Discrete complex analysis: convergence results*

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Various discretizations of complex analysis have been actively studied since 1920s because of applications to numerical analysis, statistical physics, and integrable systems. This talk concerns complex analysis on quadrilateral lattices tracing back to the works of J. Ferrand, R. Isaacs, R. Duffin.

We solve a problem of S.K. Smirnov from 2010 on the convergence of discrete harmonic functions on planar nonrhombic lattices to their continuous counterparts under lattice refinement. This generalizes the results of R. Courant–K. Friedrichs–H. Lewy, L. Lusternik, D.S. Chelkak–S.K. Smirnov, P.G. Ciarlet–P.-A. Raviart.

We also prove convergence of discrete period matrices and discrete Abelian integrals to their continuous counterparts (this is a joint work with A.I. Bobenko). The proofs are based on energy estimates inspired by electrical network theory.

References

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