Twisted Compactifications and Black Hole Microstates

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University of Amsterdam

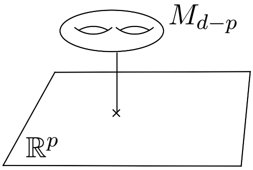
Based on
1) 1511.09462 with N. Bobev and F. Benini
2) 1708.xxxxx with N. Bobev
3) 1707.04257 with N. Bobev & V. Min, and F. Azzurli & A. Zaffaroni.

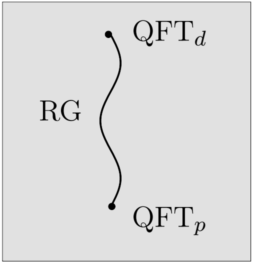
HSE, Moscow

August 1, 2017

Basic Setup

Consider a QFT on $M_d = \mathbb{R}^p \times M_{d-p}$ and flow to IR:





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2) IR theory?
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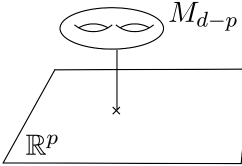

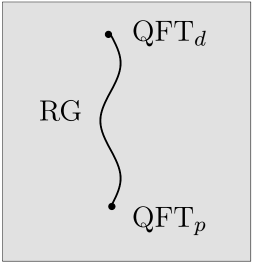

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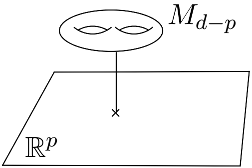

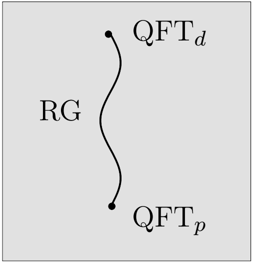

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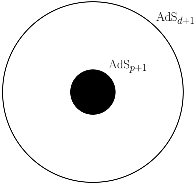

Motivation

- Recently, great interest in QFTs on M_d
- Exact results by localization in various dimensions **[Pestun, Kapustin et al., Benini et al., Doroud et al.]** Check of dualities and AdS/CFT
- Taking $M_d = M_{d-p} \times \tilde{M}_p$ leads to new dualities $T_{d-p} \leftrightarrow \tilde{T}_p$ **[Alday-Gaiotto-Tachikawa, Gadde-Pomini-Rastelli-Razamat, Dimofte-Gaiotto-Gukov, $\cdots$]**
- Compactification leads to large classes of SCFTs from higher dimensions **[Maldacena-Núñez, Bershadsky-Johansen-Sadov-Vafa, Gaiotto, Bah et al., Crichigno-Benini-Bobev,$\cdots$]**
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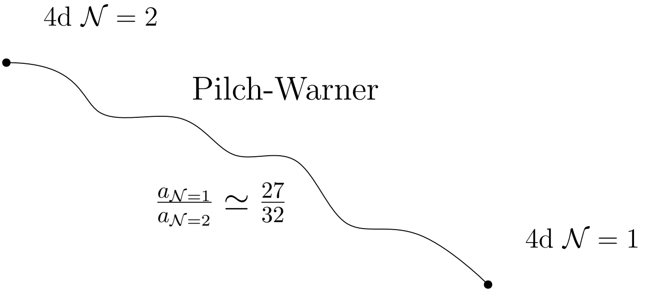
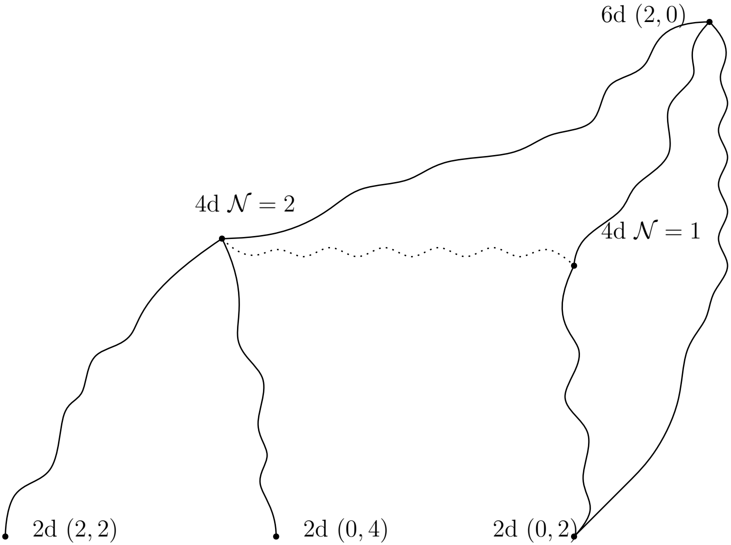
What do we mean by "universal"?

