Omega-limit sets of generic points of partially hyperbolic diffeomorphisms

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We prove that for any $E^u \oplus E^{cs}$-partially hyperbolic C^2-diffeomorphism the ω-limit set of a generic (with respect to the Lebesgue measure) point is saturated by the unstable leaves. As a corollary we prove a conjecture from a paper by Ilyashenko (2011) that the Milnor attractor is saturated by the unstable leaves. This property was used by Ilyashenko to prove that there exists a locally generic set of boundary preserving diffeomorphisms of $[0,1] \times \mathbb{T}^2$ with "thick" Milnor attractors.

Definition 1. For a diffeomorphism F of a Riemannian manifold X the Milnor attractor (notation: $A_M(F)$ or A_M) is the smallest invariant closed set that contains the ω-limit sets of almost all points with respect to the Lebesgue measure on X.

Consider is a Riemannian manifold M, possibly with a boundary. The metric induces the Lebesgue measure on M.

Definition 2. A diffeomorphism $F\colon M \to M$ is called $E^u \oplus E^{cs}$-*partially hyperbolic* if there exist $\lambda > 1, \mu < \lambda$, $c > 0$ and two invariant distributions $E^{cs}_x \subset T_xM$ and $E^u_x \subset T_xM$, (i.e., $dF_x(E^{cs,u}_x) = E^{cs,u}_{F(x)}$) and

$$T_xM = E^{cs}_x \oplus E^u_x,$$

$$\|dF^n_x|_{E^{cs}_x}\| \leq c\mu^n, \quad \|dF^{-n}|_{E^u_x}\| \leq c\lambda^{-n}.$$

Theorem. *Let $F\colon M \to M$ be a $E^u \oplus E^{cs}$ partially hyperbolic C^2-diffeomorphism. Then the ω-limit set of almost any point with respect to the Lebesgue measure is saturated by the unstable leaves (i.e. it either contains an entire leaf or does not intersect it).*

References

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