Closures of group actions and Poisson measures

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Consider the group G of all transformations of a Lebesgue measure (M, μ) space (M, μ) leaving the measure μ quasiinvariant. Denote by $\mathbb{R}^+$ the half-line $t > 0$. We say that a *polymorphism* (see [3]) of M is a measure $\varkappa$ on $M \times M \times \mathbb{R}^+$ satisfying two conditions:

1) The pushforward of $\varkappa$ to the first factor M is μ.

2) The pushforward of $t \cdot \varkappa$ to the second factor M is μ.

For any $g \in G(M)$ we define a polymorphism as a pushforward of the measure μ under the map $m \mapsto (m, g(m), g'(m))$. Denote by $\mathrm{Pol}(M)$ the set of all polymorphisms. It has a structure of a semigroup with a topology, and the group $G(M)$ is dense in $\mathrm{Pol}(M)$ (so $\mathrm{Pol}(M)$ is a 'natural completion' of $G(M)$).

Therefore, for any action of a group Γ on M there arise a problem about closure of Γ in $\mathrm{Pol}(M)$. This question is interesting at least for infinite-dimensional ('large') groups Γ, several examples were examined in [1–5].

The talk contains a solution of this question for the space (Ω, ω) of Poisson configurations on a space N with non-atomic σ-finite measure and the group Γ of measurable transformations leaving ω quasiinvariant.

References

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