- Prototypical example of a (4d→4d) universal flow:



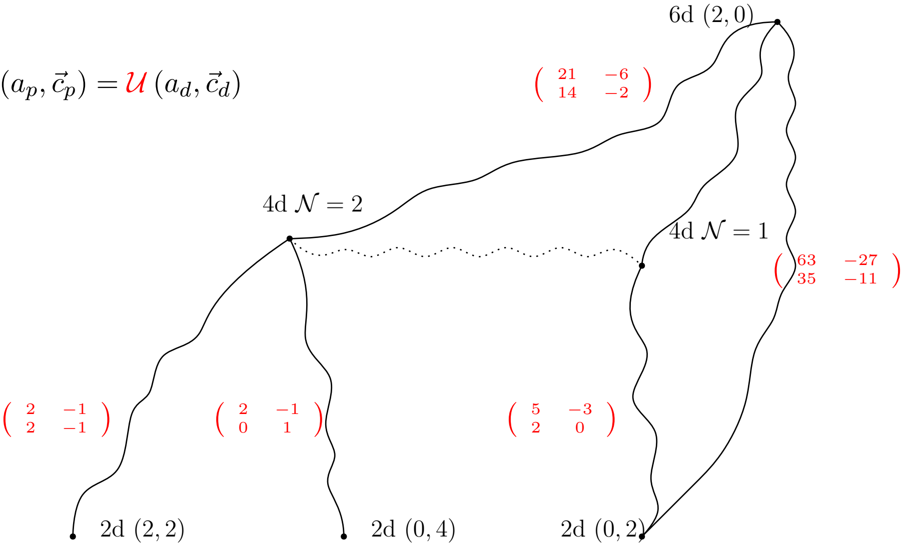
- First found for $\mathcal{N}=4 \to \mathcal{N}=1$ flow **[Anselmi-Freedman et al. 1997]**
- Later proven in more generality for $\mathcal{N}=2 \to \mathcal{N}=1$ flows **[Tachikawa-Wecht 2009]**

Universal flows across (even) dimensions



Universal flows across (even) dimensions

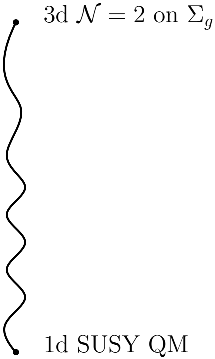
$$(a_p, \vec{c}_p) = \mathcal{U}\,(a_d, \vec{c}_d)$$



Universal flows across (odd) dimensions

- In even d anomalies nicely behaved under RG, but more general story. In 3d:

$$F_{\mathrm{IR}} = (g-1)\, F_{\mathrm{UV}}$$



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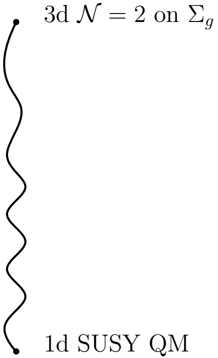

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Review tools

Tool 1: Topological twist [Witten 1988]

To preserve SUSY on M_d:

$$(\partial_\mu + \omega_\mu)\epsilon = 0$$

Generically, no solutions. But, if global R-symmetry, turn background on:

$$(\partial_\mu + \omega_\mu + A_\mu^{back})\epsilon = 0 \quad \xRightarrow{A_\mu^{back} = -\omega_\mu} \quad \partial_\mu \epsilon = 0$$

