Practical optimization in control problems

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Sources

- Optimal Control Theory
- Theory of finite-dimensional optimization
- Theory of global optimization
- Experience in developing optimization software
- Experience in solving applied problems
- Libraries of algorithms for computing problems
- …

Software for OCP Russia

- DISO (CS RAS)
- CONTROL (IAM RAS)
- PAOEM (CEMI RAS)
- «PC for OCP» (IPS RAS)
- MAPR (IDSTU SB RAS)
- KONUS (IDSTU SB RAS)
- OPTCON (IDSTU SB RAS)

Software for OCP

- MISER (Australia)
- SOCS, RIOTS-95, GPOPS-II, SNOPT_SNCTRL, KNITRO, DIDO, OTIS (USA)
- PDECON, EASY-FIT, DIRSOL, MINOPT, LOTOS, JBENDGE, ACADO (Germany)
- TOMLAB (Norway)
- PSOPT (Netherlands)
- BASILE (France)
- ROMEO (China)
- …

Optimal Control Problem

Box control restrictions

$$\dot{x} = f(x,u,t)$$

$$x(t_0) = x^0, \quad t \in T = [t_0, t_1]$$

$$u(t) \in U = \{u \in R^r : \underline{u}_i \le u_i \le \overline{u}_i\}$$

$$I(u) = \varphi(x(t_1)) \to \min$$

Optimal Control Problem
Terminal restrictions

$$\dot{x} = f(x,u,t)$$

$$x(t_0) = x^0, \quad t \in T = [t_0, t_1]$$

$$u(t) \in U = \{u \in R^r : \underline{u}_i \le u_i \le \overline{u}_i\}$$

$$I_0(u) = \varphi^0(x(t_1)) \to \min$$

$$I_j(u) = \varphi^j(x(t_1)) = 0, \; j = \overline{1,m}$$

Discrete Optimal Control Problem
Box control restrictions

$$x(t+1) = f(x(t), u(t), t)$$

$$x(t_0) = x^0, \;\; t \in T = [t_0, t_0+1, \ldots, t_1]$$

$$u(t) \in U = \{u \in R^r : \underline{u}_i \le u_i \le \overline{u}_i\}$$

$$I_0(u) = \varphi^0(x(t_1)) \to \min$$

Test problems

Nonlinear stabilization of pendulum

$$\dot{x}_2 = u_1 - \sin x_1$$

$$x(0) = (5, 0)$$

$$| u_1(t) | \le 1, \ t \in [0, 5]$$

$$I(u) = x_1^2(5) + x_2^2(5) \to \min$$

Test problem 01

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u_1 - \sin x_1$$

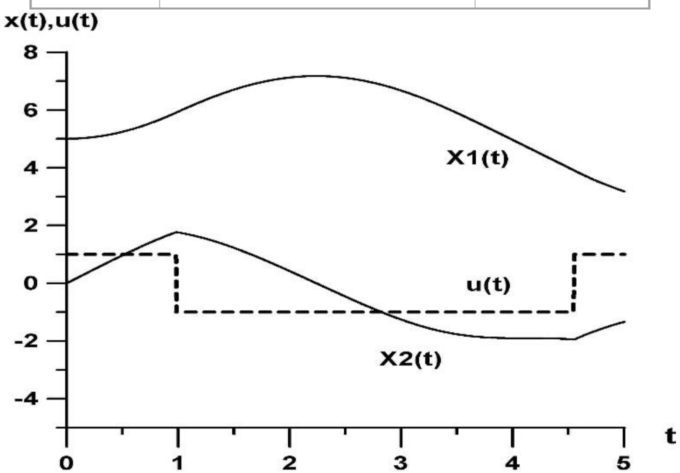
$$x(0) = (5, 0) \quad t \in [0, 5] \quad |u_1(t)| \le 1$$

