Приведем результаты вычисления траекторий задачи (1)–(3) при управлении (5). На рисунках показаны траектории системы (1) для $x \in \mathbb{R}^3$, $a_1 = (-2, 2, 3)$, $a_2 = (2, -1, 4)$, $a_3 = (-2, -2, 2)$, $x(0) = (-5, 1, 2)$, $x(T) = (6, -1, 1)$, $T = 3$, $a = 2$, $\dot{x}(0) = (1, 1, -1)$, $\dot{x}(T) = (0, -1, 1)$. Левый соответствует $m = 2$, правый $m = 3$. Показаны траектории, отвечающие векторам $\xi_1 = (\cos(8\pi/5), \sin(8\pi/5), 0)$, $\xi_2 = (\cos(7\pi/5), \sin(7\pi/5), 0)$, $\xi_3 = (\cos(9\pi/5, \sin(9\pi/5), \sin(2\pi/5))$, $\xi_4 = (\cos(\pi/5, \sin(\pi/5), \sin(2\pi/5))$, $\xi_5 = (\cos(4\pi/5, \sin(4\pi/5), \sin(3\pi/5))$. При ξ_3, ξ_4, ξ_2 траектории пересекают единичные окрестности точек a_1, a_2, a_3. Векторы ξ_1, ξ_5 выбраны по алгоритму леммы 1. Шары единичного радиуса на рисунках приведены для иллюстрации расстояния от траектории до точек a_1, a_2, a_3.

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Parallel Fair–Taylor algorithm for dynamic general equilibrium models

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Dynamic computable general equilibrium (CGE) models are widely used for estimating the effects of demographic and technological changes on energy use and carbon dioxide (CO_2) emissions (see, e.g., [1]). The equilibrium

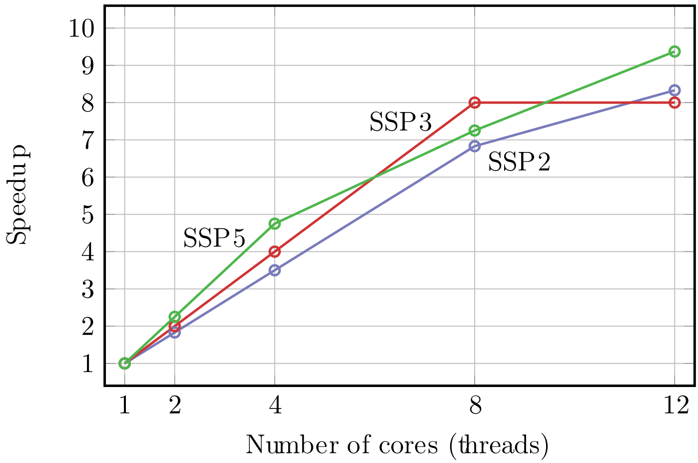

Figure 1. Speedup of the model runs for different SSPs.

is described in the framework of the Arrow–Debreu theory, which leads to a systems of nonlinear equations.

The solution to the dynamic general equilibrium model is described by a large-scale system of nonlinear equations. The Krylov methods (see, e.g., [2]) become increasingly more popular than the stationary iterative method of the Gauss–Seidel type [3].

In [4], we suggested a parallel version of the Gauss–Seidel method that uses the block structure of the nonlinear system, which describes a dynamic general equilibrium model. We implemented the parallel algorithm in the one-region model and showed that its computation time is comparable to the Krylov methods.

To demonstrate the effectiveness of the parallel algorithm, we use the PET model calibrated to reproduce major outcomes for the socioeconomic scenarios from the Shared Socioeconomic Pathways (SSP) database (see, e.g., [6]). The PET model is a forward-looking CGE model with tree types of agents: consumers, producers, and government. Consumers maximize their lifetime utility function taking prices as given. Producers maximize profits supported by the prices. Government redistributes capital through taxes and transfers. International trade is described by the Armington model [5]. Prices are determined by the markets clearing conditions for production factors, intermediate and final goods. The first-order optimality conditions for the agents and supply-equals-demand conditions for markets

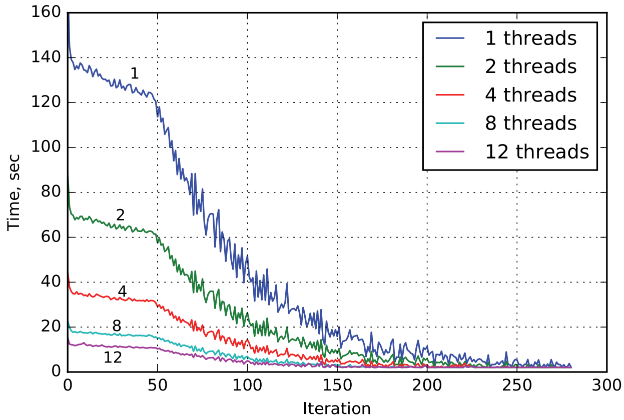

Figure 2. Timing for iterations.

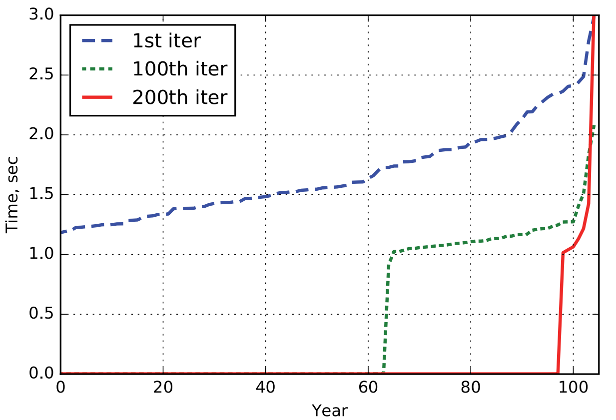

Figure 3. Timing of year-blocks.

form a system of nonlinear equations that determines the general equilibrium. The algorithm for calculating the equilibrium has been implemented using OpenMP and tested at the Lomonosov supercomputer [7].

To study strong scalability of the parallel algorithm, we need to increase the computing power while keeping the total problem size constant. This is achieved by running the model with the same initial approximations and same set of numerical parameters (for each SSP) with increasing number of threads. Figure 1 shows that the speedup of the parallel algorithm grows almost linearly as the number of threads grows from 1 to 12 (for calculations with more threads, see [8]).

Figure 2 shows that, for the number of threads from about one to ten, there is a visible monotone decreases in timing of the iteration as the algorithm converges. This effect can be explained if we look at the timings of different year-blocks of the inner loop (Fig. 3).

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