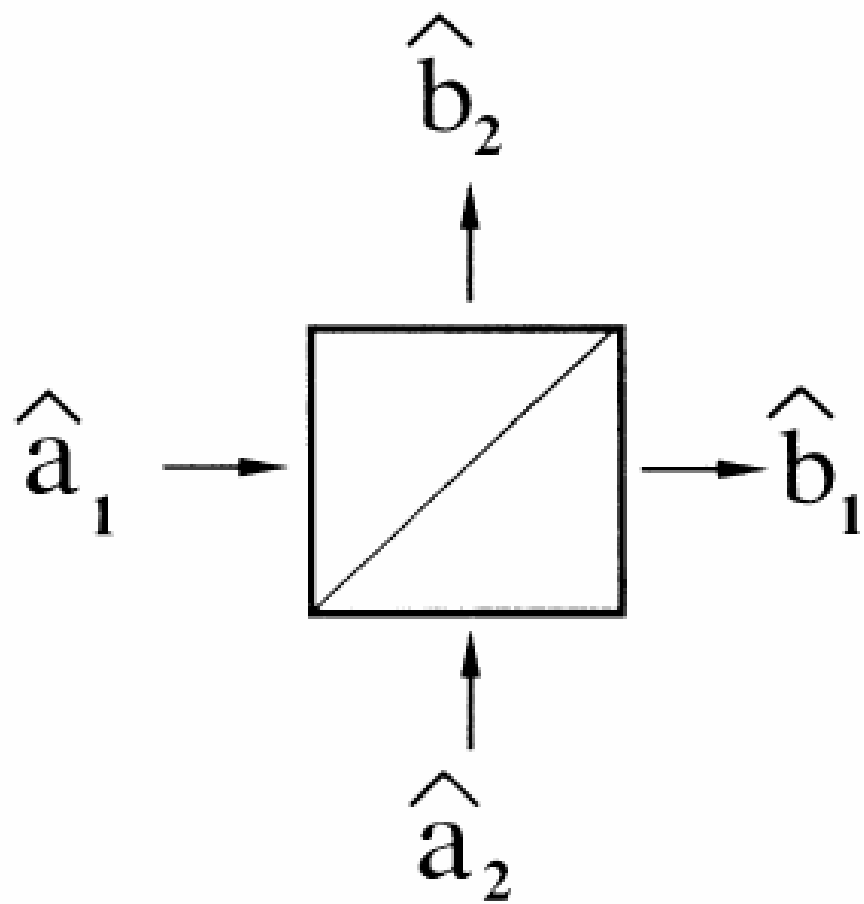
$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

$$\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle$$



R.A. Campos et al, PRA 40 1371 (1989)

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

$$B_{ij} = |B_{ij}| e^{i\phi_{ij}}, \quad i, j = 1, 2$$

$$[\hat{b}_i, \hat{b}_j^\dagger] = \hat{b}_i \hat{b}_j^\dagger - \hat{b}_j^\dagger \hat{b}_i = \delta_{ij}$$

$$|B_{11}|^2 + |B_{12}|^2 = 1$$

$$|B_{21}|^2 + |B_{22}|^2 = 1$$

$$B_{11} B_{21}^* + B_{12} B_{22}^* = 0$$

$$|B_{11}|^2 = |B_{22}|^2 = \tau \equiv \cos^2\theta$$

$$|B_{12}|^2 = |B_{21}|^2 = \rho = \sin^2\theta$$

$$\phi_\tau \equiv \tfrac{1}{2}(\phi_{11} - \phi_{22})$$

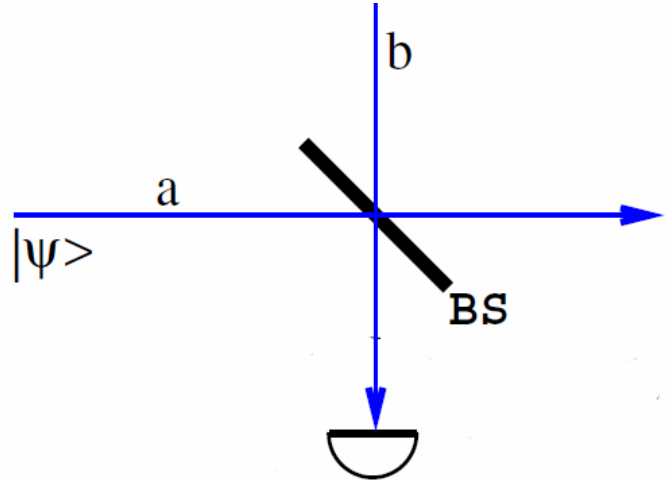
$$\phi_\rho \equiv \tfrac{1}{2}(\phi_{12} - \phi_{21} \mp \pi)$$

$$\phi_0 \equiv \tfrac{1}{2}(\phi_{11} + \phi_{22})$$

$$\underline{\boldsymbol{B}} = e^{i\phi_0} \begin{pmatrix} \cos\theta e^{i\phi_\tau} & \sin\theta e^{i\phi_\rho} \\ -\sin\theta e^{-i\phi_\rho} & \cos\theta e^{-i\phi_\tau} \end{pmatrix}$$