Comments:

- SUSY only partially preserved
- Spin of fields *shifted*: $(\partial_\mu + (s - q)\,\omega_\mu)\phi$
- If $M_d = \mathbb{R}^p \times \mathcal{M}_{d-p}$ twist only along $\mathcal{M}_{d-p}$

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- Global symmetry $S \times G_R$ (Lorentz×R-symmetry)
- Twist amounts to choosing embedding

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Tool 2: Anomalies

Anomalies are robust nonperturbative observables. Interested in two kinds of anomalies:

- 't Hooft R-symmetry anomalies $\langle \partial_\mu j_R^\mu \rangle \neq 0$. In 4d and 2d:

$$k_{RRR}, k_{RR}, \cdots$$

- Weyl Anomalies

$$\langle T^\mu_\mu \rangle_{M_d} \sim aE + c_i W_i$$

- If SUSY $\Rightarrow \{T_{\mu\nu}, j_R^\mu\} \Rightarrow$

(a, c_i) related to $k's$

- Downside: Only available in *even* dimensions

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Field Theory

$$\text{4d } \mathcal{N} = 1 \text{ on } \Sigma_{g>1}$$

$$\rightsquigarrow$$

$$\text{2d } \mathcal{N} = (0,2)$$

Universal relation [Benini-Bobev-MC 2015]

To establish relation among central charges, 3 simple steps:

1) R-symmetry is $U(1)_R$. Assume *no Abelian flavor symmetry*:

$$I_6 = \frac{k_{RRR}}{6} c_1(R)^3 - \frac{k_R}{24} c_1(R)\, p_1(T_4)$$

2) Twist on $\Sigma_{\mathfrak{g}}$: $U(1)_\Sigma \subset U(1)_R \Rightarrow$

$$c_1(R) \to c_1(R) + \frac{1}{2}\mathrm{dVol}(\Sigma_g)$$

3) Integrate $\int_{\Sigma_g} I_6$ and compare to

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This leads to (*anomaly matching*)

$$k_{RR} = (g-1)k_{RRR}\,, \qquad k = (g-1)k_R$$

Assuming fixed point in UV and IR: (*Ward identities*)

$$\begin{aligned} \text{4d}: &\qquad a_{4d} = \frac{9}{32}k_{RRR} - \frac{3}{32}k_R\,, \qquad c_{4d} = \frac{9}{32}k_{RRR} - \frac{5}{32}k_R \\ \text{2d}: &\qquad c_r = 3k_{RR}\,, \qquad c_r - c_l = k \end{aligned}$$

gives:

$$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = \frac{16(g-1)}{3}\begin{pmatrix} 5 & -3 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix}$$

- True also for compactification of $\mathcal{N} = 2, 3, 4$ theories.
- In the large–N limit:

$$c_r \simeq c_l \simeq \frac{32}{3}(g-1)a_{4d}$$

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$$4\text{d }\mathcal{N}=2\text{ on }\Sigma_{g>1}$$

$$\big\downarrow$$

$$2\text{d }\mathcal{N}=(2,2)$$

R-symmetry is now $SU(2) \times U(1)_r$. Thus, more twists are possible:

Pick $U(1)_\Sigma \subset \underbrace{U(1)_{R_3}}_{\alpha} \times \underbrace{U(1)_r}_{\beta}$ [Kapustin 2006]

Assuming IR fixed point one shows: [Bobev-MC 2017]

- For α–twist:

$$\text{2d } \mathcal{N} = (2,2): \qquad \begin{pmatrix} c_r \\ c_l \end{pmatrix} = 12\,(g-1) \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix}$$

  Note $c_r = c_l = 3(g-1)d_G$ with $d_G \equiv 4(2a_{4d} - c_{4d})$ dim. of Coulomb branch of 4d theory [Shapere-Tachikawa 2008]

