Emergence and Para-Dynamics

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Recently we showed that some degenerate bifurcations can occur robustly. Such a phenomenon enables ones to prove that some pathological dynamics are not negligible and even typical in the sense of Arnold–Kolmogorov. More precisely, we proved:

Theorem 1. *For every $\infty > r \geq 1$, for every $k \geq 0$, for every manifold of dimension ≥ 2, there exists an open set $\hat{U}$ of C^r-k-parameter families of self-mappings, so that for every topologically generic family $(f_a)_a \in \hat{U}$, for every $\|a\| \leq 1$, the mapping f_a displays infinitely many sinks.*

We will introduce the concept of Emergence which quantifies how wild the dynamics is from the statistical viewpoint, and we will conjecture the local typicality of super-polynomial ones in the space of differentiable dynamical systems.

To this end, we will develop the theory of Para-Dynamics, by giving the following negative answer to a problem of Arnold (1989):

Theorem 2. *For every $\infty > r \geq 1$, for every $k \geq 0$, for every manifold of dimension ≥ 2, there exists an open set $\hat{U}$ of C^r-k-parameter families of self-mappings, so that for every topologically generic family $(f_a)_a \in \hat{U}$, for every $\|a\| \leq 1$, the map f_a displays a fast increasing number of periodic points:*

$$\limsup \frac{\log Card\ Per_n\, f_a}{n} = \infty.$$

In order to prove this theorem, we will show an extension of theorems by Gonchenko–Shilnikov–Turaev, Kaloshin and Turaev, to give the following answer to questions asked by Smale in 1967, Bowen in 1978 and by Arnold in 1989, for manifolds of any dimension ≥ 2:

Theorem 3. *For every $\infty \geq r \geq 1$, for every manifold of dimension ≥ 2, there exists an open set U of C^r-diffeomorphisms, so that a generic $f \in U$ displays a fast growth of the number of periodic points.*

The proof involves a new object, the λ-C^r-parablender, the Renormalization for hetero-dimensional cycles, the Hirsh–Pugh–Shub theory, the parabolic renormalization for parameter family, and the KAM theory.