Anti-integrable limit and non-existence of first integral in many-dimensional systems

Sergei A. Dovbysh

Moscow State University, Moscow, Russia

sdovbysh@yandex.ru

Earlier, the author has obtained conditions which imply the non-integrability of many-dimensional systems in the strongest analytic case, i.e., the absence of a non-constant analytic and even meromorphic first integral, and a non-trivial one-parameter analytic symmetry group [1–4]. These conditions are related to transversal intersections of many-dimensional invariant manifolds (separatrices) and utilize coordinates reducing dynamics over separatrices to normal forms. So, the conditions are constructively verifiable for concrete systems but their checking could involve some difficulties.

We discuss situation of anti-integrable limit where these conditions are easily verifiable. It is practically evident that quasirandom dynamics of systems near the anti-integrable limit admits a description in terms of Smale horseshoes and corresponding homoclinic structures, though this fact seems to be not pointed out in the literature, except for some particular discussion in [5]. In the anti-integrable limit the invariant manifolds and their normalizing transformations are tending to some manifolds and mappings that admit very simple description. This easily allows to apply author's results on many-dimensional non-integrability.

Consider the simplest example, a many-dimensional generalized standard map defined as

$$y_1 = y + \varepsilon^{-1} DV(\varphi), \qquad \varphi_1 = \varphi + y_1,$$

where $(y, \varphi), (y_1, \varphi_1) \in \mathbb{R}^n \times \mathbb{T}^n$, with ε being a small parameter, and V, a potential on $\mathbb{T}^n$.

Theorem. *If C^∞-potential V possesses a non-degenerate critical point on $\mathbb{T}^n$ then the mapping is non-integrable for all small $\varepsilon \neq 0$.*

References

1. *Dovbysh S.A.* Transversal intersection of separatrices and non-existence of an analytical integral in multi-dimensional systems // Variational and Local Methods in the Study of Hamiltonian Systems: Proc. Workshop, Trieste, Oct. 24–28, 1994. World Scientific, 1995. P. 156–165.

2. *Dovbysh S.A.* Transversal intersection of separatrices, the structure of the set of quasi-stochastic motions, and the non-existence of an analytic integral in multidimensional systems // Russ. Math. Surv. 1996. V. 51, N 4. P. 730–731.
3. *Dovbysh S.A.* Transversal intersection of separatrices and branching of solutions as obstructions to the existence of an analytic integral in many-dimensional systems. I: Basic result: Separatrices of hyperbolic periodic points // Collectanea Math. 1999. V. 50, N 2. P. 119–197.
4. *Dovbysh S.A.* Intersection of separatrices and the non-integrability of multi-dimensional systems // Russ. Math. Surv. 2000. V. 55, N 3. P. 574–575.
5. *Chen Y.-Ch.* Smale horseshoe via the anti-integrability // Chaos, Solitons & Fractals. 2006. V. 28, N 2. P. 377–385.