- For β–twist:

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  Note $c_r = c_l = 3(g-1)d_G$ with $d_G \equiv 4(2a_{4d} - c_{4d})$ dim. of Coulomb branch of 4d theory [Shapere-Tachikawa 2008]

- For β–twist:

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- $\frac{1}{3}\alpha + \frac{4}{3}\beta$–twist equals $\mathcal{N} = 1$ twist

R-symmetry is now $SU(2) \times U(1)_r$. Thus, more twists are possible:

$$\text{Pick} \qquad U(1)_\Sigma \subset \underbrace{U(1)_{R_3}}_{\alpha} \times \underbrace{U(1)_r}_{\beta}$$

[Kapustin 2006]

Assuming IR fixed point one shows: [Bobev-MC 2017]

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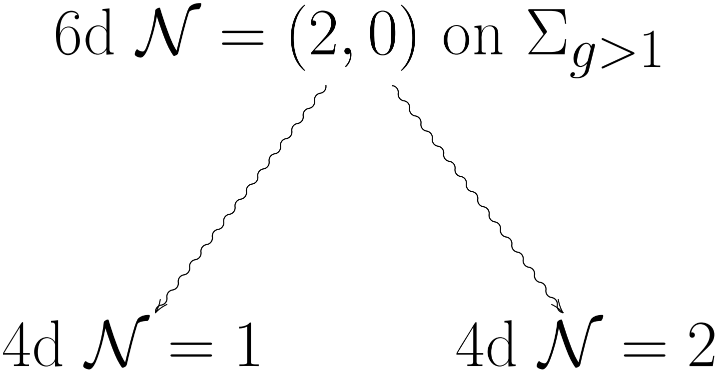
6d $\mathcal{N} = (2,0)$ on $\Sigma_{g>1}$
4d $\mathcal{N} = 1$
4d $\mathcal{N} = 2$

6d→4d and 6d→2d

Pick $SO(2)_\Sigma \subset SO(2) \times SO(2) \subset SO(5)_R$

4d $\mathcal{N} = 2$:
$$\begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix} = \frac{(g-1)}{72} \begin{pmatrix} 21 & -6 \\ 14 & -2 \end{pmatrix} \begin{pmatrix} a_{6d} \\ c_{6d} \end{pmatrix}$$

4d $\mathcal{N} = 1$:
$$\begin{pmatrix} a_{4d} \\ c_{4d} \end{pmatrix} = \frac{(g-1)}{384} \begin{pmatrix} 105 & -33 \\ 49 & -1 \end{pmatrix} \begin{pmatrix} a_{6d} \\ c_{6d} \end{pmatrix}$$

Similarly, compactifying on Kähler M_4:

2d $\mathcal{N} = (0,2)$:
$$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = \frac{(P_1 + 2\chi)}{96} \begin{pmatrix} 63 & -27 \\ 35 & -11 \end{pmatrix} \begin{pmatrix} a_{6d} \\ c_{6d} \end{pmatrix}$$

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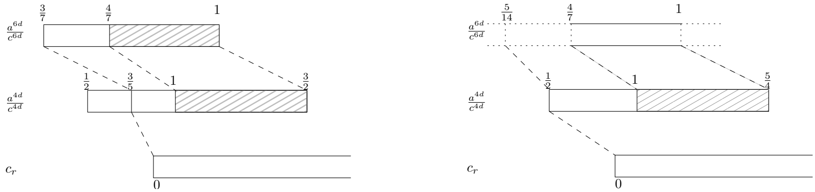
Comment: Hofman-Maldacena Bounds

These are 4d bounds on $\frac{a_{4d}}{c_{4d}}$ from energy positivity **[Hofman-Maldacena '08, '16]**



Universal flows map 4d values to (interesting?) values in 6d and 2d

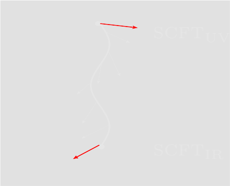
Which flows are *not* universal?