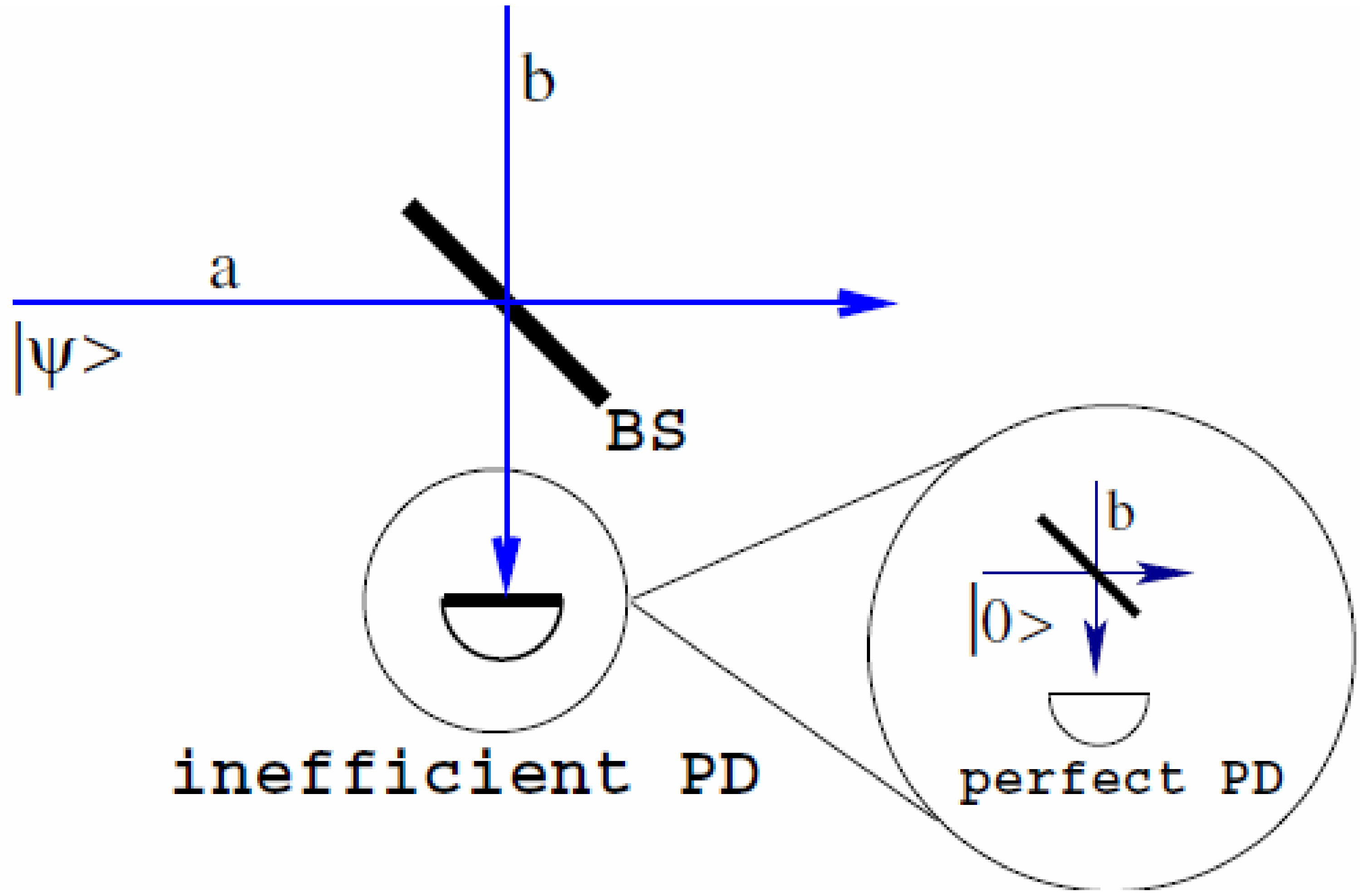
Удаление фотона

$$U = e^{\theta\left(a_1 a_2^{+} - a_1^{+} a_2\right)}$$

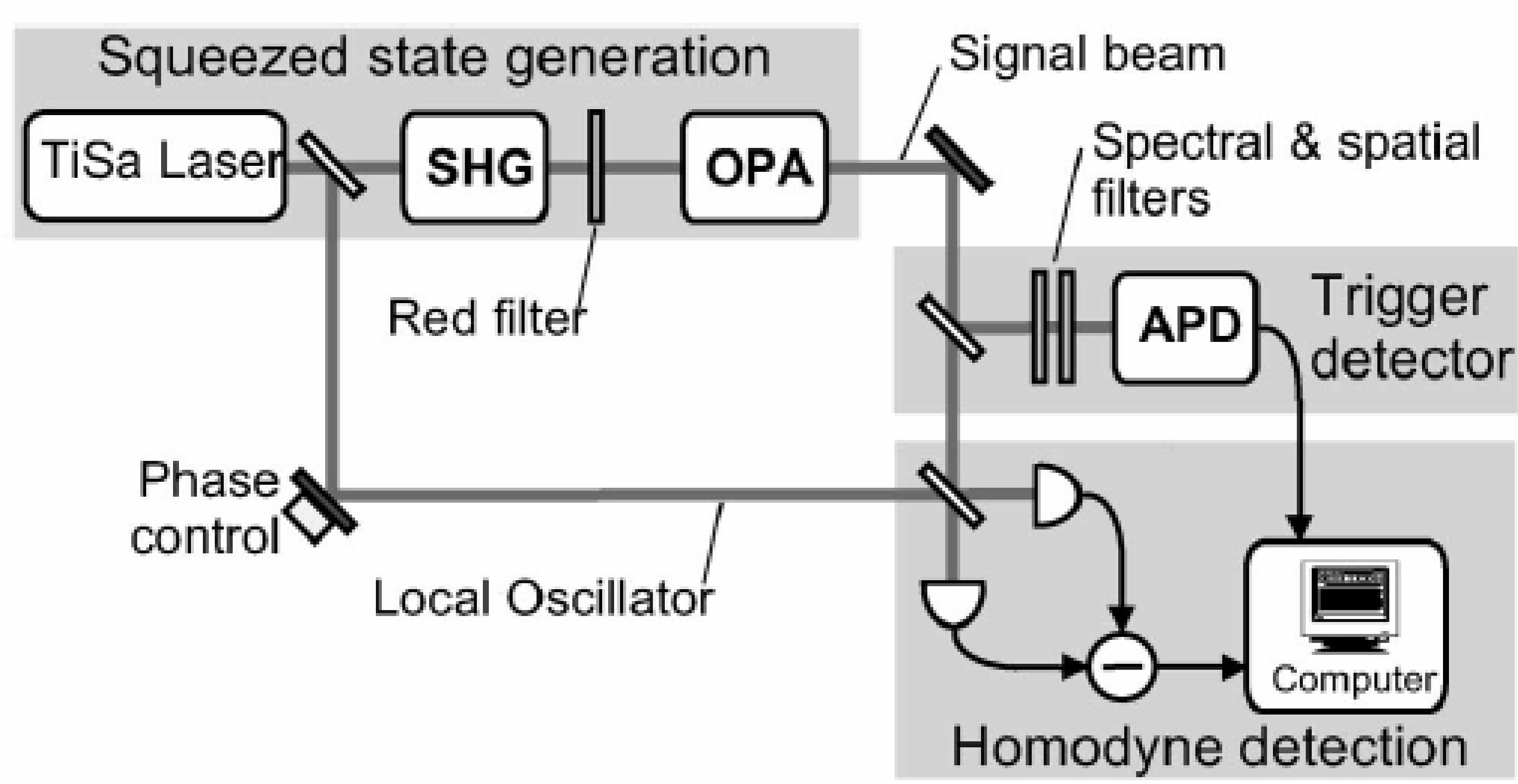


$$\rho_1 \sim \sum_{k=1}^{\infty} \langle k| U \rho_1 \otimes |0\rangle_2 \langle 0| U^{+} |k\rangle_2$$

$$\rho_1 \sim a\rho_1 a^{+}\left(\theta^2 - \frac{\theta^4}{3}\right) - \frac{\theta^4}{2}\left(a^{+}a^2\rho_1 a^{+} + a\rho_1 (a^{+})^2 a - a^2\rho(a^{+})^2\right)$$


b
a
|ψ>
BS
inefficient PD
b
|0>
perfect PD

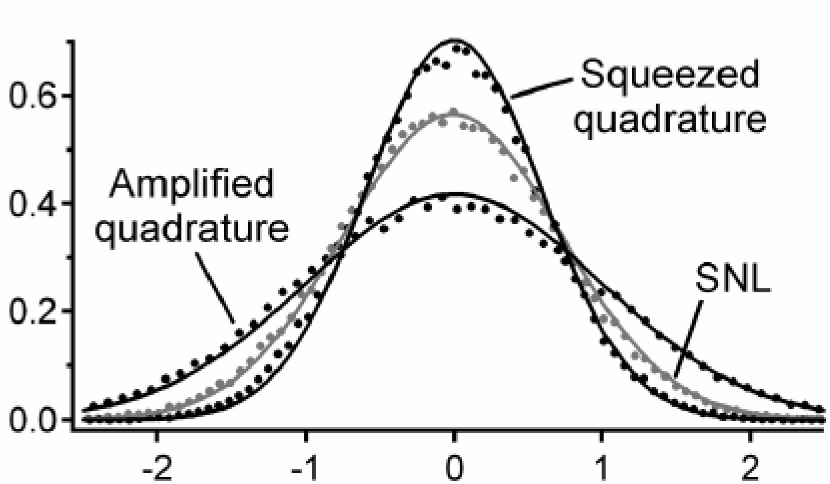
Экспериментальная реализация



J. Wenger et al. Phys. Rev. Lett. **92**, 153601 (2004)

Экспериментальная реализация

- Вход

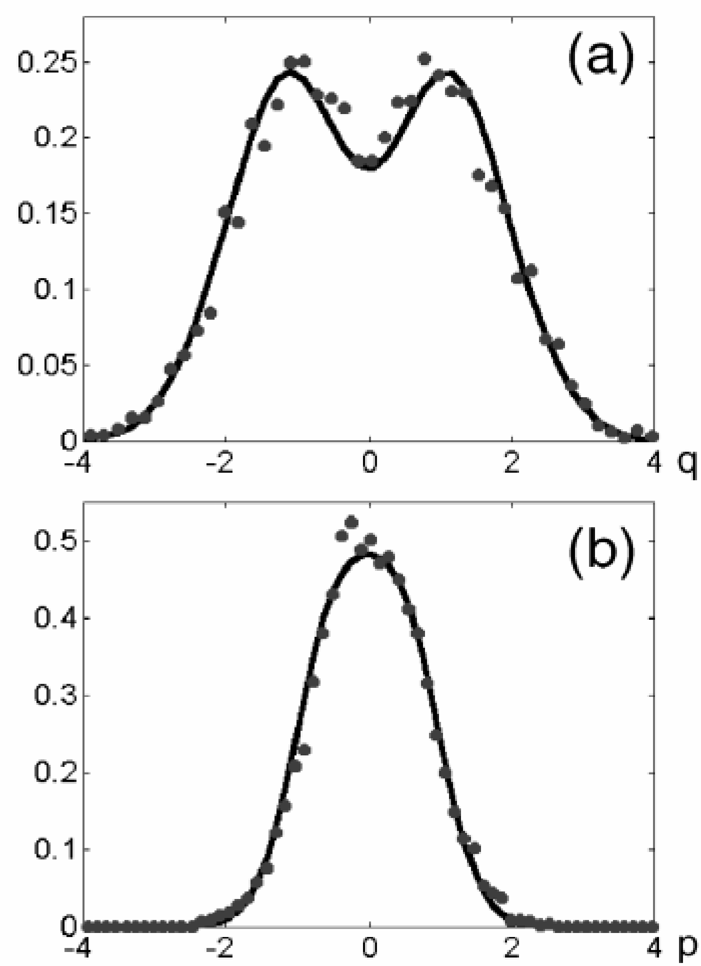


$$|\Psi_s\rangle = \alpha\,|0\rangle + \beta\,|2\rangle + \gamma\,|4\rangle$$

$$\begin{aligned}|\Psi_{s,\mathrm{out}}\rangle = &(\alpha\,|0\rangle_1 + t^2\beta\,|2\rangle_1 + t^4\gamma\,|4\rangle_1)\,|0\rangle_2 \\ &+ (\sqrt{2}rt\beta\,|1\rangle_1 + 2rt^3\gamma\,|3\rangle_1)\,|1\rangle_2 + O(2)\end{aligned}$$

$$|\Psi_{\mathrm{cond}}\rangle \propto \beta\,|1\rangle + \sqrt{2}\gamma\, t^2\,|3\rangle$$

- Выход



J. Wenger et al. Phys. Rev. Lett. **92**, 153601 (2004)

$$\hat{B}(\theta, \phi) = \exp\left\{\frac{\theta}{2}(\hat{a}^\dagger \hat{b}\, \mathrm{e}^{\mathrm{i}\phi} - \hat{a}\hat{b}^\dagger\, \mathrm{e}^{-\mathrm{i}\phi})\right\}$$

$$\phi = 0$$

$$\begin{pmatrix} \hat{a}_{\mathrm{out}} \\ \hat{b}_{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} t & -r \\ r & t \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}$$

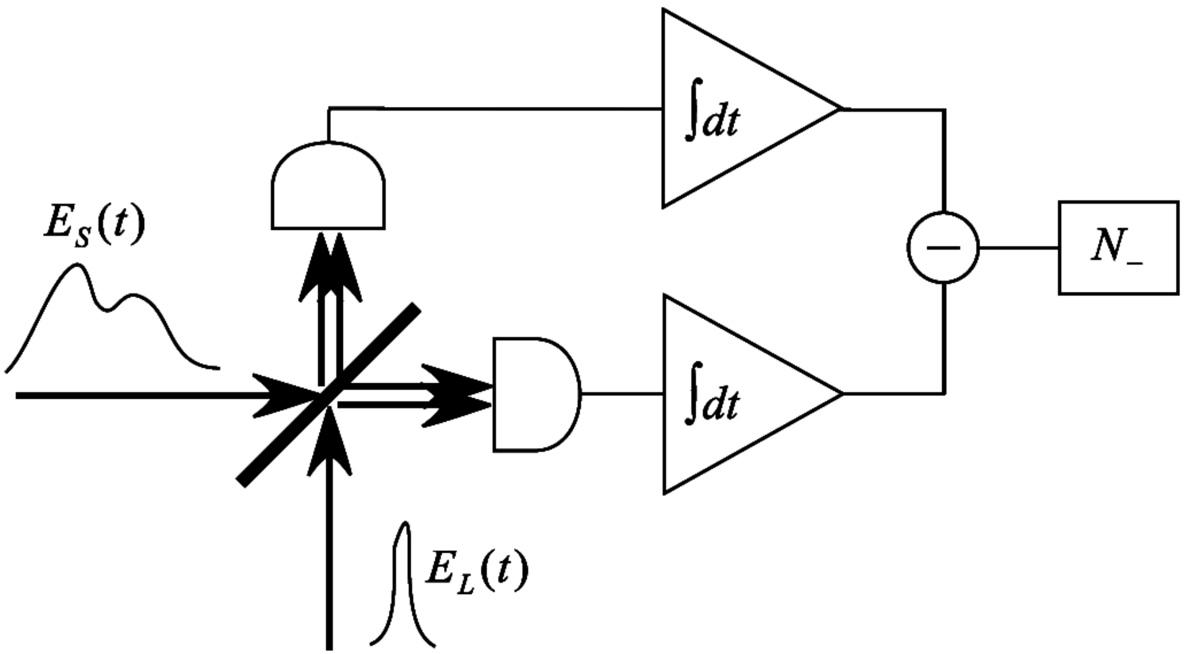
$$\hat{n}_b - \hat{n}_a = (t^2 - r^2)(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) + 2tr(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger)$$

