The significant difficulty in study of such problems lies in the fact that scenarios can bifurcate at some moments which requires the use of feedback control or of some non-anticipating strategy concept.

We demonstrate how to derive optimality condition for such optimal non-anticipating strategy in the form of some non-standard maximum principle.

The main component of this maximum principle is a new form of adjoint system which is significantly different from the classical one in Pontryagin maximum principle.

Of course, the use of non-anticipating strategies as control procedures is not very practical in applications and feedback control is preferable. The another new aspect of this work is the use of the new maximum principle and an adjoint system of equations for a design of optimal feedback control.

Certainly, such optimal feedback controls are discontinuous functions of a state vector. Important aspect of an implementation of discontinuous feedback control is its robustness properties with respect to small measurement errors of state vector and small perturbations of the dynamics. We discuss this aspect of an implementation of optimal discontinuous feedback to demonstrate its robustness.

Optimal solutions in a neighborhood of a singular extremal for a problem with two-dimensional control*

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We consider an optimal control problem that is affine in two-dimensional bounded control. We study a behavior of solutions in a neighborhood of a singular extremal. Optimal singular solutions and solutions with accumulations of control switchings (chattering solutions) are very typical for control-

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affine problems. Singular solutions for a single-input control problem with bounded control is well studied. It was proved that in the case of the general position there exists at least one parametric family of chattering solutions if the Hamiltonian system dimension is no less than 7 [1, 2]. Singular solutions for problems with multidimensional control have been studied much less.

In the present work we consider an optimal control problem that is affine in two-dimensional bounded control. Let

$$\dot{x} = \frac{\partial H}{\partial \psi}, \qquad \dot{\psi} = \frac{\partial H}{\partial x} \tag{1}$$

be the Hamiltonian system corresponding to the problem with

$$H = H_0(x, \psi) + u^1 H_1(x, \psi) + u^2 H_2(x, \psi) \tag{2}$$

where $x \in \mathbb{R}^n$, $\psi \in (\mathbb{R}^n)^*$, $u = (u^1, u^2) \in U \subset \mathbb{R}^2$, and U is some ellipse. The functions H_i $(i = 0, 1, 2)$ are assumed to be smooth. The optimal control function $\hat{u}(t)$ satisfies the maximum condition

$$\hat{u} = \arg\max_{u \in U} H(x, \psi, u). \tag{3}$$

Definition. A point $(x^0, \psi^0) \in \mathbb{R}^{2n}$ is called a *singular point of second order* if the following conditions are satisfied at (x^0, ψ^0):

1) The functions

$$H_i, \quad (\operatorname{ad} H_k) H_i, \quad (\operatorname{ad} H_l)(\operatorname{ad} H_k) H_i,$$
$$(\operatorname{ad} H_j)(\operatorname{ad} H_l)(\operatorname{ad} H_k) H_i, \qquad i = 1, 2, \quad j, k, l = 0, 1, 2,$$

vanish at the point (x^0, ψ^0). The set of the differentials of these functions at (x^0, ψ^0) has constant rank.

2) The bilinear form

$$B_{ij} = \operatorname{ad} H_i (\operatorname{ad} H_0)^3 H_j \big|_{(x^0, \psi^0)}, \qquad i, j = 1, 2,$$

has rank 2 and is symmetric and negative definite.

3) All other commutators of the fifth order from the functions H_j $(j = 0, 1, 2)$, which are independent of the aforementioned ones, vanish at the point (x^0, ψ^0).

Consider the following model problem:

$$\begin{gathered} \int_0^\infty \|x(t)\|^2 \, dt \to \inf, \\ \ddot{x} = u, \qquad \|u(t)\| \le 1, \qquad x(0) = x^0, \qquad \dot{x}(0) = y^0. \end{gathered} \tag{4}$$

Here $x, y, u \in \mathbb{R}^2$ and $\|\cdot\|$ means the standard Euclidean norm of $\mathbb{R}^2$. This problem can be considered as the Fuller problem with two-dimensional control.

It is known [2, 3] that $(x(t), \dot{x}(t)) = 0$ is a unique singular solution for the problem (4). For any point (x^0, y^0) we have the following:

(i) a unique optimal solution hits the origin in finite time $T(x^0, y^0)$ and the optimal control function $\hat{u}(t)$ does not have a limit as $t \to T(x^0, y^0) - 0$;

(ii) if x^0 and y^0 are parallel then optimal trajectories are chattering trajectories;

(iii) there exists a one-parameter family of optimal solutions that represent logarithmic spirals; they approach the origin in finite time with countable number of rotations.

Let (x^0, ψ^0) be a singular point of the second order of the control Hamiltonian system (1)–(3). We prove that in the neighborhood of (x^0, ψ^0) the behavior of optimal solutions for (1)–(3) is determined by optimal solutions of (4). Using the technique developed in [4], we show that for the problem under consideration there exists optimal spiral-similar solution which attains the singular point in finite time making a countable number of rotations.

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