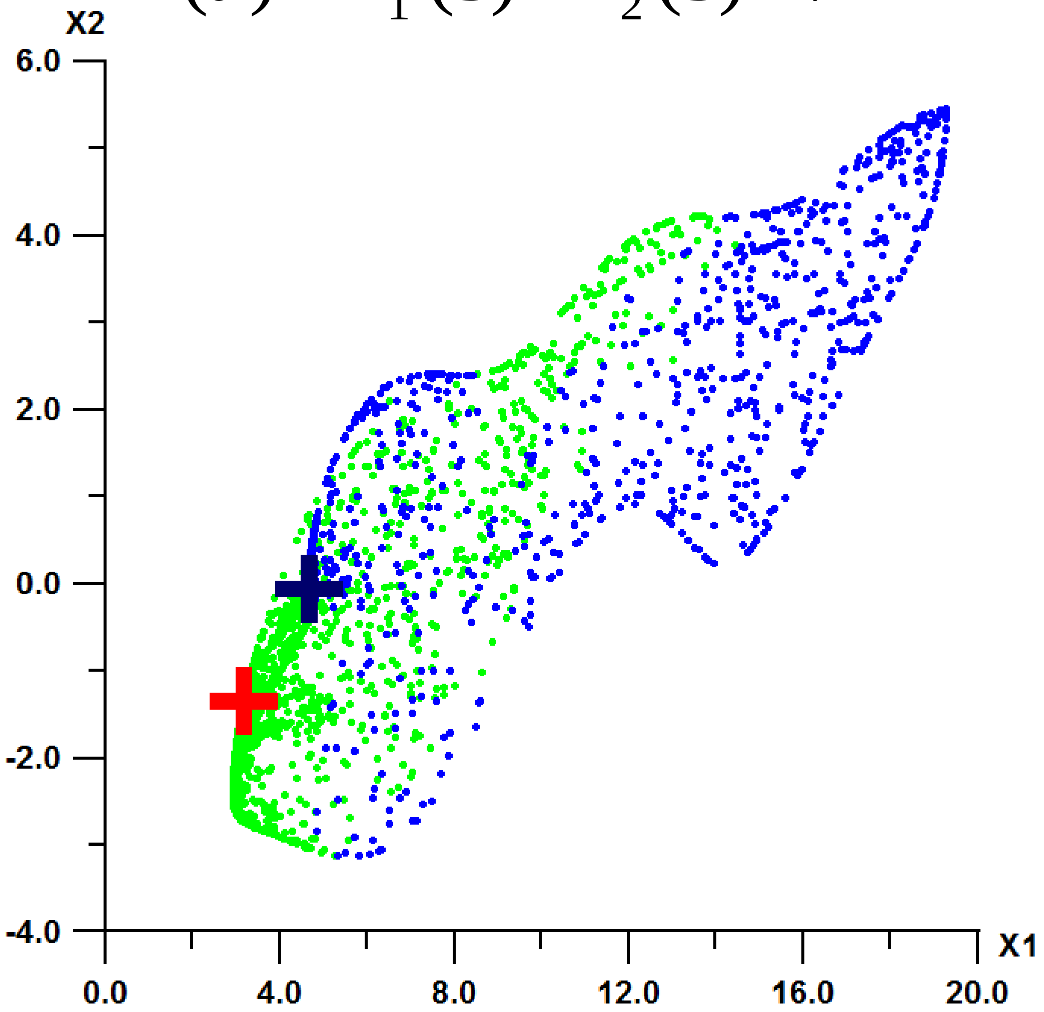
$$I(u) = x_1^2(5) + x_2^2(5) \to \min$$

ITERATIONS 1000
CP TIME 7591 сек.
STOP KRITERIA -216.8

CAUCHY PROBLEMS 186015

N	Functional	Volume
1	1.190817e+01	0.457
2	2.182900e+01	0.543





Applications

- Flight dynamics
- Space navigation
- Mechanics
- Robotics
- Electrical power engineering
- Ecology
- Economics
- …

Dimensions of solved problems

- Typical dimensions of the solved problems are 3-5 phase variables and 1-2 controls
- The maximum dimensions are 553 phase variables and 24 controls

The terms
problems types

- Test problem
- Model problem
- “The problem with meaningful meaning“
- Application problem

The terms
synonyms

- Optimal control problem
- Problem of optimization for dynamic systems
- Problem of trajectory optimization
- Problem of parametric identification (control-constants)
- …

The terms
Control actions

- Control functions
- Control constants
- Pulse controls
- Position control
- …

The terms

Target functional ("quality criterion")

- Terminal
- Integral
- «Pointed»
- …

The terms
constraints

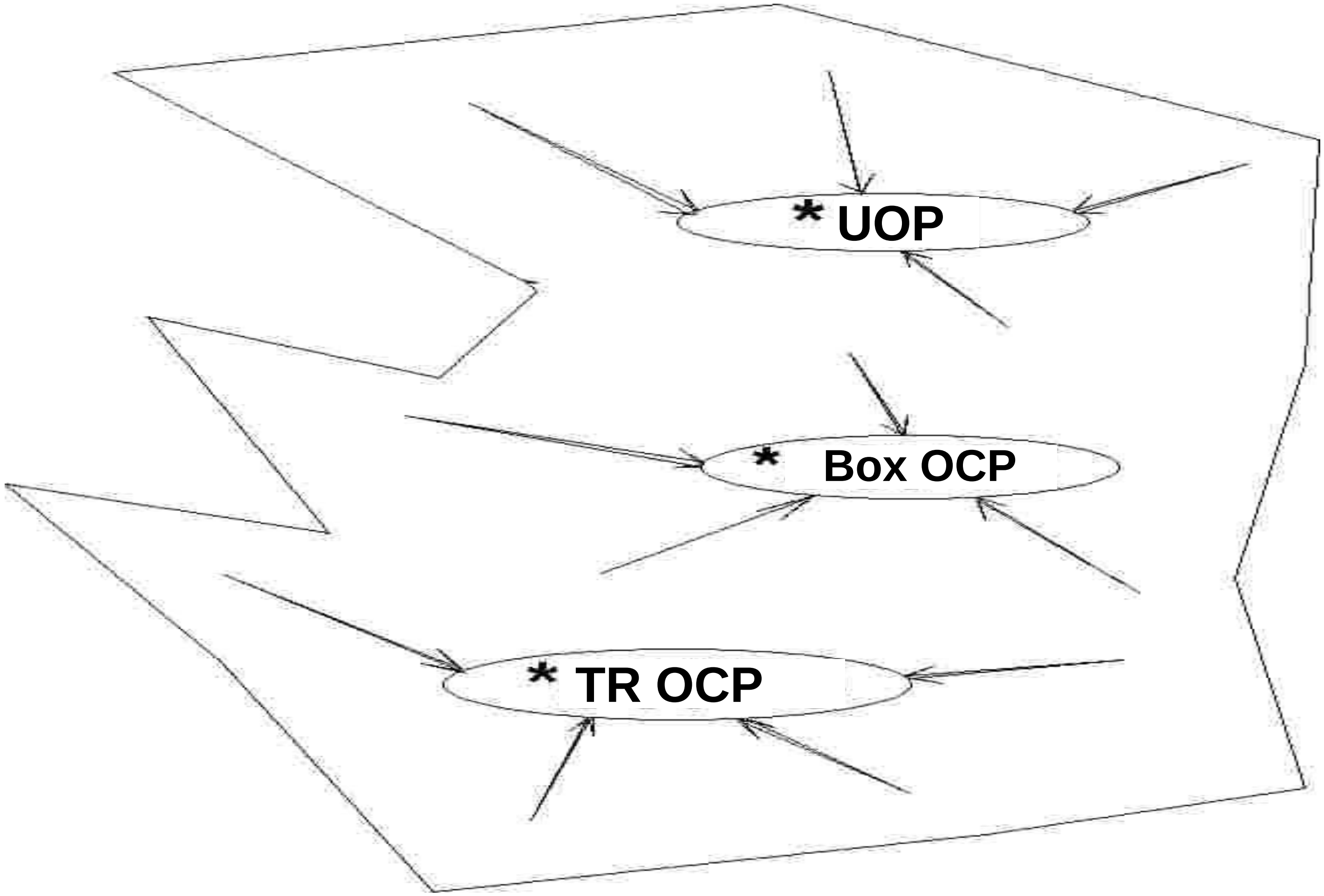
- Box constraints on the control
- Integral constraints
- Terminal constraints on the trajectory
- Phase constraints on the trajectory,
- Mixed constraints
- Intermediate constraints on the trajectory
- …

Summary

The problems of optimal control are the "four-dimensional" space of extremal problems of various structures and with essentially different properties

The problem of optimal control

General approach to the solution



The terms

The structure of optimal control problems

- The system of differential equations
- Controls
- Optimized functional ("quality criterion")
- Restrictions (direct, terminal, phase, intermediate, ...)