$$r = t = 1/\sqrt{2}$$

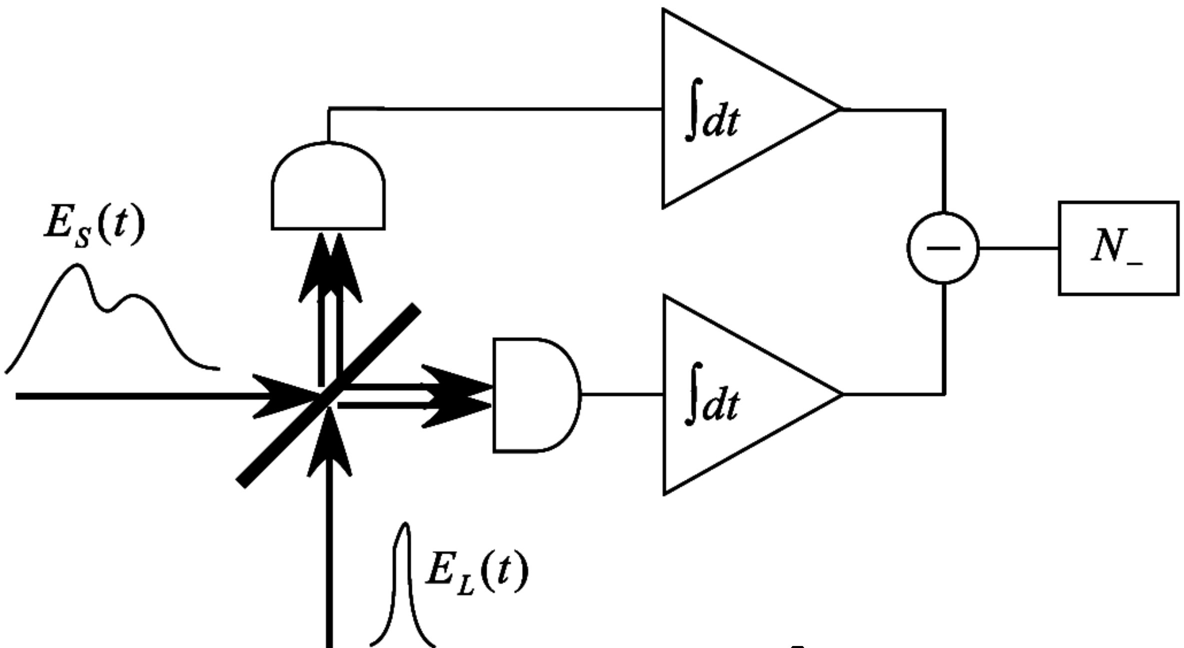
$$\hat{n}_a - \hat{n}_b = \hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger$$

$$\hat{n}_b - \hat{n}_a = |\mathcal{E}_L|(\hat{a}^\dagger \mathrm{e}^{\mathrm{i}\phi_L} + \hat{a}\, \mathrm{e}^{-\mathrm{i}\phi_L}) \equiv \hat{N}_{\phi_L}$$

Гомодинное детектирование

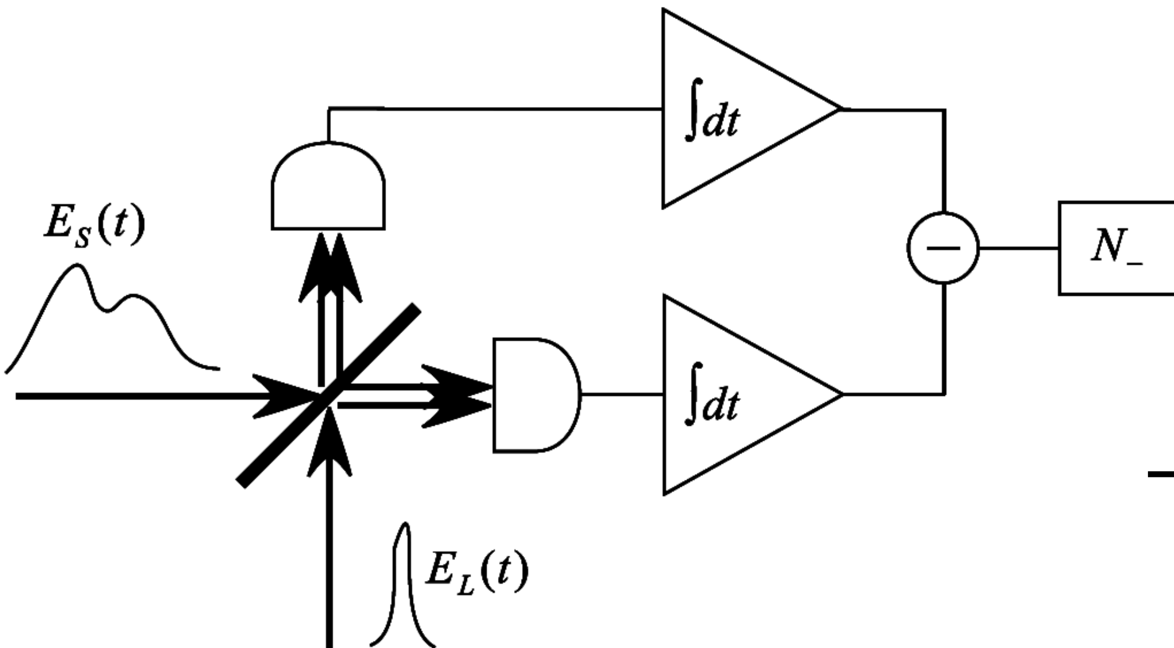


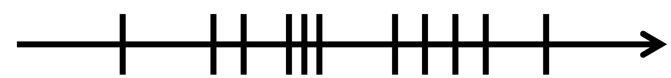
Гомодинное детектирование



$$\frac{\hat{N}_-}{\sqrt{2}\left|\alpha_L\right|} = \frac{\hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}}{\sqrt{2}} = \hat{X}_\theta$$

Гомодинное детектирование





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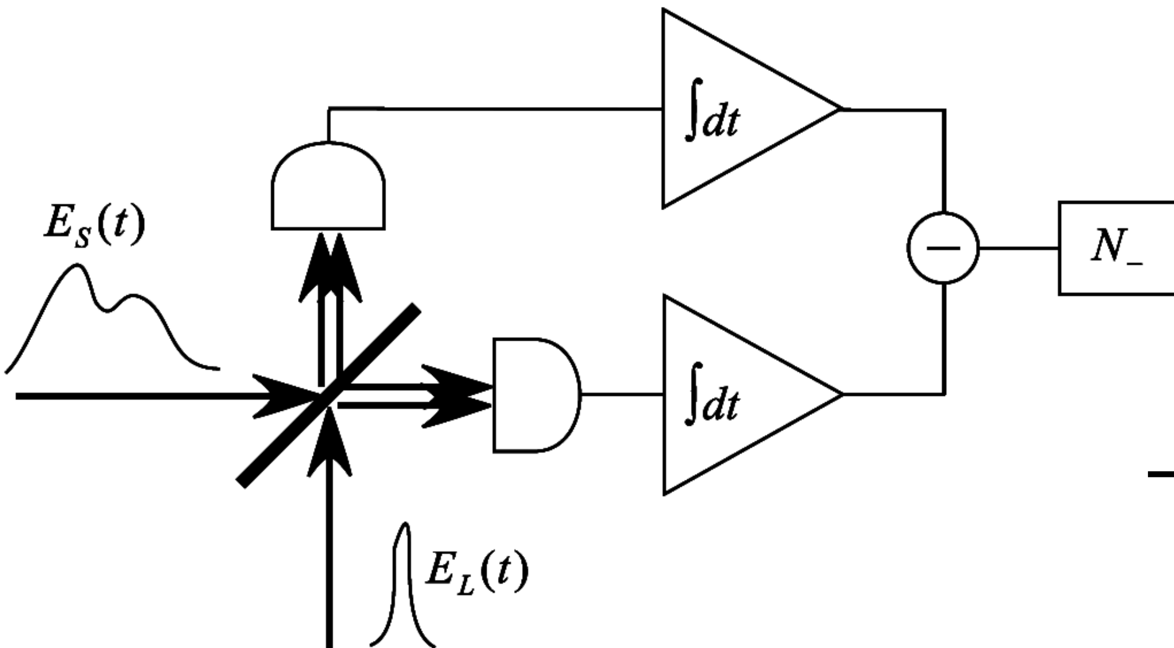