Consider theories with flavor symmetries G_F with generators F_i

- Infinite family of twists:

$$T_{back} = T_R^{\text{conf.}} + b_i F_i$$

- Complication: *Mixing* of flavor and R-symmetry along flow:



$$R_{\text{IR}} = R_{UV} + \epsilon_i(b) F_i \,, \qquad \epsilon_i = ?$$

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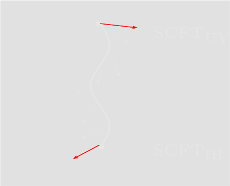

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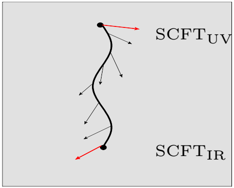

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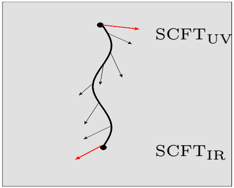

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Global symmetry $SU(2)_1 \times U(1)_2 \times U(1)_B \times U(1)_R$

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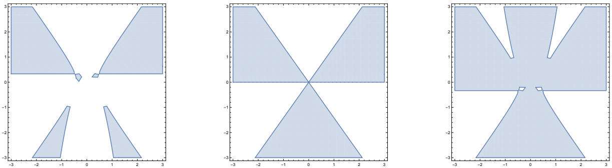
Theories unitary $(c_{l,r} > 0)$ only in regions of parameter space:



Regions in $b_{1,2}$ plane $(b = 0)$ where $c_R > 0$ (for $\kappa = \{1, 0, -1\}$)

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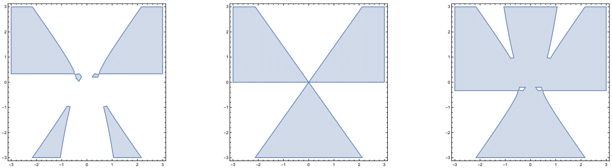

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Holography

Two reasons for holography:

1) Establish *existence* of RG flows (at large N)

2) When anomalies not available, holography predicts

$$F_{IR} = u\, F_{UV}$$

$\Rightarrow$ Nontrivial predictions for Matrix models

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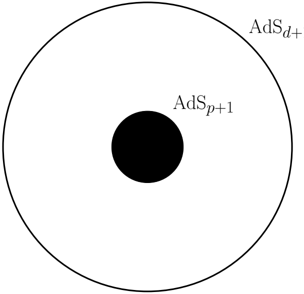
AdS/CFT associates an AdS_{D+1} geometry to a CFT_D.



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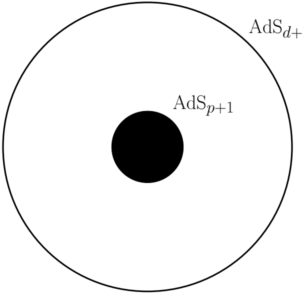

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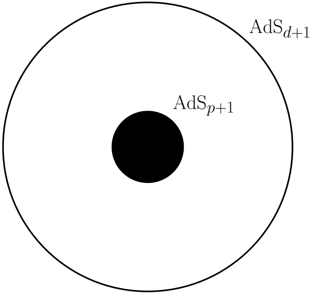

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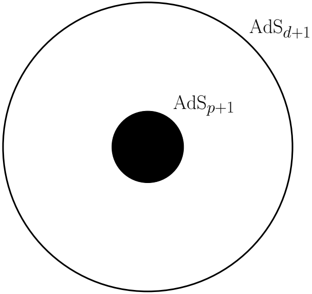

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Black strings in AdS_5

5d $\mathcal{N}=2$ *minimal* gauged SUGRA $(g_{\mu\nu}, A_\mu)$. Ansatz:

$$ds_5^2 = e^{2f(r)}(-dt^2 + dz^2 + dr^2) + e^{2g(r)} ds^2_{\Sigma_{g>1}}$$
$$F = dA = -\frac{1}{3}\mathrm{dVol}_{\Sigma_{g>1}}$$

