Discrete optimization in network routing

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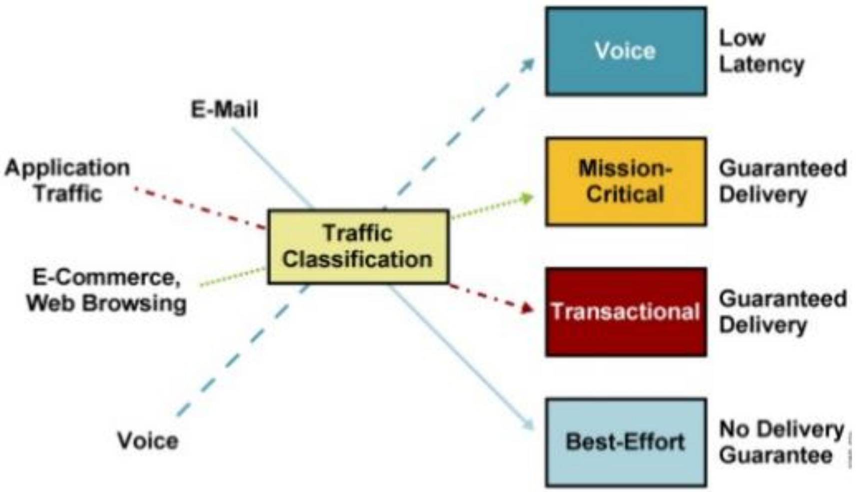
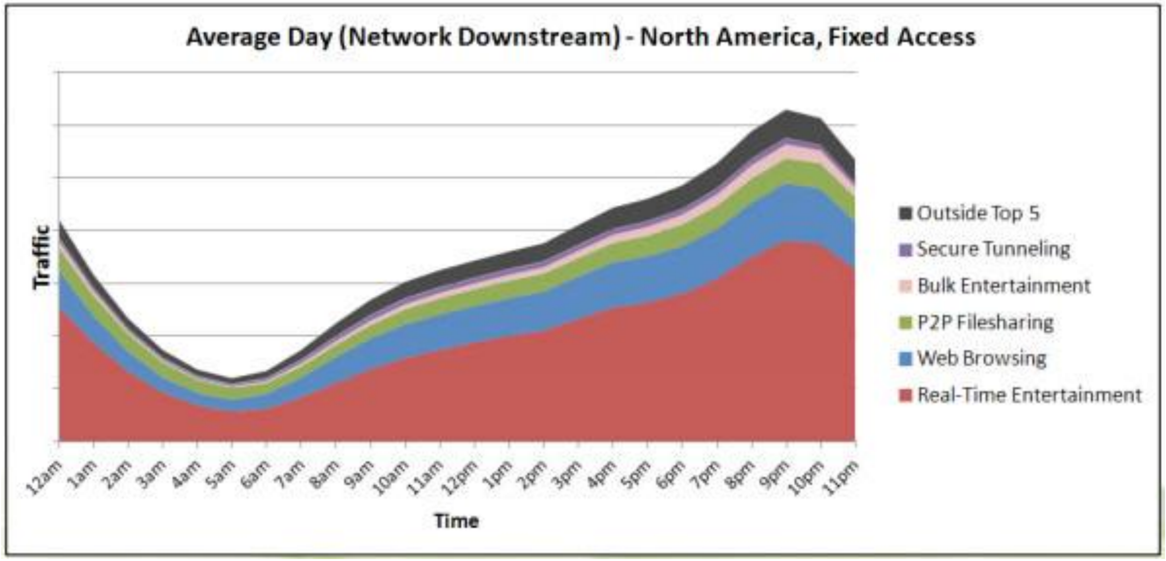
Networks Technology Challenges

New services deployment
New traffic types appear
New opportunities to measure/estimate traffic





Outline

- IGP Optimization problem statement and MILP formulation
- Multi commodity concurrent flow
- WDM setup and math model
- Basic greenfield planning problem specification

IGP network optimization task statement

Input:

- Links (source, destination, capacity, delay, metric constraint)
- Demands (source, destination, bandwidth, delay constraint)

Output:

- Links' metric

Objective:

- Minimize maximal utilization ratio (demands are sent along the shortest paths)

$$\sup_{e \in E} \frac{\sum\limits_{i=1}^{k} f_i(e)}{c(e)} \to \min$$

Constructing LP formulation (1)

- Load balance is a Multi Commodity Flow problem of MUR minimization
- Given a directed network $G=(V,E)$ with edge capacities $c(e)>0$ and commodities (services, demands) $(s_i,t_i,d_i), i=1,..K$ source, sink, and bandwidth
- For each commodity *i* and edge *e* one needs to find a flow $f_i(e)$ with constraints.

$$(1)\, capacity: \forall e \in E \sum_{i=1}^{k} f_i(e) \le c(e)$$

$$(2)\, flow conservation: \forall i, \forall u \notin \{s_i,t_i\} \sum_{e=(u,\bullet)} f_i(e) - \sum_{e=(\bullet,u)} f_i(e) = 0$$

$$(3)\, demand\ satisfaction: \forall i \sum_{e=(s_i,\bullet)} f_i(e) = \sum_{e=(\bullet,t_i)} f_i(e) = d_i$$

$$(4)\, connectivity: \forall i \sum_{e=(t_i,\bullet)} f_i(e) = \sum_{e=(\bullet,s_i)} f_i(e) = 0$$