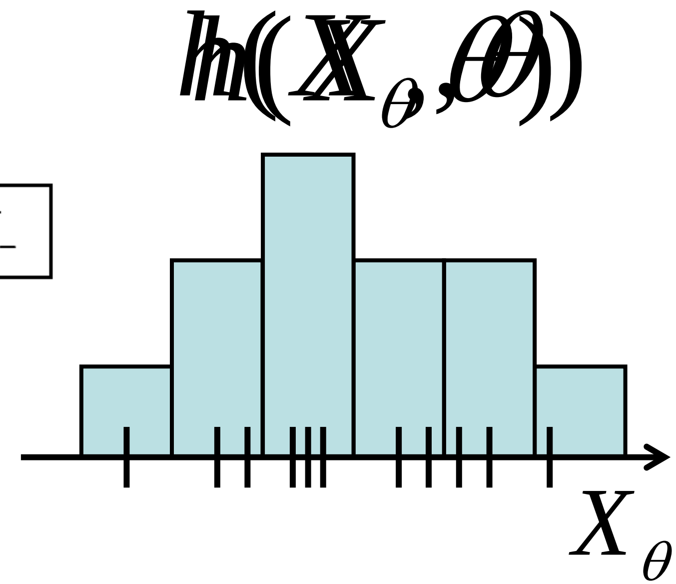

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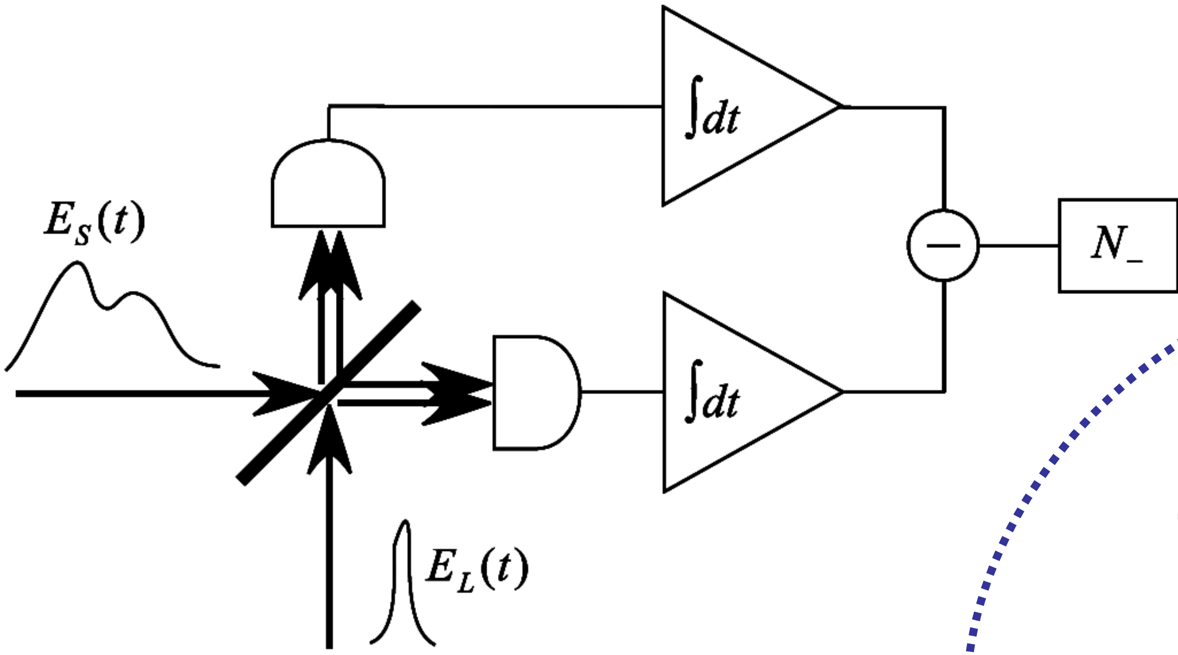
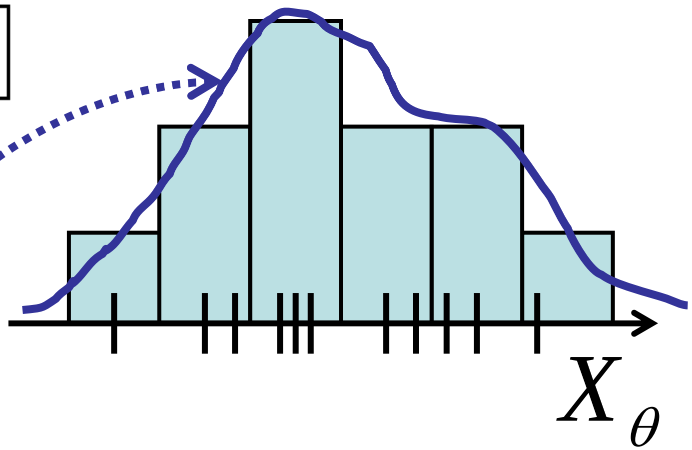




$$w(X,\theta) = \langle X_\theta|\hat{\rho}|X_\theta\rangle$$

$$\hat{X}_\theta|X_\theta\rangle = X|X_\theta\rangle$$

$$\hat{X}_\theta = \hat{Q}\cos\theta + \hat{P}\sin\theta$$

$$[\hat{Q},\hat{P}] = i$$

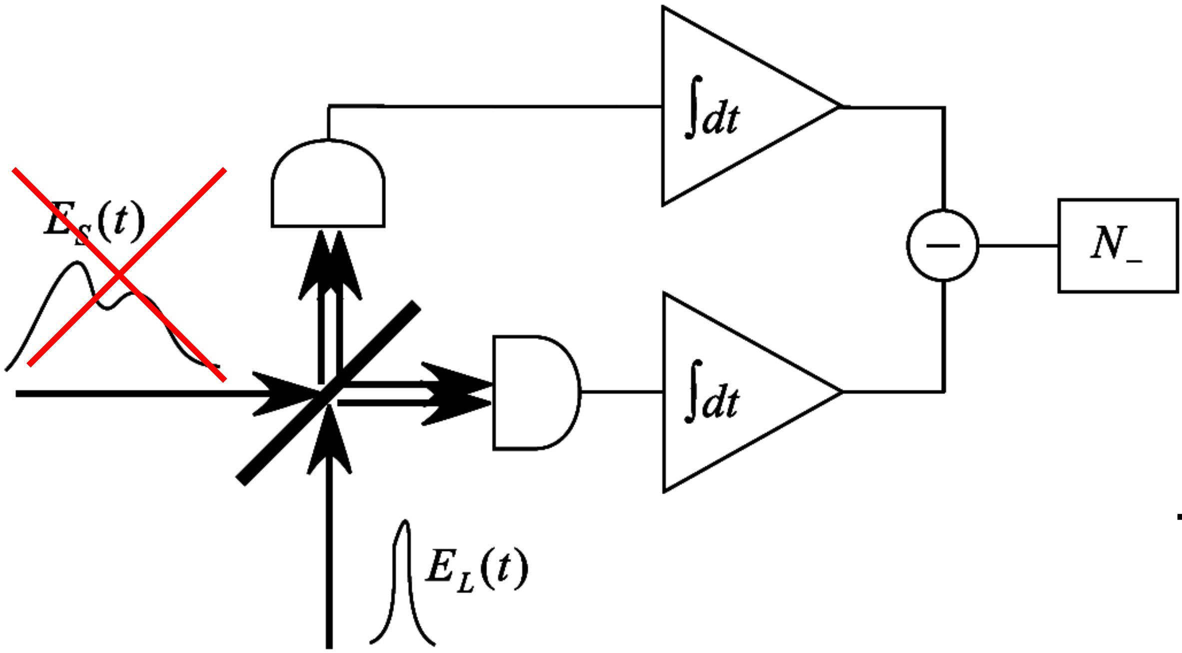
pure: $\hat{\rho} = |\psi\rangle\langle\psi|$

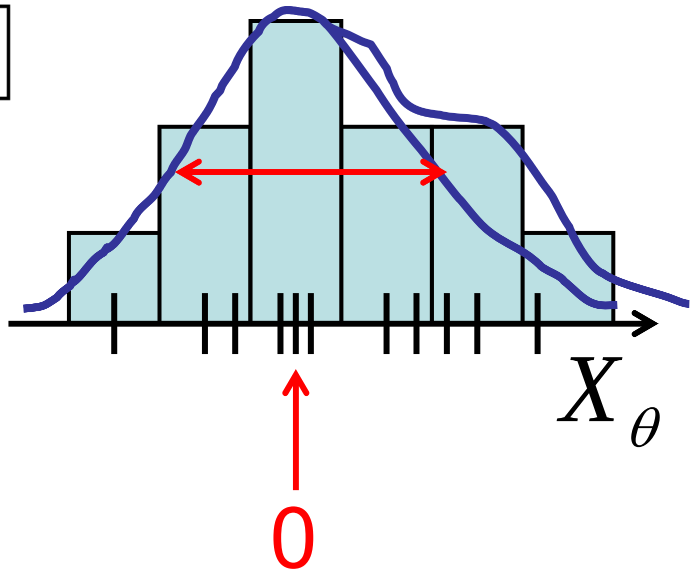
$$w(X,\theta) = |\langle X_\theta|\psi\rangle|^2$$