The terms

The types of dynamical system

- Ordinary differential equations
- Partial differential equations
- Recurrence Equations
- Systems with delay argument
- Algebraic-differential equations
- Integro-differential equations
- Functional-differential equations
- …

The numerical-analytical approach
The problems for researcher

- fine-tuning of the optimization model
- reduction of the problem to a convenient form
- choice of decision algorithms
- acceleration of payments
- verification of settlement results
- accumulation of experience in solving problems
- …

The numerical-analytical approach
Difficulties

- relatively high "qualification bar“
- undeveloped software interfaces
- non-formalization of accumulated information
- dissonance with mathematical tradition
- undeveloped systems of training and exchange of specialized information

=> WE NEED INTERACTIVE ALGORITHMS AND INSTRUMENTAL MEANS

The gap between theory and practice

- range of tasks
- hereditary problems of the theory of differential equations
- lack of practical classifications of tasks
- lack of effort to find effective algorithms
- lack of attention to technological details
- inefficiency of the axiomatic method
- ...
  => THESE STAGE NEEDS OTHER APPROACHES AND LOGICS OF ACTION

Optimal control problems
Classification 1

1. Nonlinearity
2. Rigidity
3. Ill-posed, sliding modes, chattering
4. Multi-extremality
5. Gully
6. Poor conditionality
7. Unphysicality
8. Instability of the system trajectories

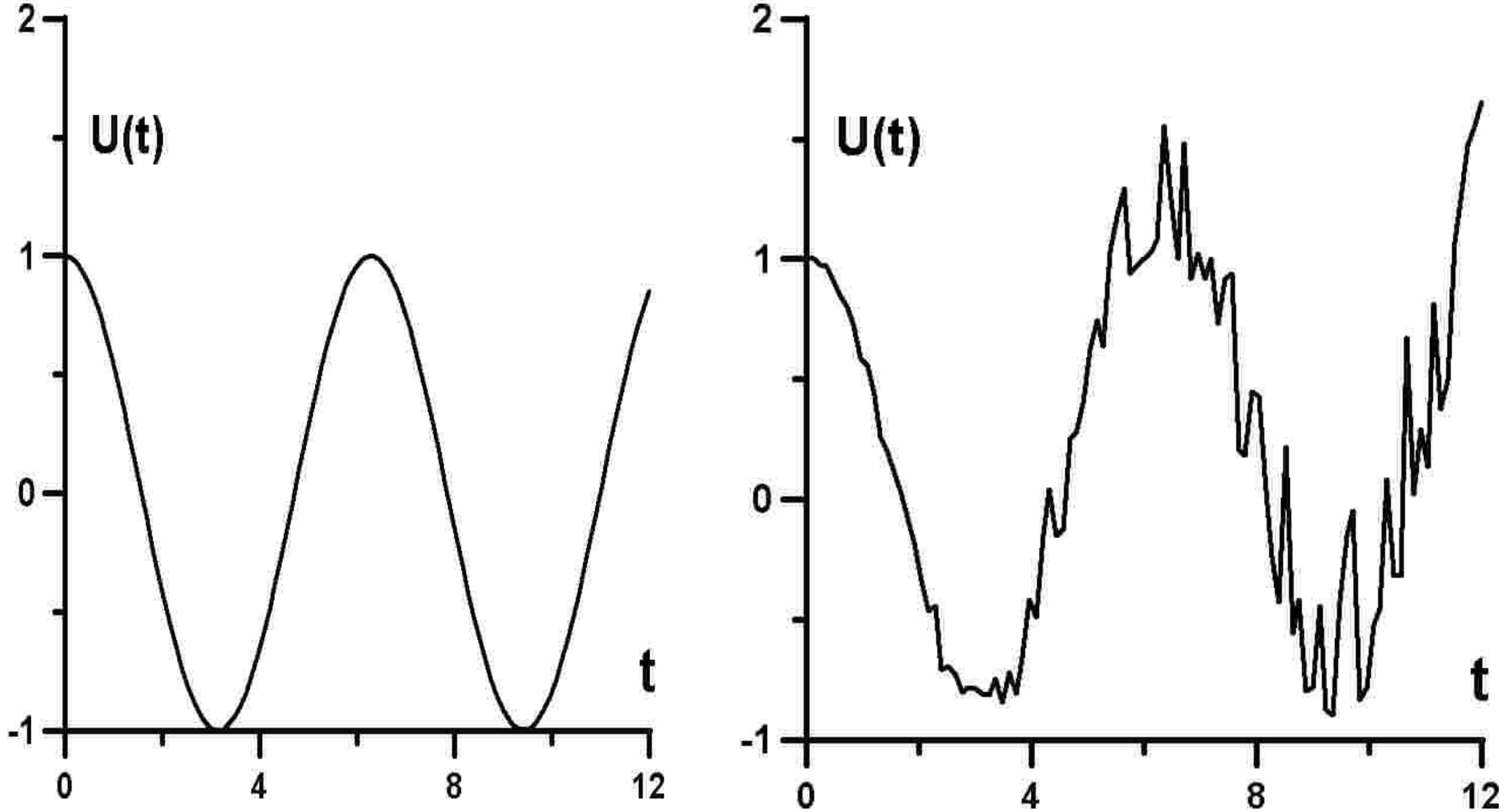
Optimal control problems
Classification 2

9. The variety of criteria
10. Incorrectness of the problem
11. Inadequacy of the model
12. The absence of an acceptable solution, unmanageable system
13. The phenomenon of “small variations”
14. The phenomenon of “tight phase constraints”
15. Singular measure on exit trajectories on phase constraints
16. The phenomenon of deterministic chaos

Optimal control problems Classification 3

17. Noise in the calculation of gradients
18. Bordered effect
19. Competition of functionals
20. “Defect of the integral functional”
21. The growth of the complexity of the set boundary reachability
22. The presence of uncontrolled subsystems
23. Catastrophic growth of errors
24. NP-difficulty
25. The phenomenon of high-frequency parasitic control disturbances

The phenomenon of high-frequency parasitic control disturbances



Numerical analysis of OCP
Applied logics

- **The logic of presentation** ("best of known"): the presented decision is considered acceptable until someone has produced the best solution
- **The logic of the credibility of Poya**: the confirmation of the investigation makes the reason more plausible
- **The logic of modeling** (the thesis of Hamming): the meaning of calculation is not a number, but an understanding
- **Logic of "last error"** - last error does not exist

Numerical analysis of OCP
Applied logics

- **The logic of accuracy**: a mistake is any inaccurate accuracy
- **Verification logic**: each calculation must be checked at least twice
- **The logic of heuristics**: be prepared for the lack of guarantees ("heuristic methods previously despicable, now respectable")
- **The logic of thrift**: there is always a problem that is considered too long

Numerical analysis of OCP
Applied logics

- **The logic of maximum program control (vs "the logic of ready systems")**: everything that you do not control does not work correctly
- **Logic of problem accumulation**: errors grow with time
- **The logic of refutation (vs the logic of the proof)**: nothing can be proved, but much can be refuted

Numerical analysis of OCP
Applied logics

- **The logic of high-rise ("Logic Brick")**: algorithm - a house of cards, the lower bricks must be strong
- **The logic of the series**: do not save time in the first tasks, it will pay off
- **Occam's logic**: create a system that even a fool can use, and only a fool will want to use it
- …

Numerical analysis of OCP
Applied logics

- **What can you say?** He honestly built the technology and carefully checked the results.