- We then address optimization problem (MCF-LB):

$$MUR\ F = \sup_{e \in E} \frac{\sum_{i=1}^{k} f_i(e)}{c(e)} \to \min$$

Constructing LP formulation (2)

- MCF-LB is a linear program although MUR is a sigmoid objective

We can add variable *F* and additional constraints:

$$(5)\forall e \in E, \sum_{i=1}^{K} f_i(e) \le F$$

And address the linear program $F \to \min$

However this has no value for OSPF or IGP, because we missed the main condition:

- (SP): the routing path are the shortest paths w.r.t. some integer metric function

$$w_e \in [1, 65535], e \in E$$

- In addition we have:
- (ECMP): $\forall p,q,r \in V, \forall i, f_i(p,q) > 0, f_i(p,r) > 0 \Rightarrow f_i(p,q) = f_i(p,r)$
- Fortz and Thorup proved in 2000: (1)&..&(4)&(SP)&(ECMP) is NP-hard
- Besides that we have: Delay, Peer link equal metric, Locked link constraints, etc. etc.
- We have to look for good heuristics for this NP-hard problem

Variables for MILP

To apply mixed integer linear programming to IGP network optimization one needs more variables:

x_e^t binary, whether *e* is on a shortest path to destination *t*

f_e^t flow on link e for destination t

w_e weight on link e

d_v^t shortest path distance from source v to destination t

$f_{d_v}^t$ dummy flow variables for splitting the flow

L the maximal utilization ratio

Also we need variable M which is the sufficiently big number

MILP model

Input:

$dem(v \to t)$: demand from source v to destination t;

$cap(e)$: capacity of the link e.

Range constraints

1. $\forall t \in V, e \in E : x_e^t \in \{0,1\}$
2. $\forall t \in V, e \in E : f_e^t \in \mathbb{R}_{\geqslant 0}$
3. $\forall e \in E : w_e \in \mathbb{Z}_{\geqslant 1}$
4. $\forall v, t \in V : d_v^t \in \mathbb{Z}_{\geqslant 0}$
5. $\forall v, t \in V : f_{d_v}^t \in \mathbb{R}_{\geqslant 0}$
6. $L \in \mathbb{R}$

Expression

7. $M = \sum_{v,t \in V} dem(v \to t)$

Flow conservation constraints

8. $\forall t \in V : \sum_{e=(\cdot,t)} f_e^t - \sum_{e=(t,\cdot)} f_e^t = \sum_{v \in V} dem(v \to t)$
9. $\forall v, t \in V : \sum_{e=(\cdot,v)} f_e^t - \sum_{e=(v,\cdot)} f_e^t = -dem(v \to t)$
10. $\forall t \in V, e \in E : f_e^t \leq M \cdot x_e^t$

Shortest path constraints

11. $\forall t \in V, e = (u \to v) \in E : d_u^t \leq d_v^t + w_e$
12. $\forall t \in V, e = (u \to v) \in E : d_v^t - d_u^t + w_e - M \cdot (1 - x_e^t) \leq 0$
13. $\forall t \in V, e = (u \to v) \in E : 1 - x_e^t \leq M \cdot (d_v^t - d_u^t + w_e)$
14. $\forall u \in V : d_u^u = 0$

Equal flow splitting constraints

15. $\forall v, t \in V, \forall e = (v, \cdot) : f_e^t \leq f_{d_v}^t$
16. $\forall v, t \in V, \forall e = (v, \cdot) : f_{d_v}^t - f_e^t - M \cdot (1 - x_e^t) \leq 0$

Nuclear of the task

17. $\sum_{t \in V} f_e^t \leq L \cdot cap(e)$

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MCFP theory (1)

The maximum concurrent flow problem (MCFP) is a multocommodity flow problem in which every pair on entities can send and receive flow concurrently. The ratio of the flow supplied between a pair of entities to the predefined demand for that pair is called *throughput* and must be the same for all pairs of entities for a concurrent flow. The MCFP objective is to maximize the throughput, subject to the capacity constraints.

The following LP model can be proposed for this problem:

We are given network graph $G = G(V, E)$, edges capacity $c : E \to \mathbb{R}^+$, k demands $(s_j, t_j, d(j))$. Let $\mathbb{P}_j$ be the set of paths from s_j to t_j for $1 \leqslant j \leqslant k$. Let $\mathbb{P}$ be the union $\mathbb{P} := \bigcup\limits_{1 \leqslant j \leqslant k} \mathbb{P}_j$.

$$(P) = \begin{cases} \lambda \to \max \\ \sum\limits_{p \in \mathbb{P} : e \in p} x(p) \leqslant c(e) \forall e \in E \\ \sum\limits_{p \in \mathbb{P}_j} x(p) \geqslant \lambda d(j) \\ x, \lambda \geqslant 0 \end{cases}$$

1°. If (P) is feasible then $\inf MUR = \lambda^{-1}$ and the flow $\frac{x^*}{\lambda}$ minimizes MUR, where $x^* \in ARGOPT(P)$.

MCFP theory (2)

Let us construct the dual problem. Let $l(e)$ and $z(j)$ be its dual variables of capacity and flow size constraints correspondently.