$$w(X,0) = |\psi(Q)|^2$$

$$w(X,\tfrac{\pi}{2}) = |\tilde{\psi}(P)|^2$$

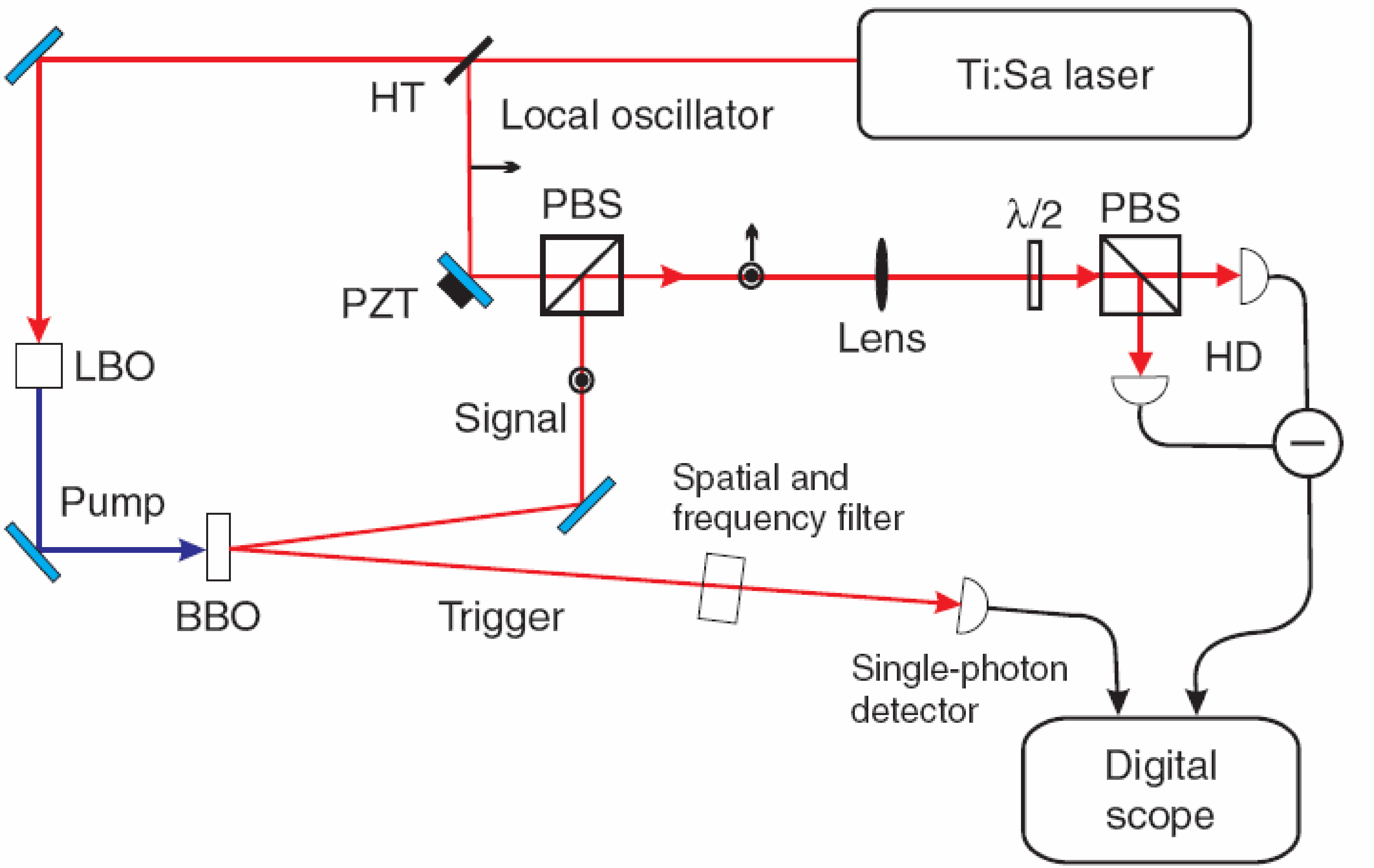
Гомодинное детектирование





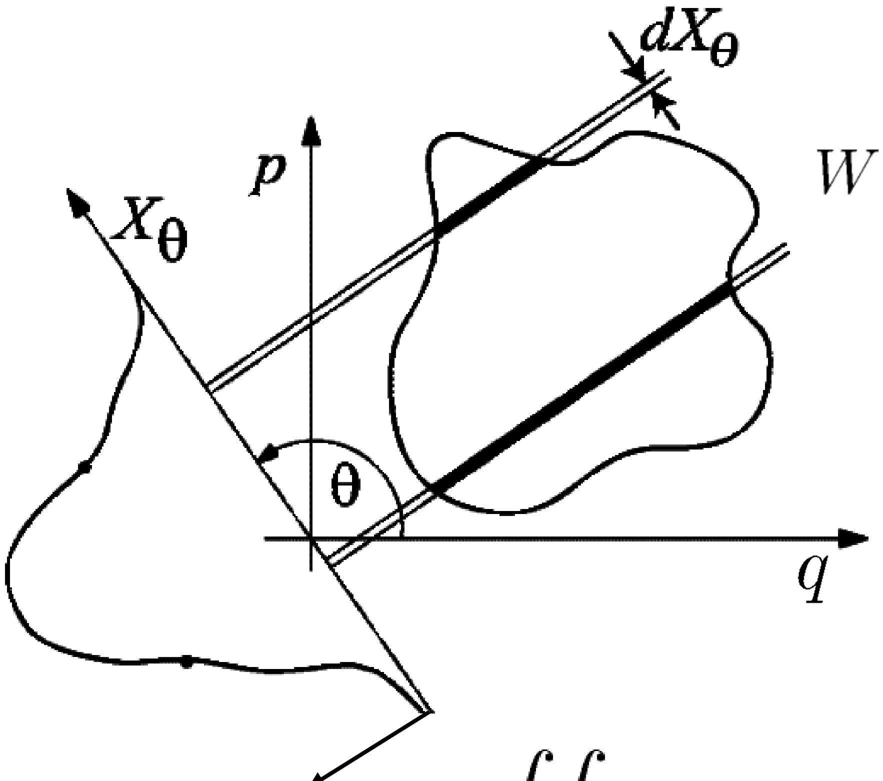
$$[\hat{Q}, \hat{P}] = i$$

$$[\hat{Q}, \hat{P}] = i\hbar$$



Ti:Sa laser
HT
Local oscillator
PBS
λ/2
PBS
PZT
Lens
HD
LBO
Signal
Spatial and
frequency filter
Pump
BBO
Trigger
Single-photon
detector
Digital
scope

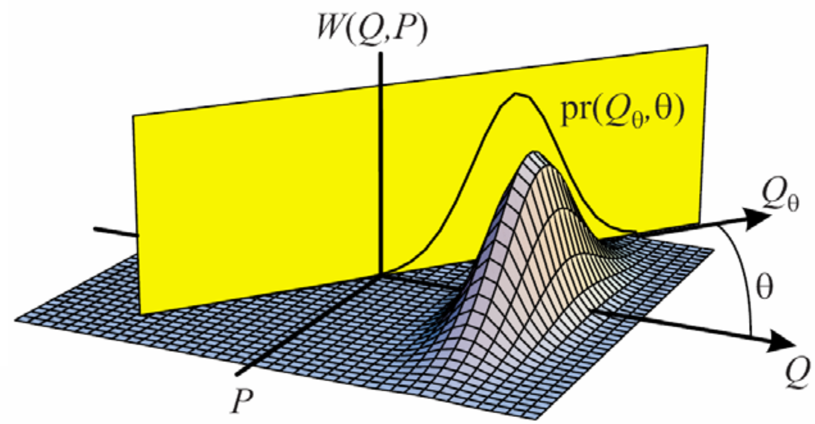
В фазовом пространстве



Функция Вигнера

$$W(q,p) = \frac{1}{2\pi}\int \rho\left(q+\frac{u}{2}, q-\frac{u}{2}\right) e^{-ipu} du$$

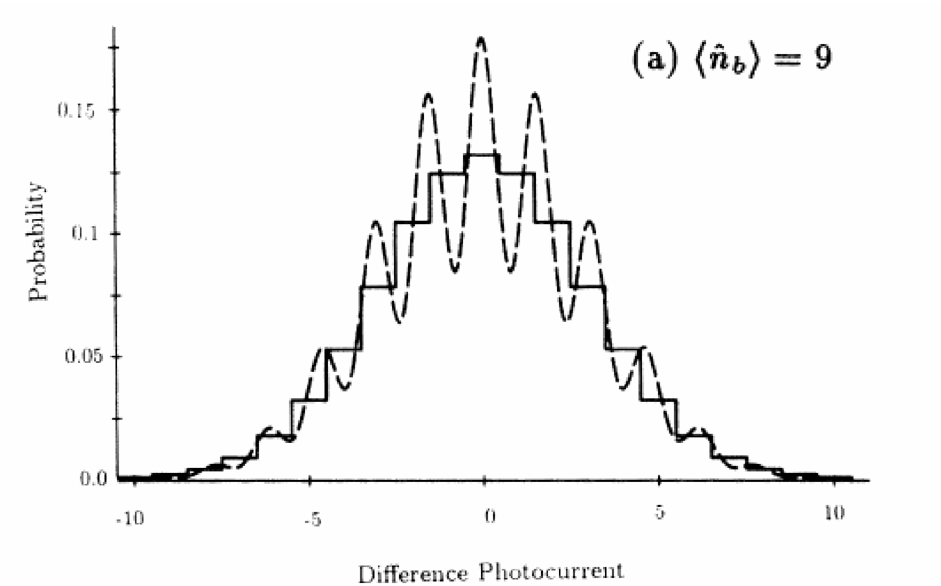
$$\rho(x,x') = \langle x|\hat{\rho}|x'\rangle$$

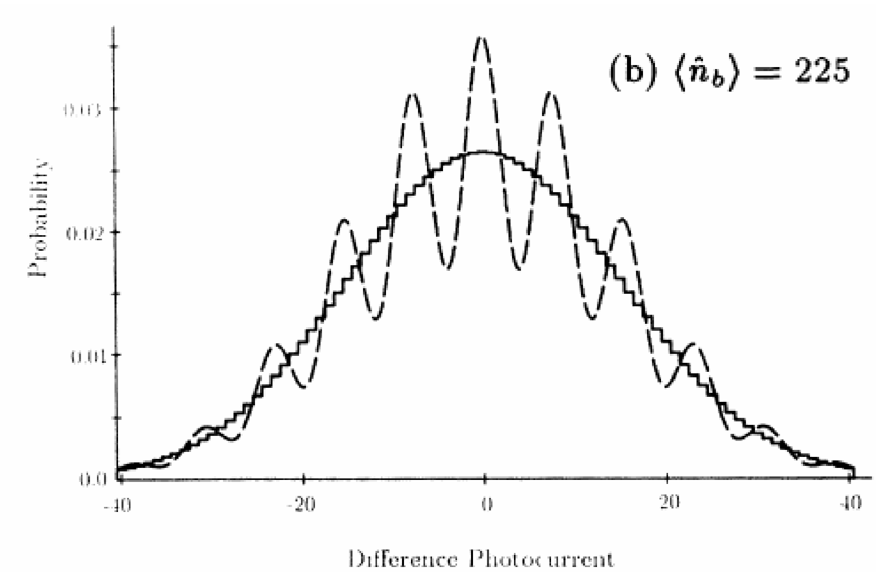


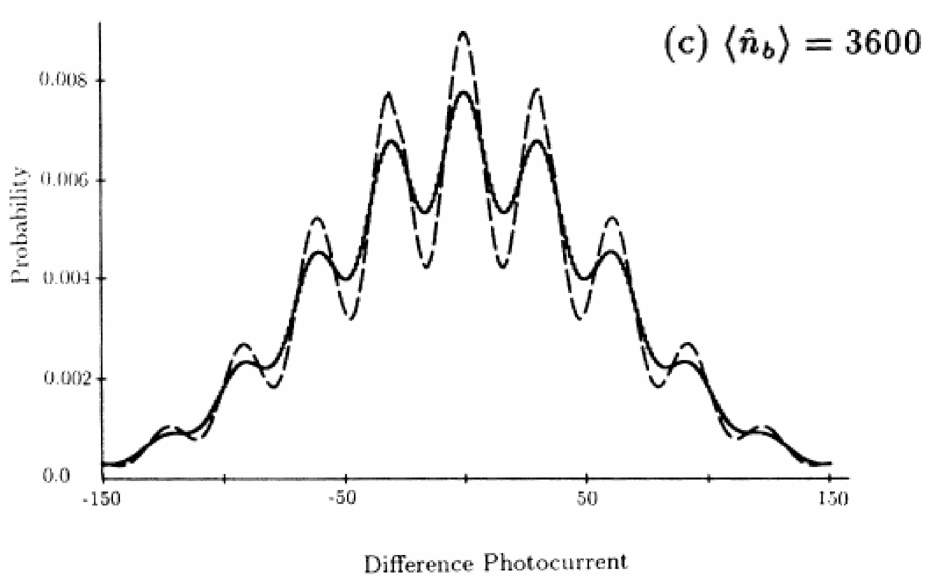
$$w(X,\theta) = \iint dq\ dp\, W(q,p)\ \delta\left(X - q\cos\theta - p\sin\theta\right)$$

$$w(X,\theta) = \mathrm{Tr}\left[\hat{\rho}\ \delta\left(X - \hat{Q}\cos\theta - \hat{P}\sin\theta\right)\right]$$

