What is the area of a finitely presented group and what does this area depend on

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The construction of a riemann metric on a manifold is easily generalized to any finite simplicial polyhedron, the notion of volume (area) and of geodesic (as a locally shortest path) conserve their meaning. This allows to use the methods of differential geometry to study, for example, abstract groups.

In the beginning of the 80s, Gromov, using the method of the filling, demonstrated that if one normalizes the metric on a two dimensional polyhedron with the condition that any closed non contractible geodesic cannot have length less than 1, then there exists a fundamental constant c such that the area of any polyhedron with a non free fundamental group is not less than c. Relatively recently the estimation of c was improved by some authors, using "elementary methods", however the hypothesis about the exact meaning of c remains completely open.

This observation of Gromov allows to relate any finitely presented group G with its area $\sigma(G)$ (sometimes called the systolic area), as the lower bound of normalized areas of 2 dimensional riemann polyhedra that have the given fundamental group G. It appears that the area of the group G is 0 if and only if G is free.

For any positive number T one can naturally define the set $G(T)$ of two by two non isomorphic groups of area not larger than T. What is the structure and the cardinality of the set $G(T)$? Although this questions were raised by Gromov almost 25 years ago, partial answers were obtained only 8 years ago.

Recently it was discovered that the area $\sigma(G)$ of the group is closely related to the purely combinatorial invariant of this group, which is equal to the minimal number of 2-simplexes required to the triangulation of some 2-complex with given fundamental group G. This invariant, called the simplicial complexity of the group, approximates well the area of the group, when this area is large. The simplicial complexity is of interest from a purely combinatorial point of view and is related to the more "economical" corepresentation of the group. The study of this invariant allowed to move forward considerably in the study of the problems formulated above.

The talk will be dedicated to the range of questions described above.

References

1. *Babenko I., Balacheff F., Bulteau G.* Systolic geometry and simplicial complexity for groups: E-print. arXiv: 1152259.