Optimal control and symplectic geometry

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Symplectic approach to the optimal control was originally inspired by the Hamiltonian form of the Pontryagin Maximum Principle but it provides a unifying language and efficient calculations' tools for high-order optimality conditions as well. In particular, symplectic interpretation of the space of Jacobi fields of the classical calculus of variations is the so called "Jacobi curve" in the Lagrange Grassmannian. This notion was generalized to certain types of singular and bang–bang extremals in our works with Prof. Gamkrelidze many years ago. Unfortunately, our theory did not work without a priori assumptions on the extremal while the Pontryagin Maximum Principle does work.

In this talk I am going to present a recent development of this old theory obtained together with my student Ivan Beschastnyi. We give an effective geometrically meaningful construction of the Jacobi curve that can be applied to all kinds of Pontryagin extremals, with regular, singular, bang–bang, chattering, etc., segments. The construction implies both pointwise necessary optimality conditions (Legendre, Goh, ...) and conjugate points-type conditions and provides a simple formula for the Morse index of the extremal.