О параметрах







$$\mathcal{N}(\cos\theta|0\rangle + \sin\theta|\alpha\rangle)$$

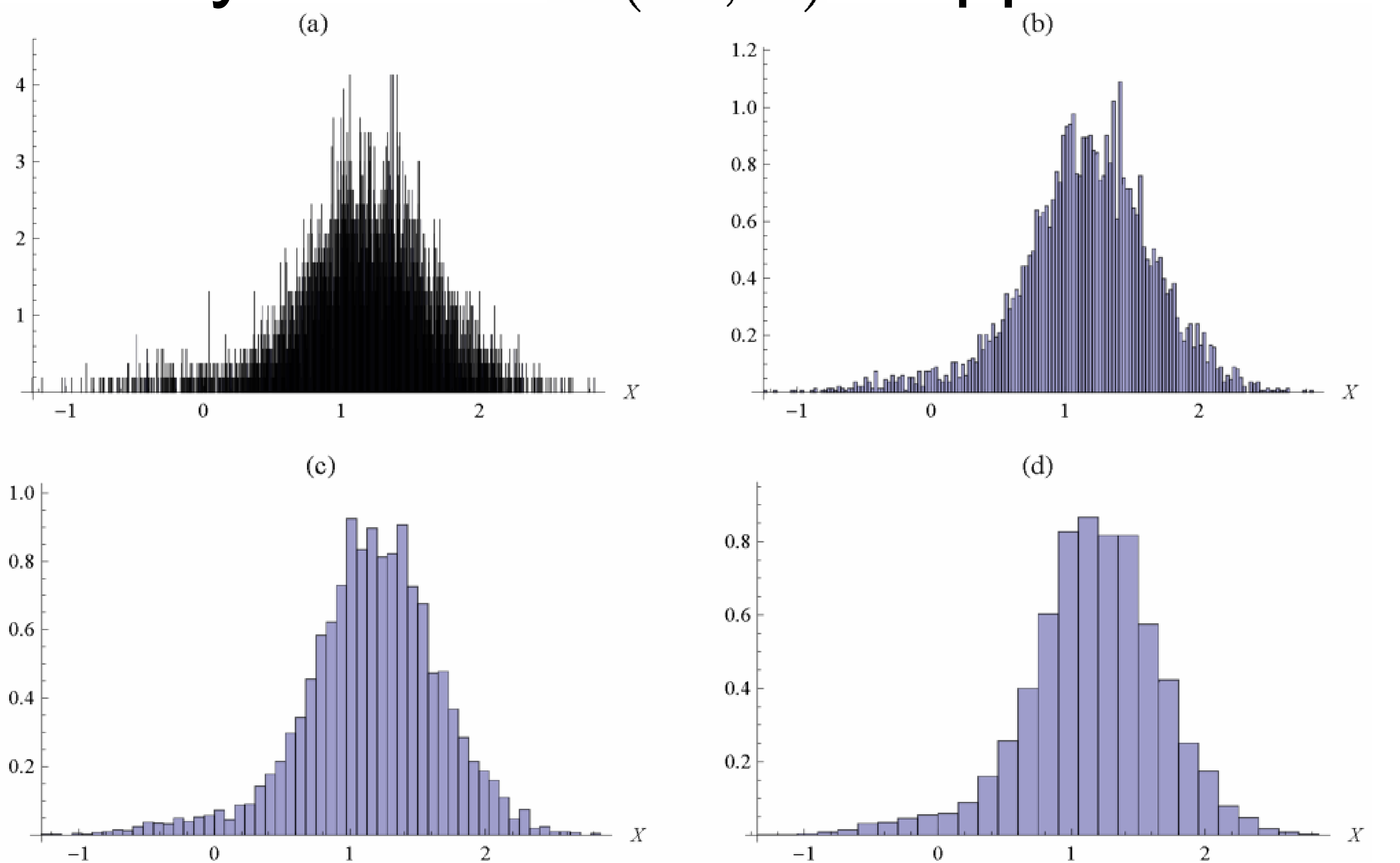
$$\theta = 10^\circ$$

$$\langle \hat{n}_a \rangle \approx 4.2$$

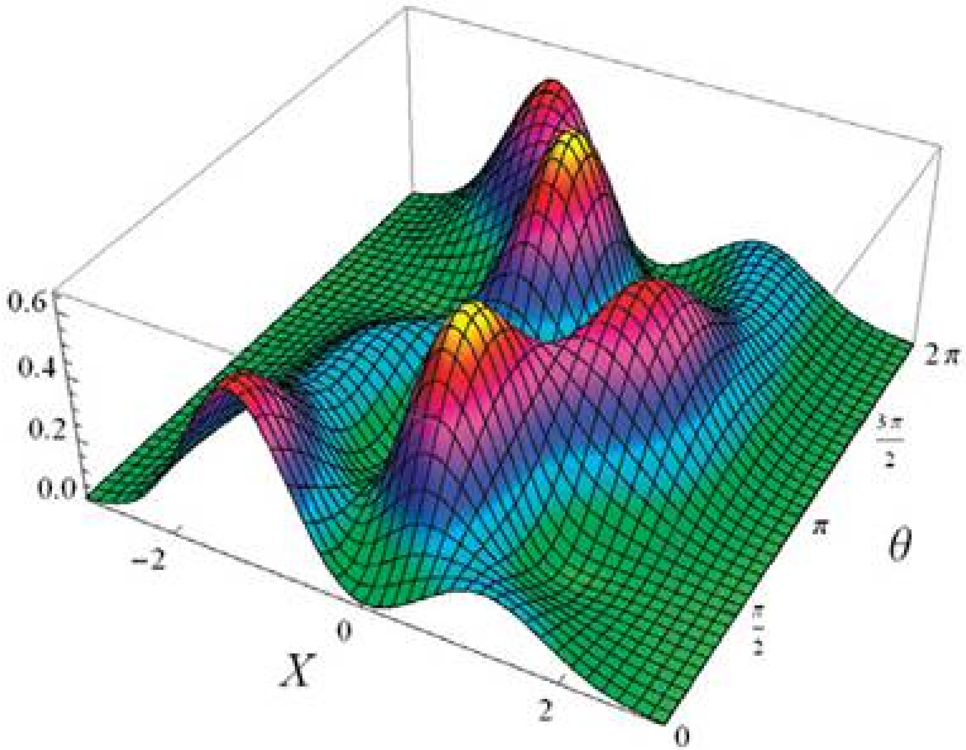
$$\langle \hat{n}_b \rangle \gg \frac{|\alpha|^4}{8} \approx \frac{\langle \hat{n}_a \rangle^2}{8\sin^4\theta}$$

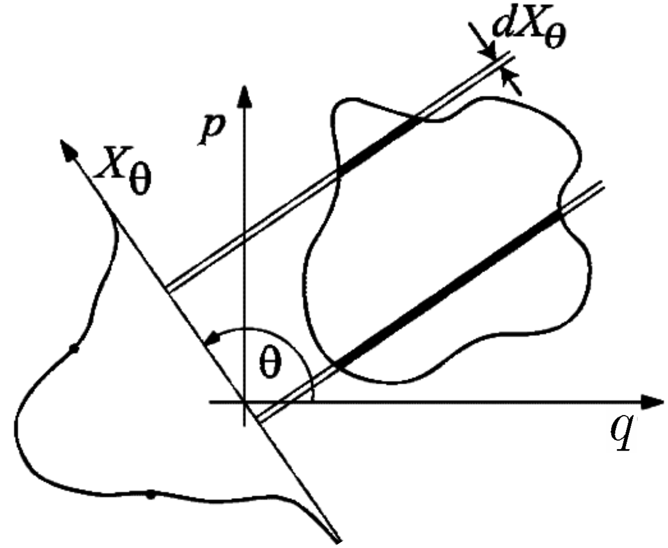
S.L. Braunstein PRA 42 474 (1989)

Экспериментальные данные: как получичить $w(X,\theta)$ корректно?



Точность



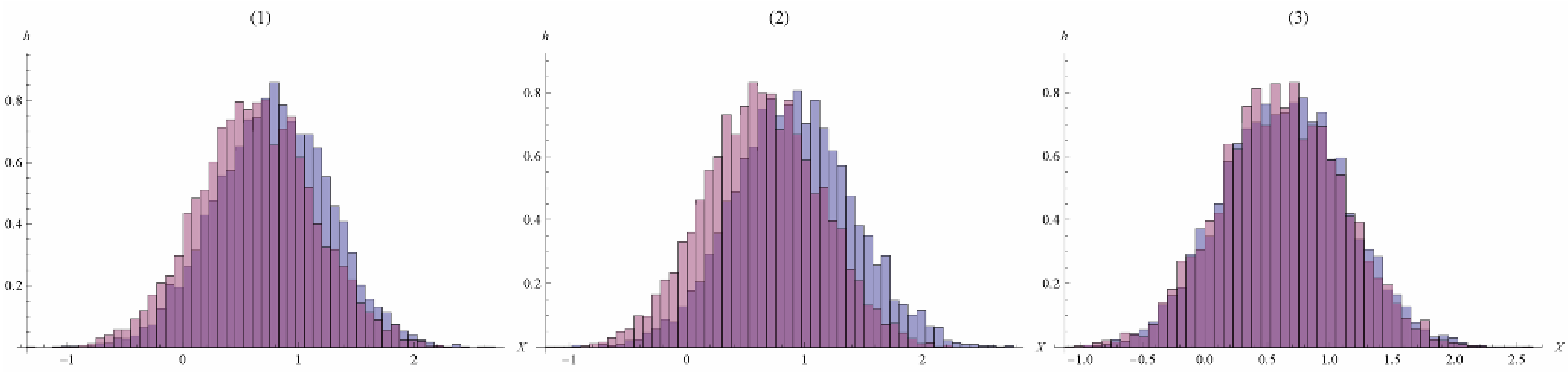


$$w(X,\theta) = w(-X,\theta+\pi)$$

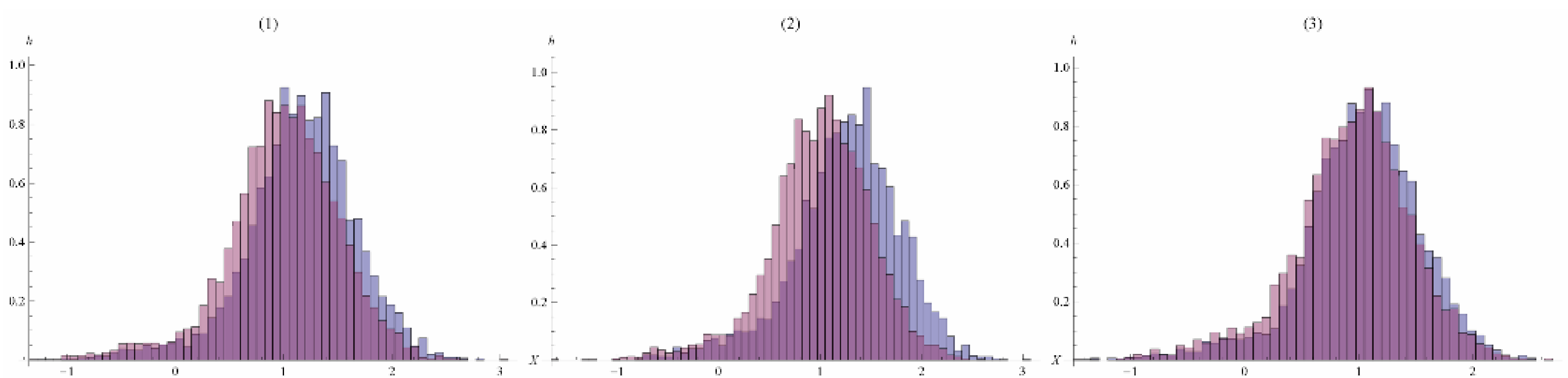
Physica Scripta 83, 058101 (2011)