- But they want some kind of guarantees ... **(The logic of reputation comes into effect)**

Numerical analysis of OCP
What is this all for?

- The scope of numerical analysis is inconclusive more than the field of problems available for analytical research

- Using numerical methods, it is possible, in some cases, for **MINUTES** to obtain a solution that can be analytically obtained in **YEARS**

- **Rational methodology of research - first numerical methods, and only then - theory, and not vice versa**

Numerical analysis of OCP

Summery

- **The "logic of action" of the anthropomorphic type works**: the task is like a person, learn how to communicate with it and everything will turn out
- **The main teaching method** is "do as I“
- **Why strive?** - to increase the reliability
- Numerical analysis - walking along the slippery streets: most often you will not fall, although city services had to provide good roads ...

The classical concept of solving applications

1) Mathematician **MUST** understand "Physical sense" of the problem and only then begin to address tasks.

2) You must first create a theory, then taken to solve the problems.

The proposed concept of solving applications

1) Mathematician **DO NOT MUST** to understand "Physical sense" of the problem, and can rely on the expertise and qualified partners of work.

2) It is necessary to try solving the problem by available techniqies, and only then trying to create theory.

Phenomenological approach ru.wikipedia.org

- The formulation of regularities that determine the relationship between various observations of phenomena (effects) in accordance with the fundamental theory, but directly from this theory are not the following.
- In fact, it results from the processing of experiments, the results of which can not yet be described by existing theories.
- Phenomenological theories develop in those cases when the observed phenomena can not be explained by the general laws of nature, either due to the absence of a proper mathematical apparatus, or due to ignorance of the corresponding laws.

The problem of optimal control

Specific features

$$P\left(\frac{\partial f}{\partial x_i} \approx \frac{\partial f}{\partial x_{i+1}}\right) = 0 \qquad f(x) \to \min$$

$$P\left(\frac{\partial I}{\partial u(t_i)} \approx \frac{\partial I}{\partial u(t_{i+1})}\right) = 1 \qquad I(u) \to \min$$

The problem of optimal control
Specific features

- “The thesis of simplification”
- The sequence of solved problems becomes easier as we approach the adequate model

The problem of optimal control
Specific features

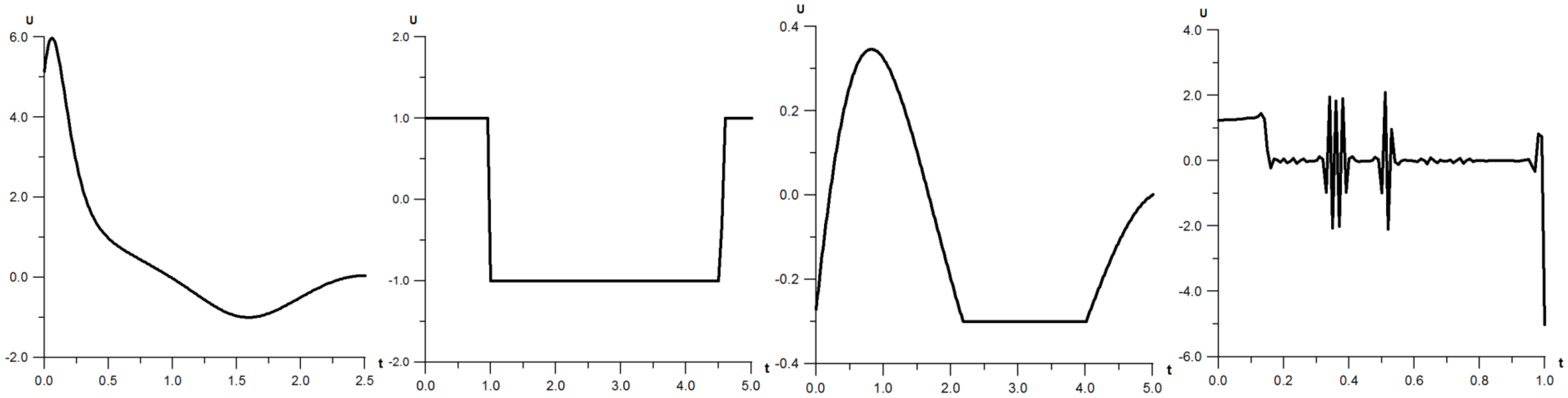
- “The thesis of simplicity”

- The problems of optimal control are simple

- The real computational complexity of the optimal control-function search problem can correspond to the complexity of solving the finite-dimensional optimization problem with a small number of variables (2-5)

The problems of optimal control

Types of optimal solutions



The problem of optimal control
Specific features

- "The thesis of linear growth”

- computing costs

- There are algorithms for most classes of OCPs for which the dependence of the growth of computational costs on the number of nodes of the discretization grid is linear