AdS_3 fixed point: $e^{2f} = \frac{1}{r^2}e^{2f_0}\,, e^{2g} = e^{2g_0}$. Central charge:

[Brown-Henneaux]

$$c_R \simeq c_L \simeq \frac{3L_{\mathrm{AdS}_3}}{2G_N^{(3)}} \simeq \frac{32}{3}(g-1)a_{4d}\,!$$

(used holographic dictionary $a_{4d} \simeq c_{4d} \simeq \frac{\pi}{8G_N^{(5)}}$)

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Odd dimensions: Black Holes in AdS_4

Black holes in AdS_4 describe flows from 3d $\mathcal{N} = 2$ to 1d SUSY QM

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4d $\mathcal{N} = 2$ (8 supercharges) gauged supergravity: $(g_{\mu\nu}, A_\mu)$

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Uplifts

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$$ds^2_{11} = L^2 \left(ds^2_4 + 16\, ds^2_{\mathrm{SE}_7} \right) ,$$

with $ds^2_4 = -\left(\rho - \frac{1}{2\rho}\right)^2 dt^2 + \left(\rho - \frac{1}{2\rho}\right)^{-2} d\rho^2 + \rho^2 ds^2_{\Sigma_\mathfrak{g}}$ and $G_{(4)} \neq 0$.

In *massive IIA*, new solution:

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and $(A_1, A_2, A_3) \neq 0$ and

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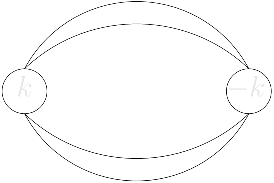
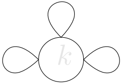
The power of the universal flow

Uplifting to either M-theory or massive IIA, dual CFT_3's very different:





M-theory: $F_{S^3} \sim N^{3/2}$ massive IIA: $F_{S^3} \sim N^{5/3}$

- Universal flow predicts: $F_{S^1\times\Sigma_g}/F_{S^3} \simeq (g-1)$.
- Nontrivial prediction of holography for corresponding Matrix Models!
- Entropy of BH given by $S_{\mathrm{BH}} = -F_{S^1\times\Sigma_g}$

The power of the universal flow

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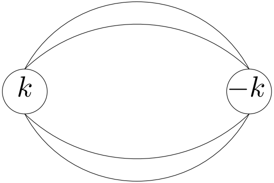

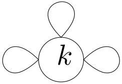

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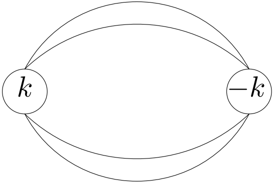

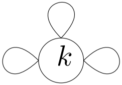

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Two explicit examples

- Taking $Y_7 = S^7/\mathbb{Z}_k$, the quantized horizon area reads:

$$\textit{Gravity:} \qquad S_{\text{BH}} = (g-1)\frac{\pi\sqrt{2}}{3}k^{1/2}N^{3/2}$$

  BH asymptotic to $\text{AdS}_4 \times S^7/\mathbb{Z}_k$–ABJM.

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- Localization on S^3: [Kapustin-Willett-Yaakov 2009]

$$Z_{S^3} = \int du e^{-ik\pi \mathrm{Tr} a^2} \prod_{\alpha} 2\sinh(\pi\alpha(a)) \prod_{\rho\in\mathcal{R}} \frac{1}{\cosh(\pi\rho(a))}$$

where $a = u + i(\Delta - 1/2)$.

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Summary & Outlook

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- Existence of $\mathcal{U}$niversal RG flows across (even) dimensions (exact, finite N)
- Strong evidence across all dimensions (large N, for now)
- Holography predicts nontrivial matrix model relations
- Microstate counting of infinite number of black holes in AdS_4
- Subleading corrections?
- 5d $\to$ 3d, 1d? New relations among matrix models? $F_{S^5}/F_{\Sigma_g \times S^3}$?
- Concrete proposal for counting entropy of black branes in AdS

THANK YOU!