Экспериментальные данные: проверка точности

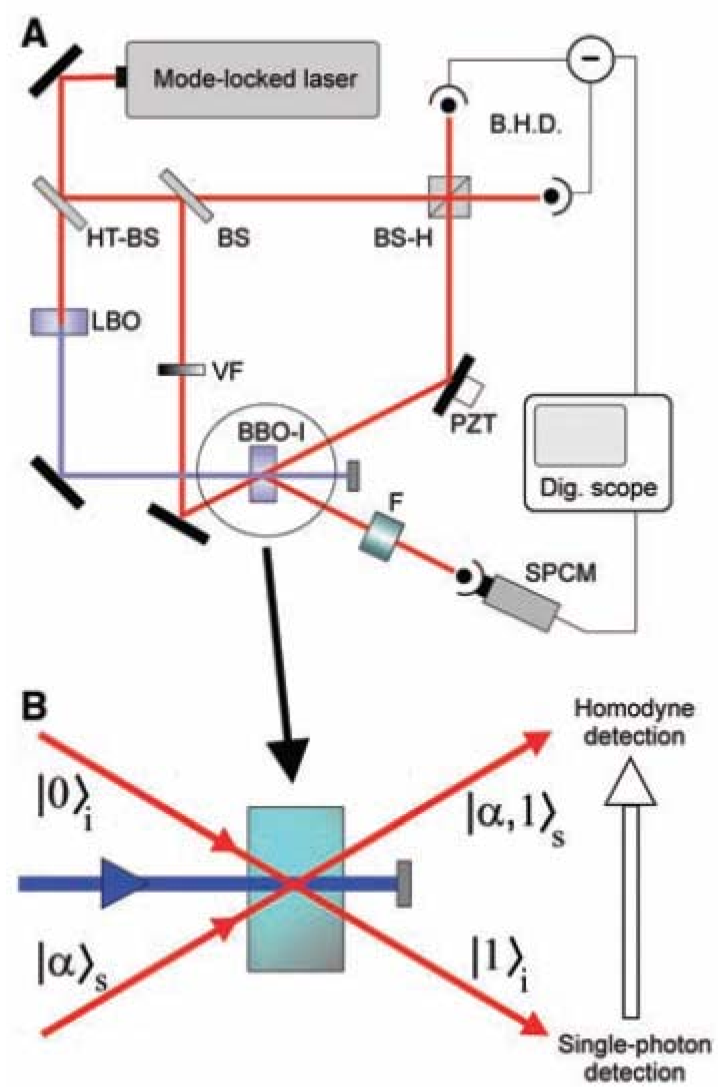
- Когерентное состояние



- Когерентное состояние с добавленным фотоном

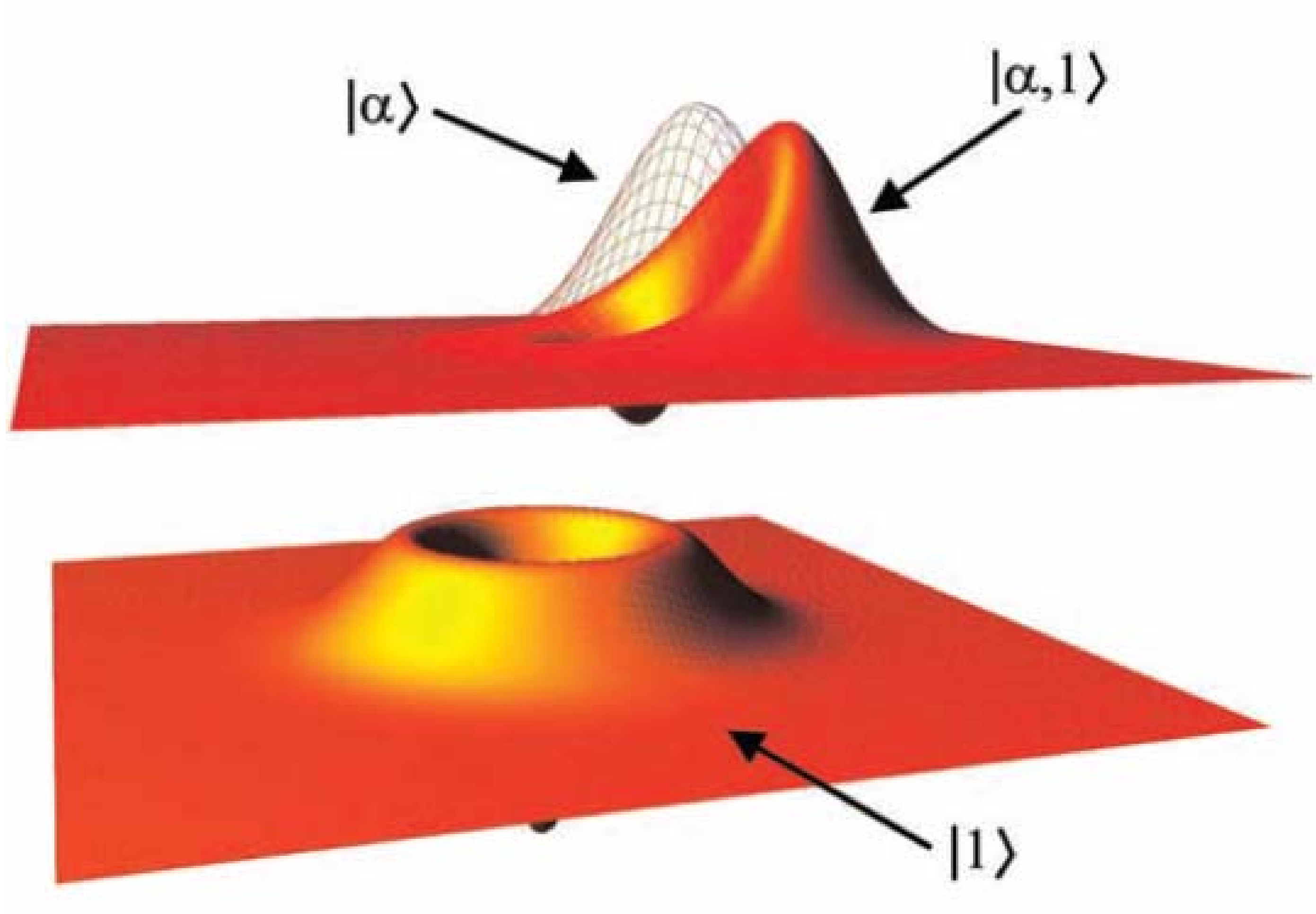


Добавление фотона



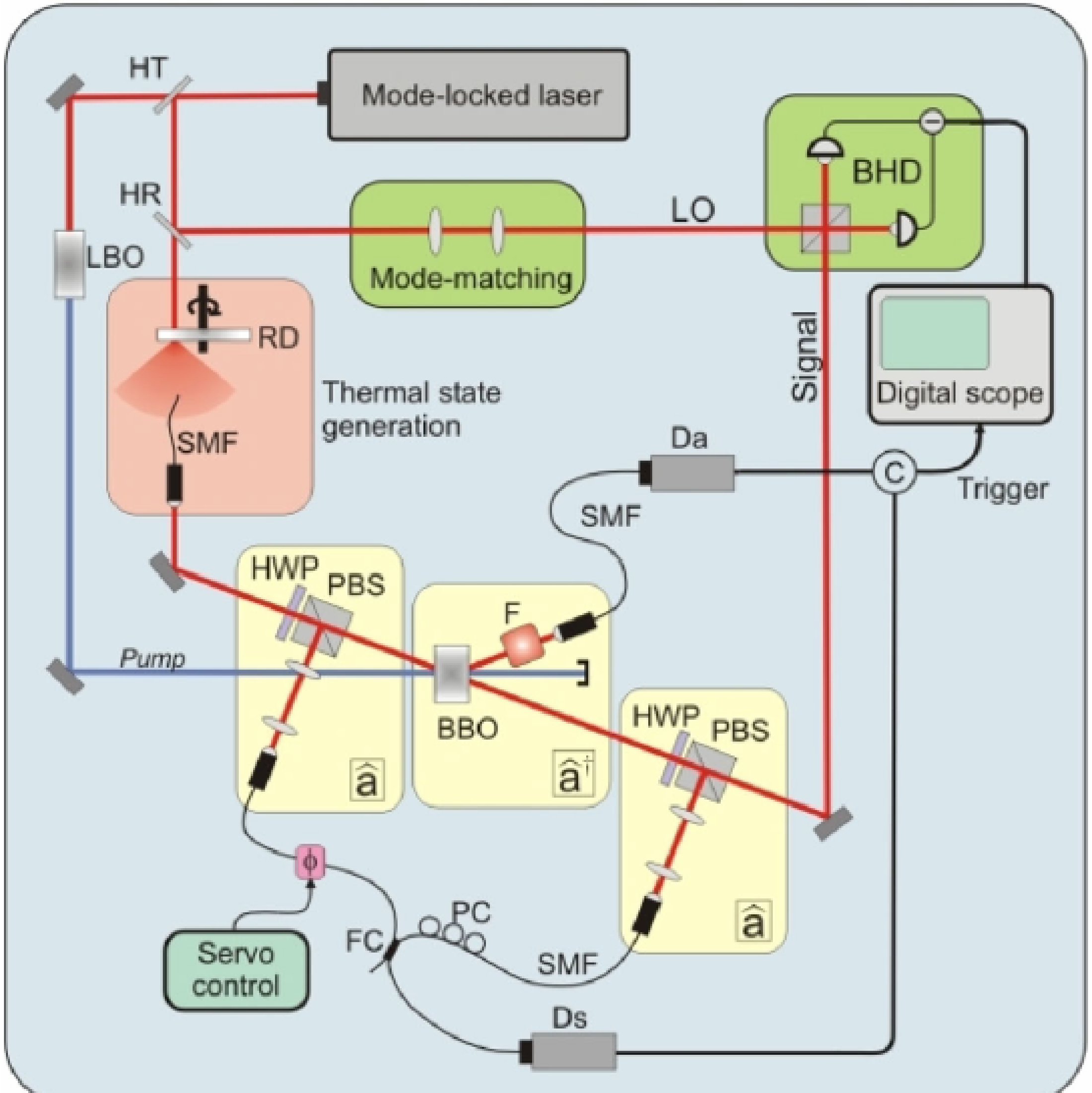
A. Zavatta et al. Science 306 660 (2004)