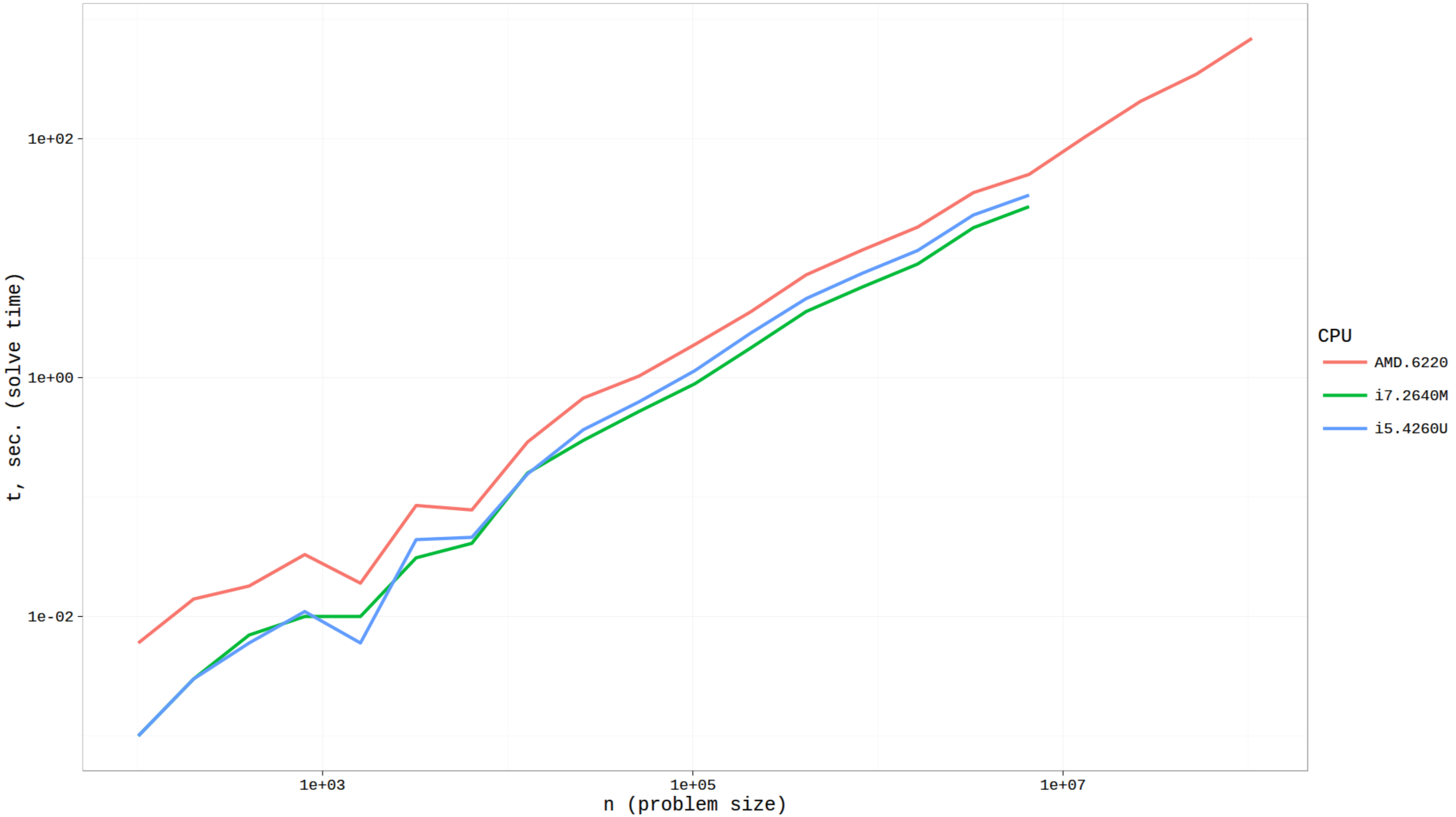
Computational experiment

Pendulum problem, Polyak's method

N	AMD 6220 Linux icc 14.0.0	i7-2640M Linux icc 15.0.1	i7-2640M Linux gcc 4.8.2	i7-2640M Linux clang 3.4.2	i7-2640M Mac OS X gcc 4.8.3	i7-2640M Mac OS X clang 3.4.2	i5 4260U Mac OS X gcc 4.8.3	i5 4260U Mac OS X clang 3.5SVN
101	0.006	0.001	0.006	0.009	0.001	0.001	0.002	0.002
201	0,014	0,003	0,012	0,005	0,003	0,003	0,004	0,004
401	0,018	0,007	0,014	0,014	0,006	0,006	0,009	0,008
801	0,033	0,010	0,023	0,028	0,011	0,010	0,015	0,015
1601	0,019	0,010	0,019	0,014	0,006	0,007	0,008	0,008
3201	0,085	0,031	0,065	0,070	0,044	0,034	0,045	0,046
6401	0,078	0,041	0,099	0,100	0,046	0,051	0,066	0,069
12801	0,289	0,159	0,271	0,311	0,156	0,154	0,220	0,255
25601	0,674	0,298	0,620	0,687	0,366	0,380	0,500	0,495
51201	1,030	0,521	1,060	1,234	0,627	0,613	0,870	0,942
102401	1,890	0,887	1,799	2,023	1,142	1,160	1,505	2,100
204801	3,546	1,765	4,415	4,015	2,352	2,319	2,866	3,044
409601	7,251	3,576	7,133	8,034	4,588	4,511	5,509	5,938
819201	11,662	5,714	11,608	12,883	7,447	7,315	8,752	8,682
1638401	18,197	8,929	18,203	20,443	11,588	11,381	13,547	13,313
3276801	35,353	17,975	35,947	40,622	22,972	25,862	27,110	26,504
6553601	50,279	26,991	52,610	59,321	33,680	33,090	39,626	39,432
13107201	103,237							
26214401	205,603							
52428801	346,577							
104857601	692,295							

Computational experiment

Pendulum problem, Polyak's method



The problem of optimal control Specific features

- "The thesis of asymmetry of relays is smooth“

- Varying smooth controls to build a good approximation of relay control **EASY**
- Varying the relay controls to build a good approximation of smooth control **VERY HARD**

Optimal control problems
Criteria for the quality of algorithms

"Three rules" of a good algorithm

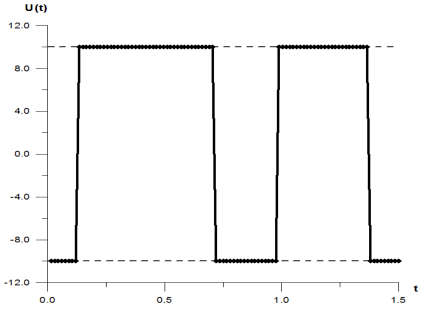
- The algorithm must maintain the superlinear convergence rate
- The algorithm must adequately handle the direct constraints
- The algorithm must preserve the smoothness of the variations

Arsenal funds
Reductions

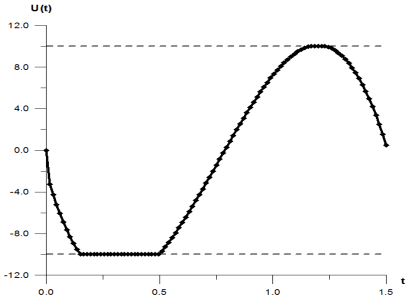
- Reduction to an equivalent problem
- Reduction to the problems sequence
- Non-equivalent reductions

Arsenal of funds
Discretization of OCP

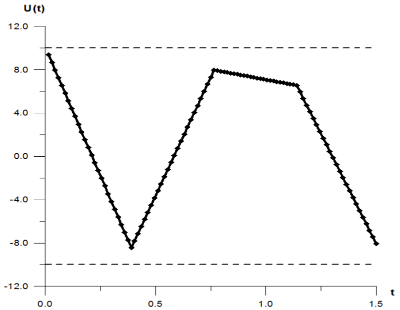
- Algorithm for constructing relay functions;



- Algorithm for constructing spline functions;