So the dual problem is

$$(D) = \begin{cases} \sum\limits_{e \in E} c(e) l(e) \to \min \\ \sum\limits_{e \in p} l(e) \geqslant z(j) \forall 1 \leqslant j \leqslant k, \forall p \in \mathbb{P}_j \\ \sum\limits_{j=1}^{k} d(j) z(j) \geqslant 1 \\ l, z \geqslant 0 \end{cases}$$

1°. If $(l, z) \in ARGOPT(D)$ then $\inf\limits_{p \in \mathbb{P}_j} \sum\limits_{e \in p} l(e) = z(j) \forall 1 \leqslant j \leqslant k$.

2°. If $(l, z) \in ARGOPT(D)$ then $\sum\limits_{j=1}^{k} d(j) z(j) = 1$.

Let us consider the following problem (D'):

$$(D') = \begin{cases} \dfrac{\sum\limits_{e \in E} c(e) l(e)}{\sum\limits_{1 \leqslant j \leqslant k} d(j) \inf\limits_{p \in \mathbb{P}_j} \sum\limits_{e \in p} l(e)} \to \min \\ l, z \geqslant 0 \end{cases}$$

3°. $OPT(D') = OPT(D)$.

4°. If $l \in ARGOPT(D')$ then $\left(\dfrac{l}{\sum\limits_{1 \leqslant j \leqslant k} d(j) \inf\limits_{p \in \mathbb{P}_j} \sum\limits_{e \in p} l(e)}, z(j) = \inf\limits_{p \in \mathbb{P}_j} \sum\limits_{e \in p} l(e) \right) \in ARGOPT(D)$.

MCFP theory (3)

So the problem is to assign weights z on demands and weights l to commodities such that each demand weight is its shortest path length and $\frac{\sum\limits_{e\in E} c(e)l(e)}{\sum\limits_{1\leqslant j\leqslant k} d(j) \inf\limits_{p\in\mathbb{P}_j} \sum\limits_{e\in p} l(e)}$ is minimal.

Suppose that x^* and (l^*, z^*) - are optimal solutions of (P) and (D) respectively. From the complementary stackness we have $supp(x^*) \subseteq \{\text{shortest paths over metric } l^*\}$.

Garg-Köenemann 1998, Fleisher 2000, Karakostas 2002 proposed the algorithm with the following general scheme:

1. Initialize l, x.
2. Choose some subset $J \subset \{1, 2, \ldots, k\}$.
3. Change $x \to x + x'$ such that $supp(x') = supp(x'(p) | \exists j \in J : p \in (P)_j) \subseteq \{\text{shortest paths over metric } l\}$.
4. Change l on links which are in paths from $supp(x')$.
5. If termination condition is not satisfied then return to 2.
6. Normalize x due to capacity conditions.

Algorithm

Karakostas algorithm:

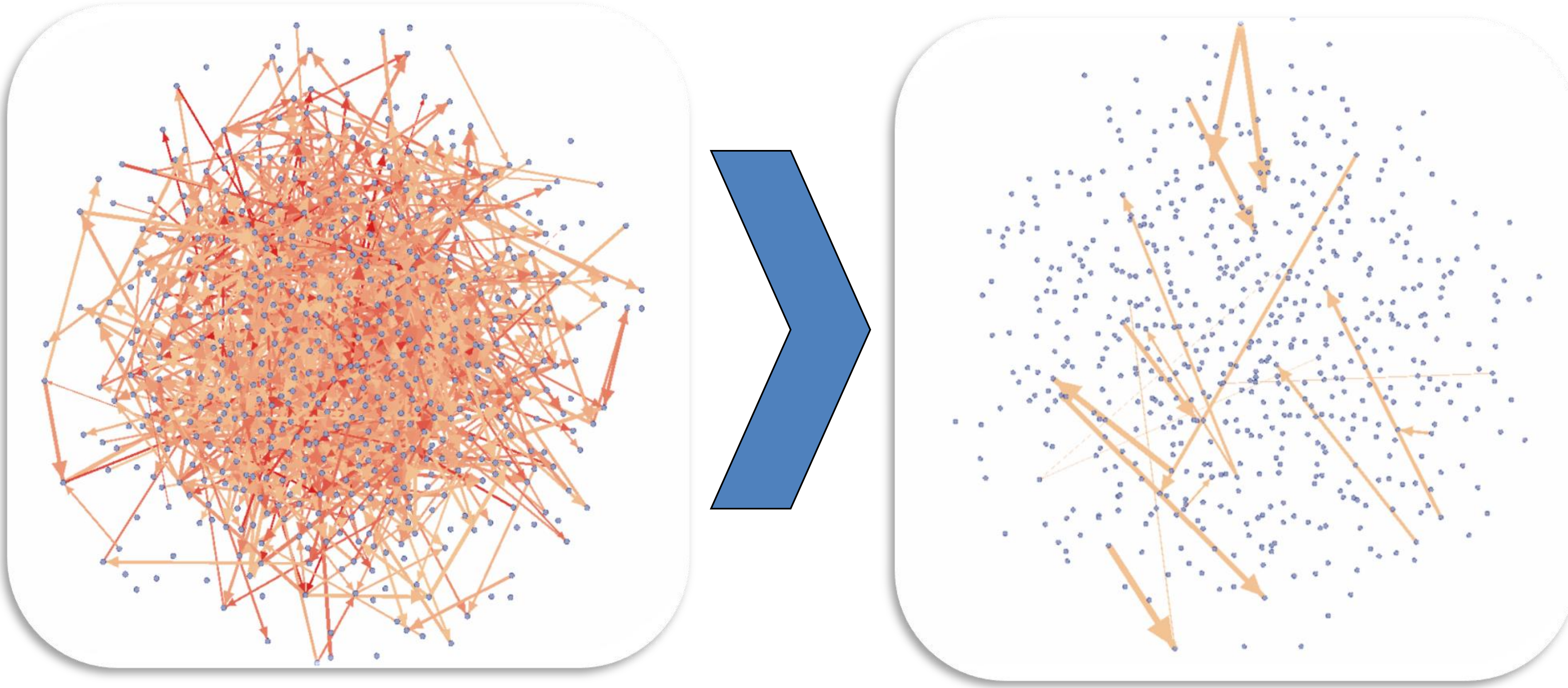
- **Input:** Network $G=(V,E)$, capacities $c(e), e\in E$, demands $(s_i,t_i,d_i), i=1...k$, accuracy ε
- **Output:** ε-optimal flow $f_i(e), e\in E$
- **Initialization:** $w(e)=\delta/c(e),\ f_i(e)=0, e\in E, i=1..k, \delta=(1+\varepsilon)^{1-1/\varepsilon}(1-\varepsilon)^{1/\varepsilon}|E|^{-1/\varepsilon}$
- while $\sum_{i=1..k} d_i l_i^w < 1$ do//phase
- for each source $s\in V$ take all demands $I_s=\{i: s_i=s\}$ //iteration
- set $d_i^{res}=d_i, i\in I_s$
- while $\sum_{i=1..k} d_i l_i^w < 1$ and $I_s^{res}=\{i\in I_s : d_i^{res}>0\}\neq\varnothing$ do
- Call $[d_i^{res}, w, f] = Route(I_s^{res}, d_i^{res}, w, f)$ //step
- Return: $w(e), e\in E$