Добавление фотона



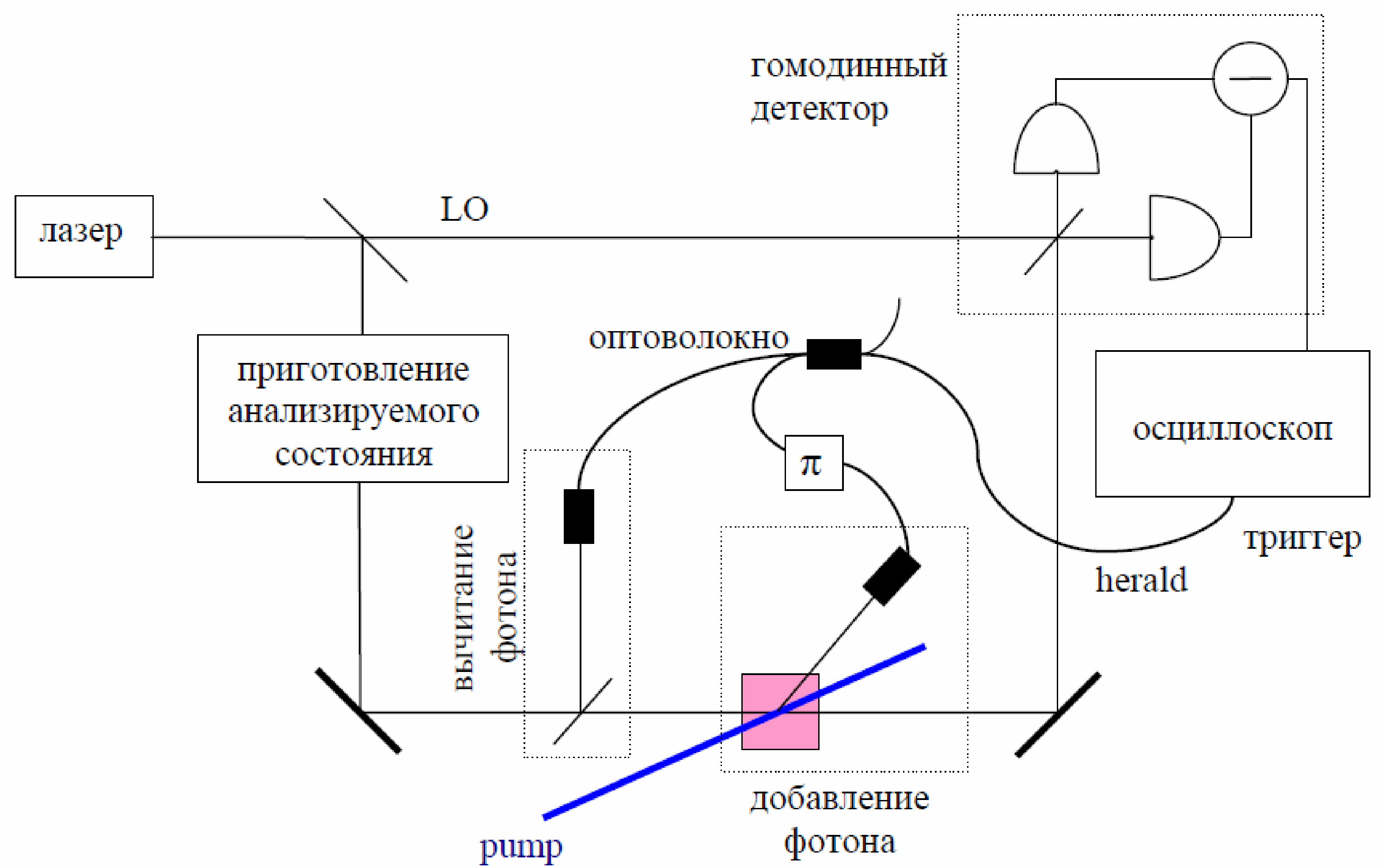
A. Zavatta et al. Science 306 660 (2004)

$$[\hat{a}, \hat{a}^\dagger] = \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \mathbb{1}$$



A. Zavatta et al. PRL 103, 140406 (2009)

Дифференциатор волновой функции



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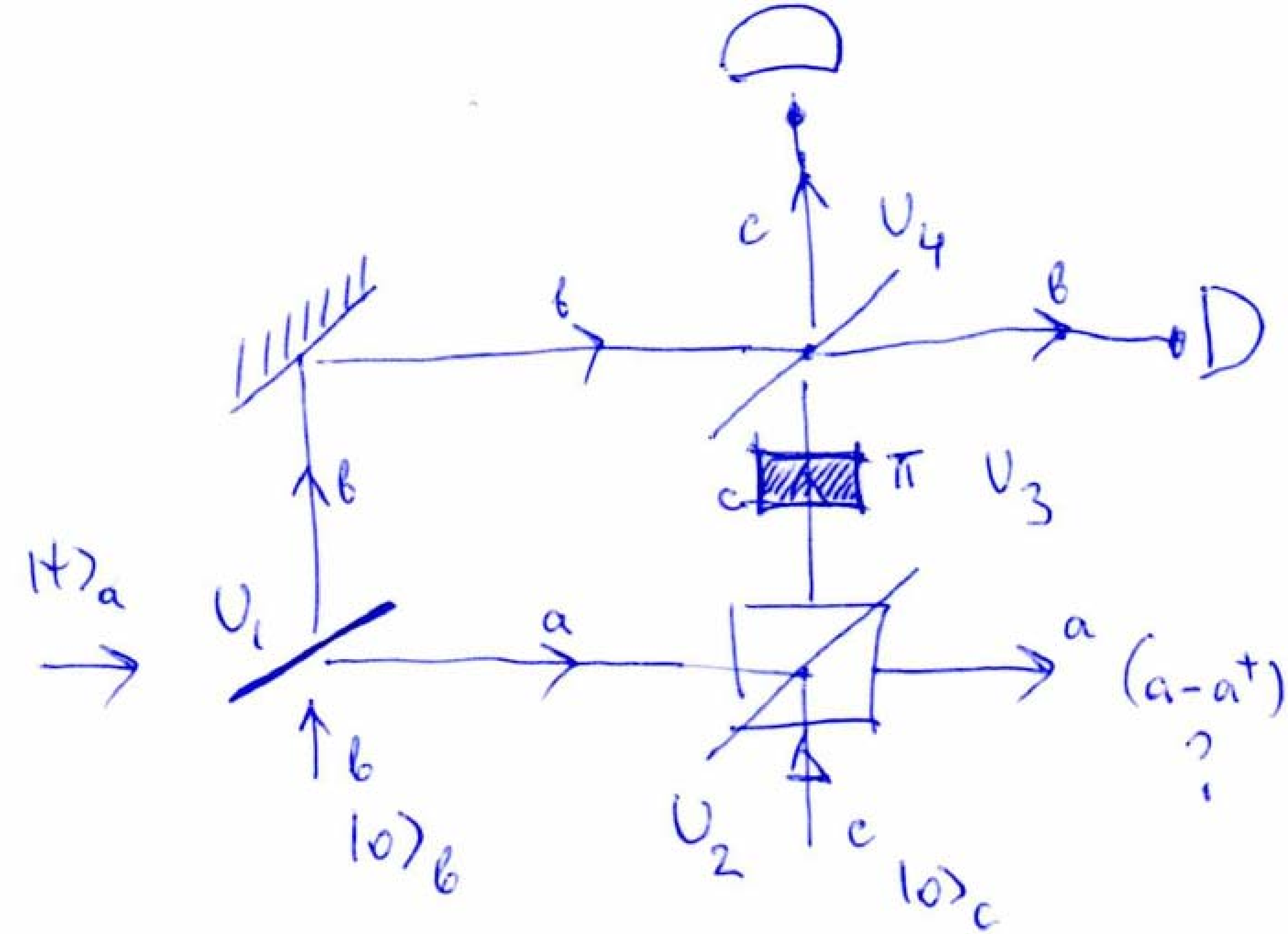

Дифференциатор волновой функции

$$U = e^{\frac{\pi}{4}\left(a_2 a_3^+ - a_2^+ a_3\right)} e^{i\pi a_3^+ a_3} e^{\theta_2\left(\alpha a_1^+ a_3^+ - \alpha^* a_1 a_3\right)} e^{\theta_1\left(a_1 a_2^+ - a_1^+ a_2\right)}$$

$$\theta_1 \approx \theta_2 \ll 1$$

$$\rho_1 \sim \frac{1}{2}\left(\theta_1 a_1 + \theta_2 \alpha a_1^+\right) \rho_1 \left(\theta_1 a_1^+ + \theta_2 \alpha^* a_1\right)$$

$$\theta_2 \alpha / \theta_1 = -1$$

$$\rho_1 \sim (a - a^+)\rho(a - a^+)$$

$$w_1 = \mathrm{tr}(\rho E) = \langle x | \psi \rangle \langle \psi | x \rangle = |\psi(x)|^2$$

$$w_2 = \mathrm{tr}(P \rho P\, E) = \langle x | P | \psi \rangle \langle \psi | P | x \rangle = \left| (-i \tfrac{\partial}{\partial x}) \psi(x) \right|^2$$

$$\psi(x) = f(x) e^{ig(x)}$$

$$f(x) = \sqrt{w_1(x)}\,,$$

$$g(x) = \int \sqrt{\frac{w_2(x)}{w_1(x)} - \left(\frac{w_1\,'(x)}{2 w_1(x)} \right)^2}\; dx\,.$$

Заключение

- Когерентная суперпозиция операций добавления в удаления фотонов позволяет выполнить операцию дифференцирования волновой функции, что может быть использовано для восстановления чистых квантовых состояний.