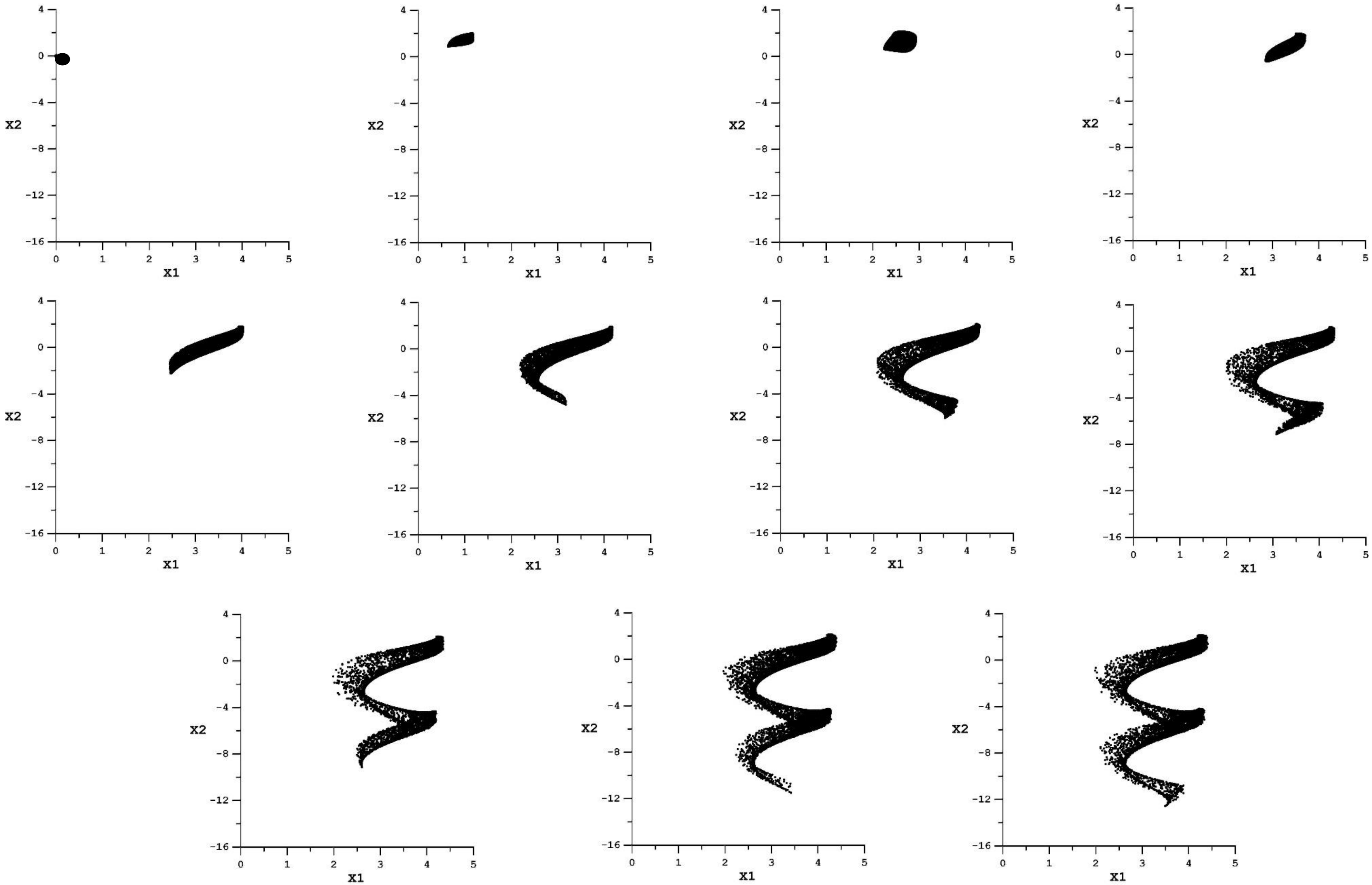
- Algorithm for constructing piecewise linear functions.



Arsenal of funds
Phase estimation

- Algorithms for the approximation of attainability set
- “Method of grids”
- Methods of stochastic approximation