Core step of FPTAS: $[d_i^{res}, w, f] = Route(I_s^{res}, d_i^{res}, w, f)$

- Find the shortest paths: $P_i : s\to t_i, i\in I_s^{res}$
- Set: $\sigma = \max\left[1, \max_{e\in\bigcup P_{i,i\in I_s^{res}}} \dfrac{\sum_{i\in I_s^{res}:e\in P_i} d_i^{res}}{c(e)}\right]$
- Set: $\gamma_i = d_i^{res}/\sigma;\ f_i(e) += \gamma_i;\ d_i^{res} -= \gamma_i; i\in I_s^{res}$
- Set: $w_i(e) = w_i(e)\left(1+\varepsilon \sum_{i\in I_s^{res}:e\in P_i} \gamma_i \Big/ c(e)\right), e\in \bigcup_{i\in I_s^{res}} P_i$

Complexity: The algorithm stops after at most $\log_{1+\varepsilon}\left(\dfrac{1+\varepsilon}{\delta}\right)$ phases.

Optimization results



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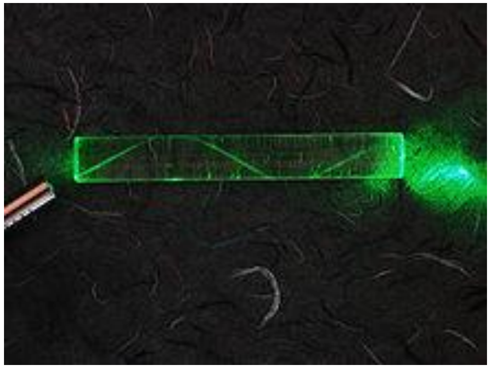
Optical transport networks

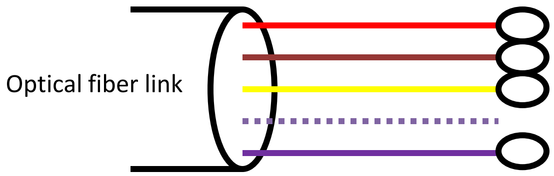
One of the main network goals is to transport data from one place to another. Firstly data is coded by electric signal with some bit rate. Then one transforms electric signal to optical (near-infrared) channel. Optical fiber links are used to represent optical channels. Optical Transport Network (OTN) is a set of nodes which are connected by optical fiber links (see ITU-T standard).

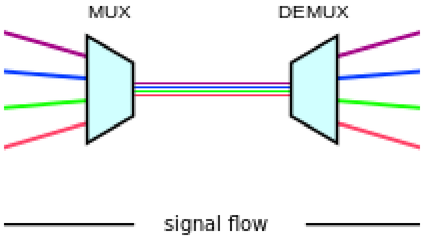


In old technologies an optical fiber link was used for only one optical channel. Nowadays there is a technology which provides using one optical fiber link for several optical channels. This technology is called WDM.

W Wavelength
D Division
M Multiplexing

WDM is a technology which multiplexes a number of optical carrier signals onto a single optical fiber by using different wavelengths (i.e., colors). This technique enables bidirectional communications over one strand of fiber, as well as multiplication of capacity. A WDM system uses a multiplexer (MUX) to join the several signals together, and a demultiplexer (DEMUX) at the receiver to split them apart. These devices are in OADMs (optical add-drop multiplexers).





