Generic mixing actions

Sergey Tikhonov

Plekhanov Russian University of Economics, Moscow, Russia

tikhonovc@mail.ru

In this talk a transformation means an invertible measure preserving transformation of a nonatomic finite Lebesgue space. A group of transformations $\{T^g\}_{g\in G}$ is called a G-action if it is isomorphic to group G.

A G-action T is called Γ-*mixing* for a given set $\Gamma \subset G$ if for any $\varepsilon > 0$ and measurable sets A, B there exists a bounded subset $C \subset G$ such that

$$|\mu(T^g A \cap B) - \mu(A)\mu(B)| < \varepsilon,$$

for every $g \in \Gamma \setminus C$. If $\Gamma = G$ then we say that T is *mixing*.

Theorem 1. *Let G be a local compact Hausdorff group with a countable neighbourhood basis and Γ an unbounded subset of G. Then the set $\mathcal{M}_{G,\Gamma}$ of all Γ-mixing G-actions is a separable metric space for certain appropriate metric m.*

We call m the *leash-metric*. It was introduced independently by S. Alpern for mixing actions [1] and by the author for all cases [2, 3].

We have several directions for using the metric.

Generic properties. A subset of a metric space is called massive if it is a countable intersection of dense open sets.

Theorem 2. *(Bashtanov* [4, 5]*) The set of rank one $\mathbb{Z}^d$-actions is massive.*

Hence generic mixing $\mathbb{Z}^d$-actions have a zero entropy and a trivial centralizer.

k-fold mixing. An action T is called *k-fold mixing* if for any $\varepsilon > 0$ and measurable sets $A_1, \ldots, A_k$ there exists a bounded set $C \subset G$ such that for any $D = \{g_1, \ldots, g_k\} \subset G$, $D^{-1}D \cap C = \varnothing$ we have

$$\left|\mu\left(\bigcap_i T^{g_i} A_i\right) - \prod_i \mu(A_i)\right| < \varepsilon.$$

Rokhlin problem. For a given group, is arbitrary 2-fold mixing action necessarily any-fold mixing?

Alpern's conjecture. Generic mixing transformation is not 3-fold mixing.

It is proven to be false by next theorem.

Theorem 3. *Let G be a direct product of an amenable monotilable group and $\mathbb{Z}$. Then generic mixing G-action is any-fold mixing.*

Spectral properties of mixing transformations. A spectral multiplicity of a transformation T is the spectral multiplicity of operator U, defined by the formula $Uf(x) = f(Tx)$.

Theorem 4. *For any finite collection $\{n_i\}$ of natural numbers there exists a mixing transformation T with*

$$M(T) = \bigcup_{I \neq \varnothing} \left\{ \prod_{i \in I} n_i \right\}.$$

In particular, there exist mixing transformations with homogeneous spectrum of any multiplicity.

This is an analog of results [6, 7] for mixing transformations.

References

1. *Alpern S.* Conjecture: In general a mixing transformation is not two-fold mixing // Ann. Probab. 1985. V. 13, N 1. P. 310–313.
2. *Tikhonov S. V.* Complete metric on mixing actions of general groups // JDCS. 2013. V. 19, N 1. P. 17–31.
3. *Tikhonov S. V.* A complete metric in the set of mixing transformations // Sb. Math. 2007. V. 198, N 4. P. 575–596.
4. *Bashtanov A.I.* Conjugacy classes are dense in the space of mixing Zd-actions // Math. Notes. 2016. V. 99, N 1–2. P. 9–23.
5. *Bashtanov A.I.* Generic mixing transformations are rank 1 // Math. Notes. 2013. V. 93, N 2. P. 209–216.
6. *Ageev O.* The homogeneous spectrum problem in ergodic theory // Invent. Math. 2005. V. 160, N 1. P. 417–446.
7. *Ryzhikov V. V.* Spectral multiplicity for powers of weakly mixing automorphisms // Sb. Math. 20012. V. 200, N 7. P. 1065–1076.