Reachable set. Example 1



Reachable set. Example 1

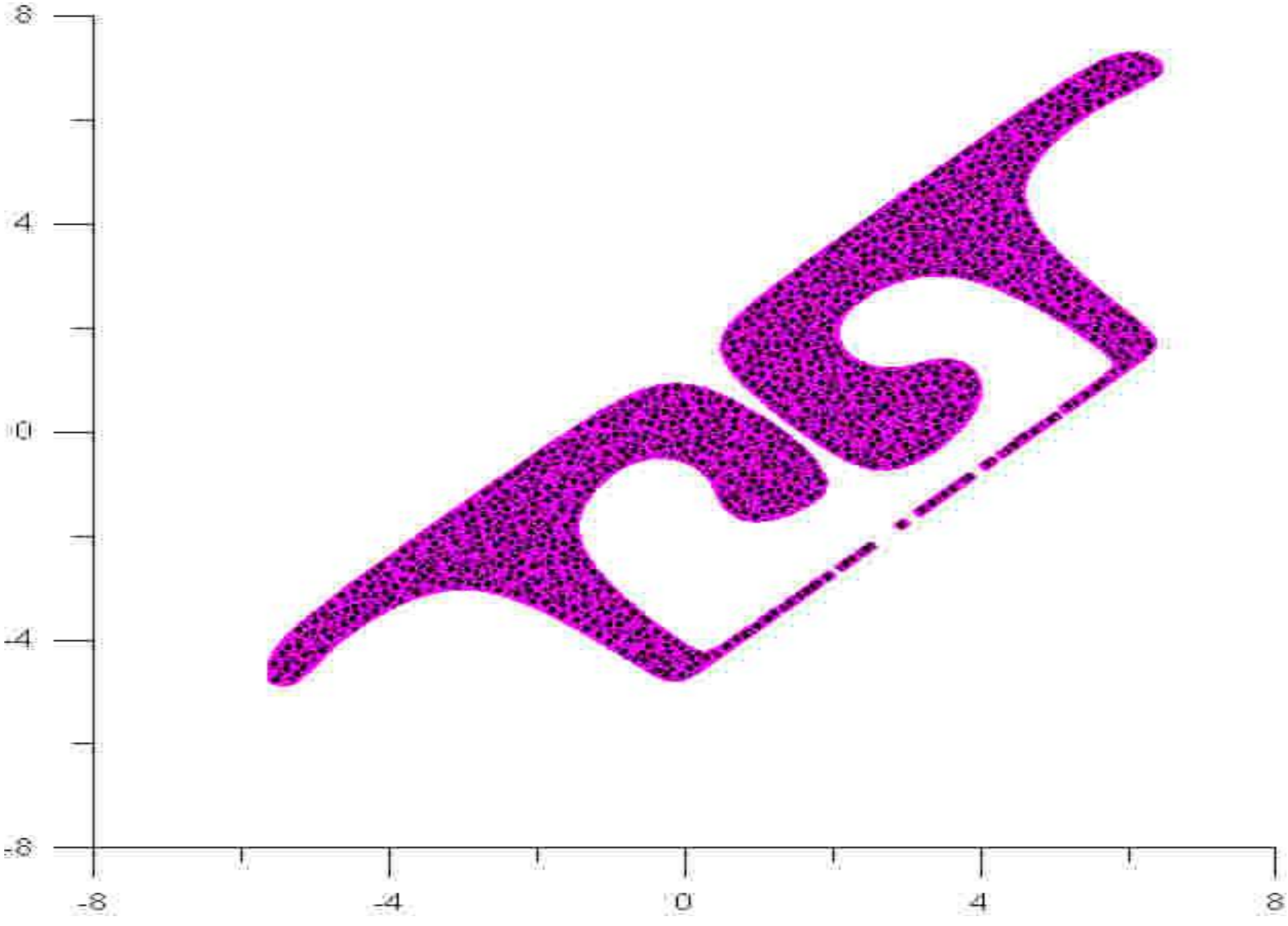
$$\dot{x}_1 = \sin(x_1) + \sin(x_2)$$

$$\dot{x}_2 = \cos(x_1) + \cos(x_2) + u$$

$$x_0 = (0,0) \qquad | \; u(t) \; | \le 1.0$$

$$t \in [0, \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}]$$

Reachable set. Example 2 “two sheeps”



Reachable set. Example 2 “two sheeps”

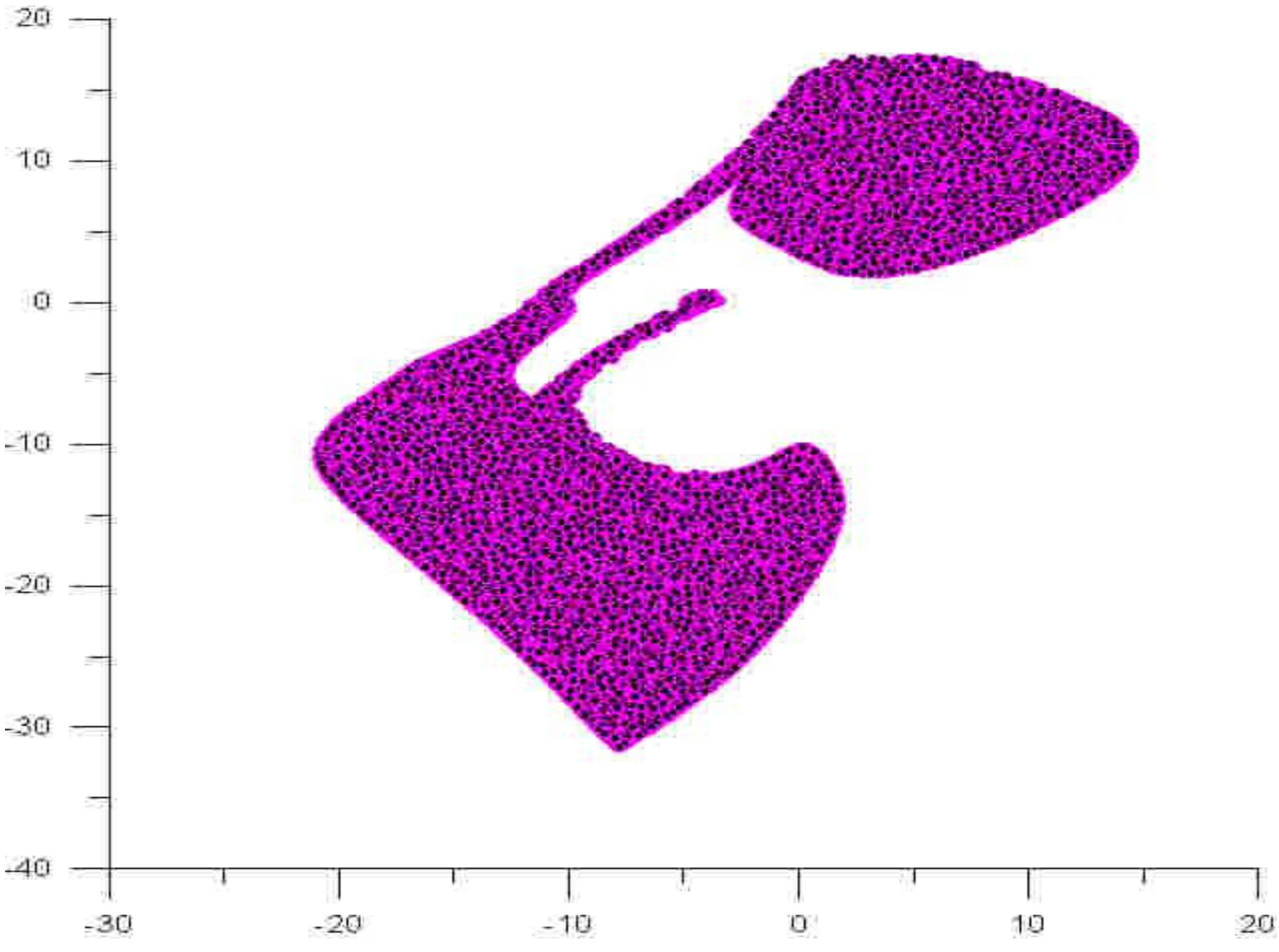
$$\dot{x}_1 = u + \cos x_2$$

$$\dot{x}_2 = u - \sin x_1$$

$$x(0) = (0.8, 1) \qquad |u| \le 0.2$$

$$t \in [0, 10]$$

Reachable set. Example 3 “hatchet”



Reachable set. Example 3 “hatchet”

$$\dot{x}_1 = u - \sin\sqrt{|x_2|}$$

$$\dot{x}_2 = u + \cos\sqrt{|x_1|}$$

$$x(0) = (1,1) \qquad |u| \le 0.45$$

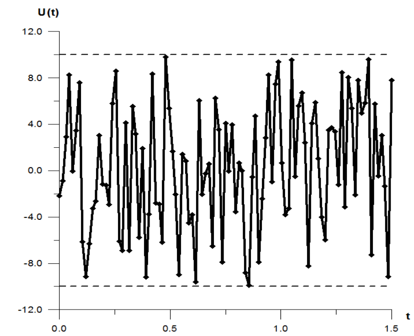
$$t \in [0, 27]$$

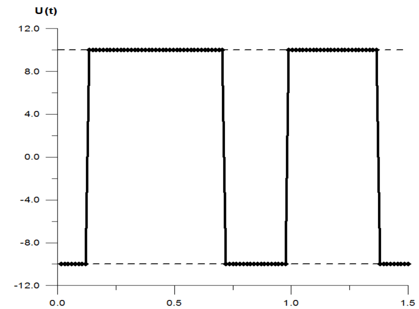
Arsenal of means
Control Sampling Schemes