Setup: OMS links

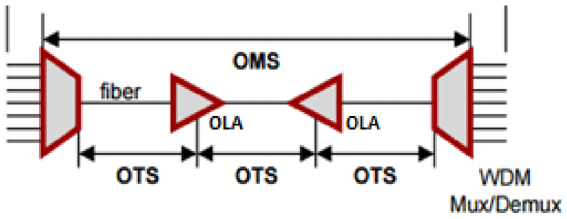
Typically in WDM there are eighty 100Gbit/sec channels with 50 GHz spacing in one optical fiber link (ITU-T G.694.1). In large networks (which are the most interesting for Huawei customers) there is a very common situation when an optical fiber link is so long that optical signal attenuates. So we need to add some devices (named OLA in the picture below), which increase signal power. These devices have neither MUX nor DEMUX. Each OLA delays the signal.

Let OMS link be a sequence of optical fiber links which

a) has MUX and DEMUX in the ends which support 80 channels with 50 GHz multiplexing

b) has OLA in each connection of two fibers

Each OMS link has delay (information about its counting and typical values).



Setup: OADM nodes

Each OADM node connects with several OMS links by DEMUXs and MUXs. DEMUX is used for incoming signal and MUX for outgoing one. Also there can be connection to a user in a node. OADM node can do with optical channels the following:

a) drop from node to

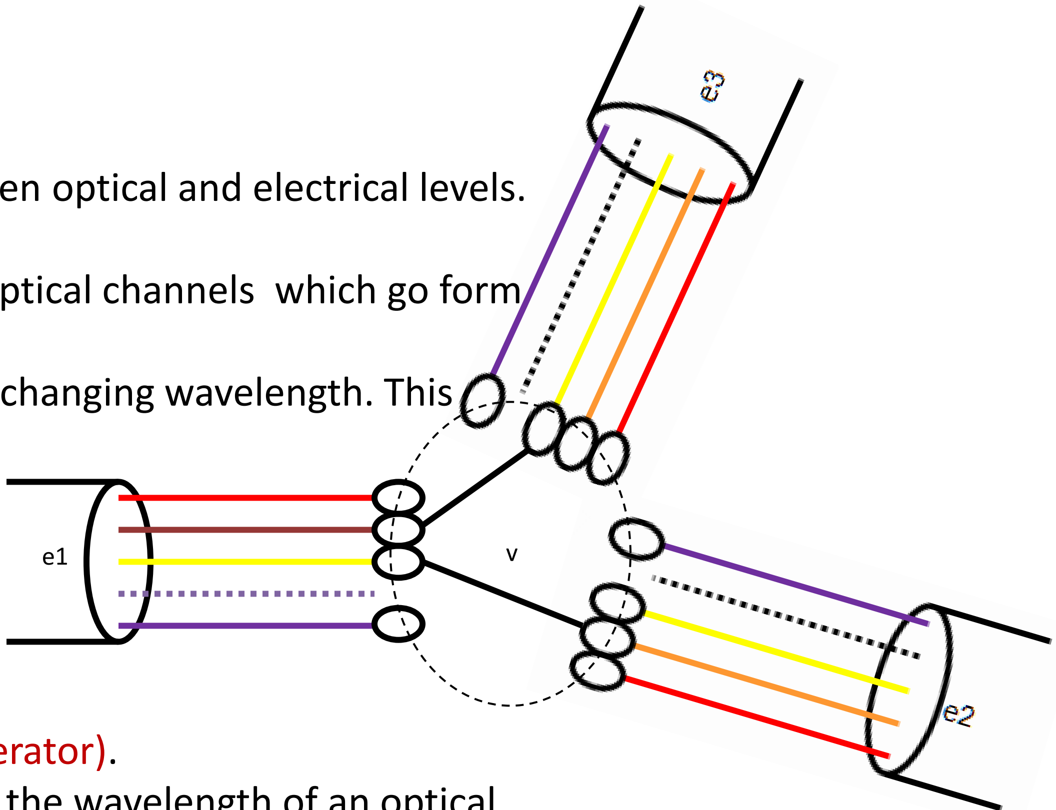
b) add to node from

a user. It transforms these signals between optical and electrical levels.

OADM node can do the following with optical channels which go form DEMUX

a) transport to MUX without changing wavelength. This operation is made by connector and we can assume that it is for free.

b) transport to MUX but change its wavelength. This operation can be made by so called regenerator/converter (hereafter regenerator).



A regenerator is a device which changes the wavelength of an optical channel. It isn't for free.

