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Simultaneous control of ensembles of nonlinear control systems

Andrey Sarychev

Dip. di Matematica e Informatica U.Dini, University of Florence, Italy

asarychev@unifi.it

Over the last decade there is a growing interest with regard to the *control of ensembles* (*parameterized families*) *of nonlinear control systems*

$$\dot{x}^\theta = f^\theta(x^\theta, u), \qquad \theta \in \Theta \subset \mathbb{R}^\nu, \tag{1}$$

by a single θ-independent control $u(\cdot)$. Such problem arises for example, when one seeks for a control, which may compensate a dispersion of parameters.

One of notable examples is the Bloch model in NMR spectroscopy, seen as a bilinear control system in $SO(3)$ with a parameter subject to dispersion. Partial controllability results for this model have been obtained by N. Khaneja and S. Li, who also suggested applying the Campbell–Hausdorff formula for "generating higher order Lie brackets ... which carry higher order powers of the dispersion parameters."

An alternative problem setting amounts to *finding for a control system $\dot{x} = f(x, u)$ a "simultaneous control" $u(t)$ which (approximately) drives an ensemble of points $x(\theta)$, $\theta \in \Theta$, to a target $z(\theta)$.* In our presentation we opt for L_p-approximate controllability: $\int_\Theta \|x(T;\theta) - z(\theta)\|^p \, d\theta < \epsilon^p$.

In a recent publication [1] with A. Agrachev and Yu. Baryshnikov we aimed at introducing a Lie algebraic ("geometric control") approach to the controllability of (1).

We started with finite ensembles (finite Θ), to which Lie rank criteria of *exact controllability* can be applied after proper modification. We proved that the property of global controllability for a finite ensemble of control-linear systems is generic and, as an example, established global controllability by means of a single scalar control for a finite ensemble of rigid bodies with generic inertial parameters.

For *continual* ensembles (1) achieving exact controllability would require, in general, infinite-dimensional set of control parameters. Instead we fix the dimension of control and study L_1-approximate controllability.

In the setting we first considered a model example of "controlling the holonomy" for an ensemble of 2-distributions in $\mathbb{R}^3$, where we get necessary and sufficient conditions for approximate controllability. We proceed then to a general ensemble of r-distributions (control-linear systems) on a manifold, for which we formulate sufficient approximate controllability criteria in terms of the Lie algebraic span. This is a version of the Rashevsky–Chow theorem for control-linear ensembles.

In what regards the "simultaneous control" of ensembles of points $x(\theta)$, the controllability criteria are similar in spirit to the previously mentioned results, but the formulations and the proofs differ. We establish genericity of the property of exact global controllability for *finite point ensembles*, and advance with a formulation of a version of the Rashevsky–Chow theorem for continual point ensembles.

References

1. *Agrachev A., Baryshnikov Yu., Sarychev A.* Ensemble controllability by Lie algebraic methods // ESAIM COCV. 2016. V. 22. P. 921–938.