Setup: demands

As we discussed before, one of the main network goals is to transport data with some bit rate from a source to a destination. We add a channel to OADM, transport data by WDM network and drop a channel from OADM. So 3 main parameters of a network demand are

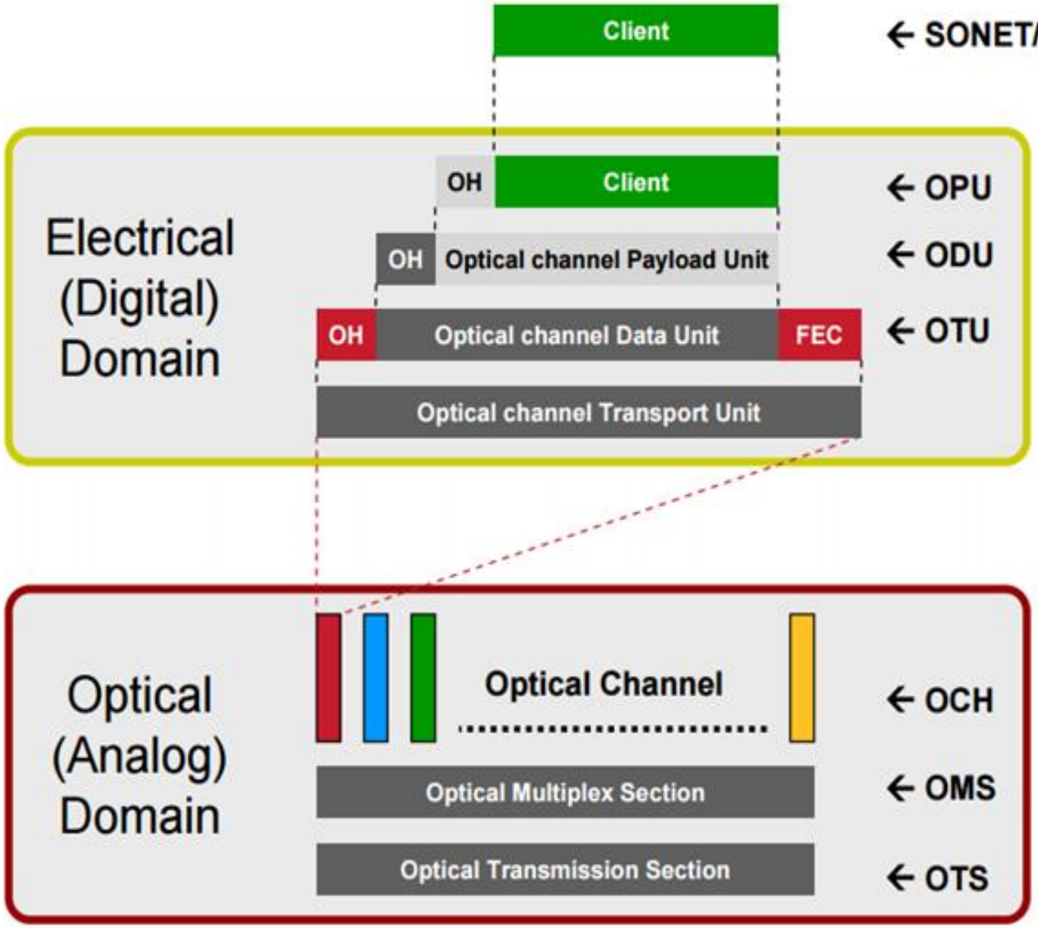
1) Source OADM
2) Bandwidth (in Gbit/sec)
3) Destination OADM

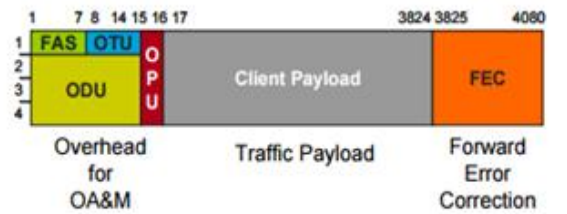
The demand should be embedded to an optical channel so its bit rate should be less than 100Gbit/sec. There are 2 general ways for embedding:

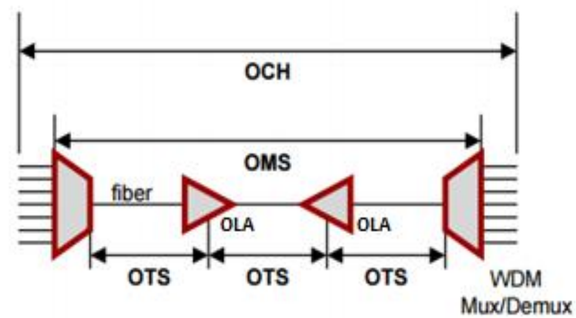
Basic: One optical channel for one demand

Grooming: Embed several demands to one optical channel

General setup







OTN standards defined by ITU-T G.872 and G.709

A network model

Network:

- Links *OMS* : source, destination, number of OLAs
- Nodes *OADM* : some number of regenerators
  - Each Regenerator connects two wavelength of two adjacent links
- Demands (source, destination, bandwidth, routing)

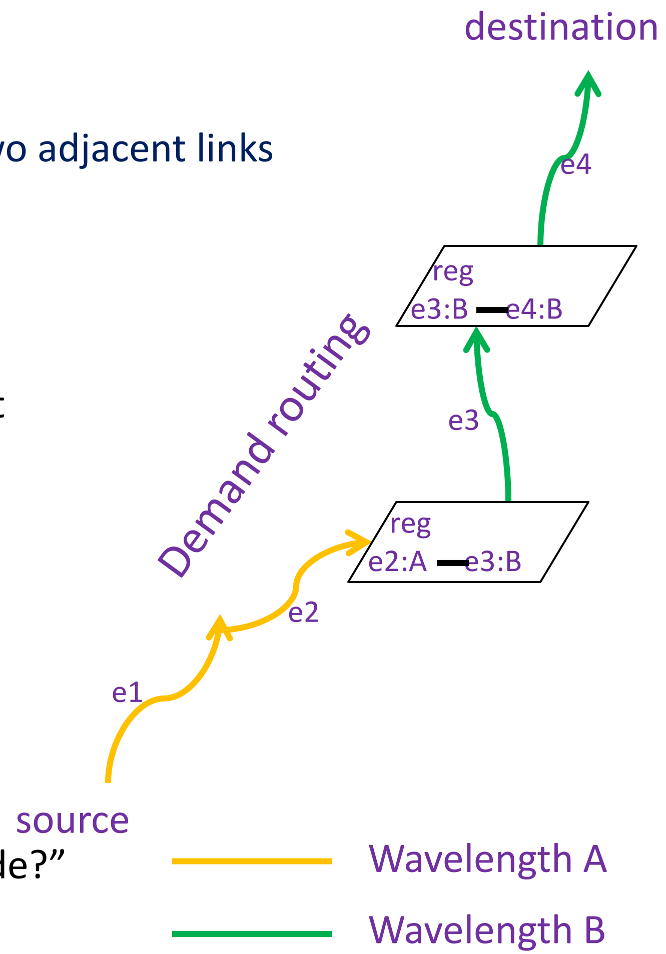
Number of **OLAs** is measure of (OMS) link delay.

Each (OMS) link has 80 channels of the same capacity at different wavelength

The **routing** of each demand is a sequence of pairs **{link, lambda}**, where lambda is wavelength of the channel used by the demand.
Lambda can be changed **only by a regenerator**.

Recoloring is a Boolean variable which means “Is it allowed to change demands wavelength in the node?”

Regenerator routes a demand between selected channels of adjacent links

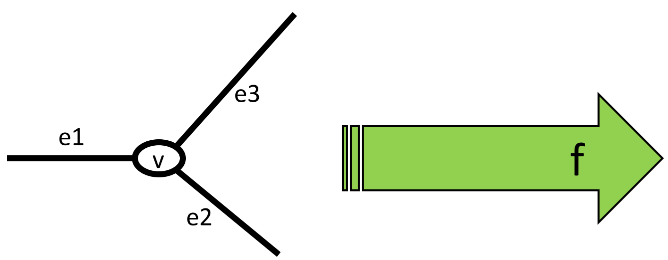


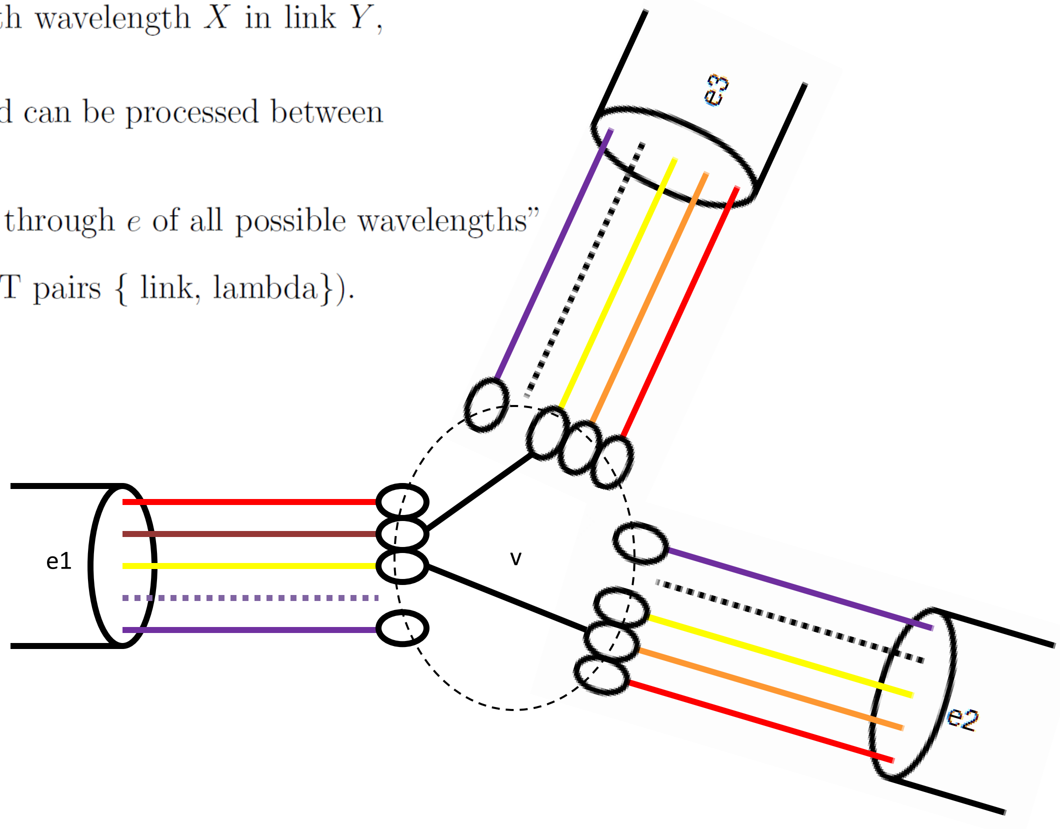
A network mathematical model

Let G be an input network.
Let G_{80} be the graph which is induced from G by the following map $f : G \to G_{80}$.

1. $\forall v \in G.V : f(v) := G_v$, where G_v is the subgraph of G_{80}.
   (a) $|G_v.V| = 80 \cdot deg(v)$.
   (b) each node in G_v means a port to the channel with wavelength X in link Y, where Y is incident to the node v.
   (c) let us connect by edge two ports in G_v if a demand can be processed between these ports.
2. $\forall e \in G.E : f(e) :=$ "80 links which mean the channels through e of all possible wavelengths"

Note: in G_{80} a demand route consists of only links (NOT pairs { link, lambda}).





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Introduction

Suppose

1) we have some network with OADM nodes and OMS links.
2) there is no any regenerator in a network OADM node.

Also we have the set of demands which should be propagated by this network.

The task is to route the demands.

It’s nice if we can do it for free, i.e. without adding regenerators. Note that every demand path is unicolored in the founded routing.

Suppose that we need to add regenerators in nodes to route the demands. Then we should use it as small as possible because regenerators have cost.

Another objective is to minimize average demands delay.

Formulation

Input:

➢Network and demands to be routed

➢Every demand needs whole wavelength

Output:

➢Demands routing

What can we change?

➢Add regenerators for some channel pairs in the nodes with Recoloring == Yes

Objectives:

➢Minimize the number of added regenerators

➢Minimize average demands delay

Mathematical model formulation

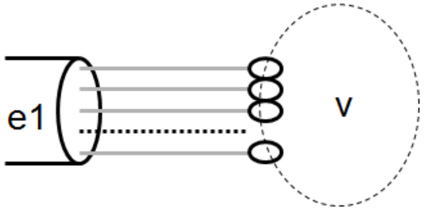
Firstly suppose that for each node v from graph G the corresponding subgraph G_v is complete. It means that there are no constraints to redirection of demands in nodes. So there is no difference what wavelegth is chosen by some demand. We get flow problem where bandwidth of each demand is 1 and capacity of each link is 80.

Let $c_1 : G.E \to \mathbb{Z}_{\geqslant 0}$ be the cost function in the graph G which means links delay. Suppose that $\{d_i\}_{i \in I}$ is the demands set. Let $P_G(d_i)$ be the path of the demand d_i.

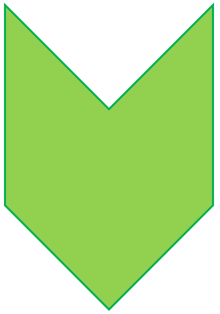
(P1) (?): $\{P_G(d_i)\}_{i \in I}$

[capacity condition] $\forall e \in G.E |\{d_i | i \in I, e \in P_G(d_i)\}| < 80$

$\sum\limits_{i \in I} c_1(P_G(d_i)) \to \min$



Let us move from the graph G to the graph G_{80}. Suppose that there are no edges in subgraphs G_v for each node $v \in G$. Now we can get the solution of **(P1)** and optimize redirecting demands in nodes of G.



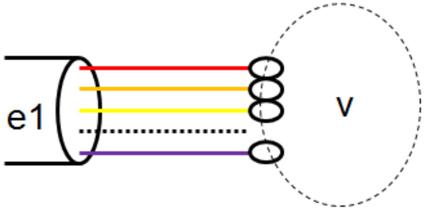
Let $c_2 : G_{80}.E \to \mathbb{Z}_{\geqslant 0}$ be the cost function in the graph G_{80} which means regenerators cost. Support of the cost function c_2 is edges in subgraphs G_v which connect nodes of G_{80} of different color. Also let $\{P_G(d_i)\}_{i \in I}$ be the given set of demands paths in the graph G (may be generated in P1). Let $\{P'_{80}(d_i)\}_{i \in I}$ be the natural image of $\{P_G(d_i)\}_{i \in I}$ into G_{80}. Note that links sets from $\{P'_{80}(d_i)\}_{i \in I}$ are not connected because of discontinuity in every subgraph G_v.

(P2) (?): $\{P_{80}(d_i)\}_{i \in I}$; the set of regenerators

[P1 routing condition] $\forall i \in I P'_{80}(d_i) \subseteq P_{80}(d_i)$

[disjoint condition] $\forall i, j \in I P_{80}(d_i) \cap P_{80}(d_j) = \emptyset$

$\sum\limits_{i \in I} c_2(P_{80}(d_i)) \to \min$



General mathematical model formulation

Such a decomposition of a WDM planning problem into 2 steps is well known (e.g. see [Zang, Jue, Mukherjee]). But also one can solve this problem without considered decomposition.

Let $c : G_{80}.E \to \mathbb{Z}_{\geqslant 0}$ be the cost function in the graph G_{80}. Suppose that $\{d_i\}_{i \in I}$ is the demands set. Let $P_{80}(d_i)$ be the path of the demand d_i.

(P3) (?): $\{P_{80}(d_i)\}_{i \in I}$; the set of regenerators

[disjoint condition] $\forall i, j \in I P_{80}(d_i) \cap P_{80}(d_j) = \emptyset$

$$\sum_{i \in I} c(P_{80}(d_i)) \to \min$$

THANK YOU