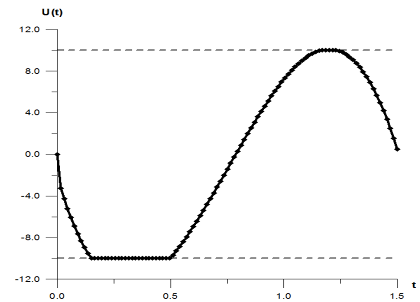
- Table Approximations
- Relay Approximations
- Piecewise-linear approximations
- Spline-approximation
- “Controlled” splines
- …

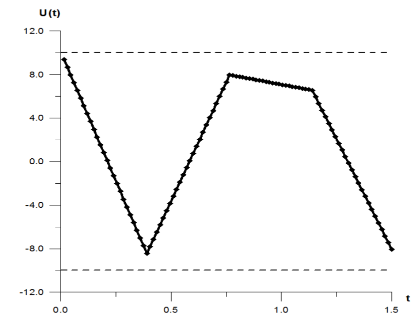
Algorithms for generation of random controls

- Algorithm for constructing random table functions
- Algorithm for constructing random relay functions
- Algorithm for constructing random spline functions
- Algorithm for constructing random piecewise-linear functions









Arsenal of means
Resampling

- Uneven Sample Sets
- Thickening is a rarefied grid
- Verification of the quality of discretization according to the Runge rule

Arsenal of means
Methods for estimating gradients

- Analytic formulas for the conjugate system
- Analytical formulas for available gradients
- Finite-difference schemes
- Fast automatic differentiation

Arsenal of means
Multimethod technologies

- Gradient methods
- Methods based on the maximum principle
- Globalizing methods
- Search methods
- Methods - "closers"

Arsenal of means
Multi-Mode

- Dialog mode
- Batch mode
- Macrointerpreter mode

Arsenal of means Visualization

Test problems 02

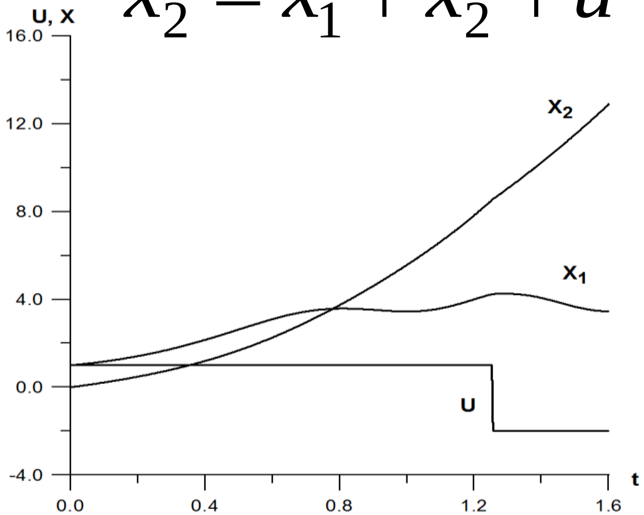
$$\dot{x}_1 = \frac{1}{\cos x_1 + 2} + 3\sin x_2 + u$$

$$\dot{x}_2 = x_1 + x_2 + u$$

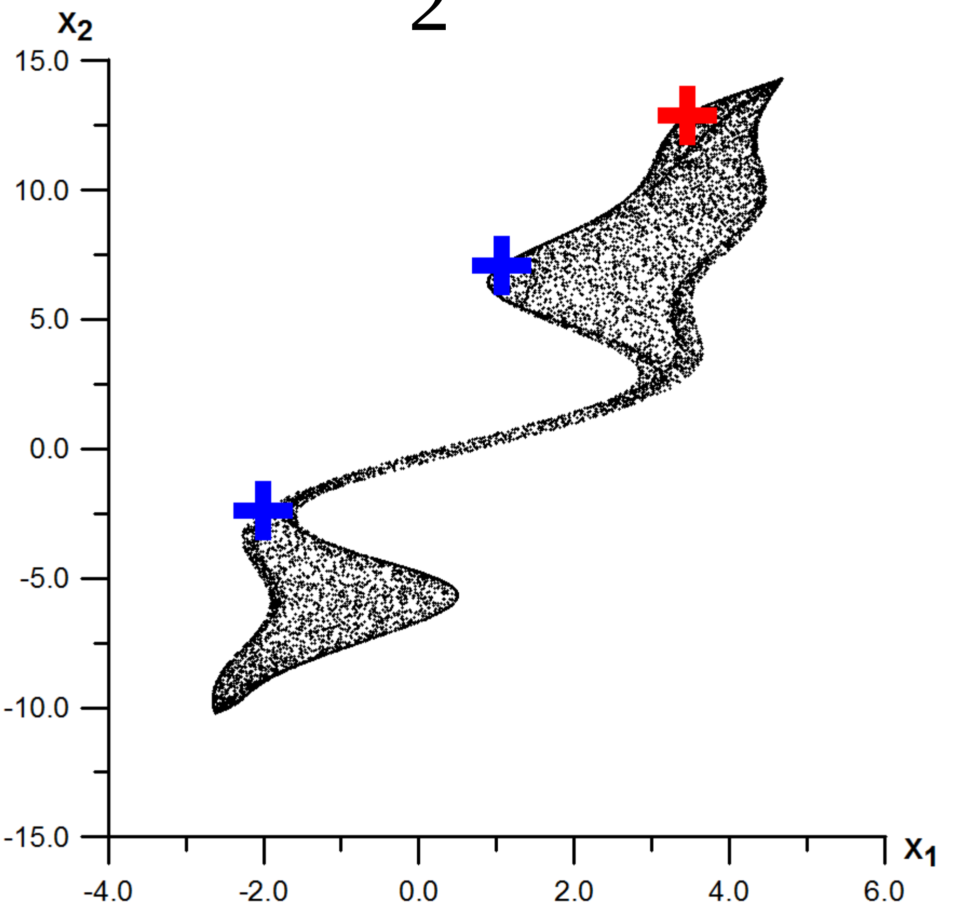
$$x(0) = (1,0) \quad u(t) \in [-2, 1]$$

$$t \in [0, 1.6]$$

$$I(u) = -x_1(t_1) + \frac{1}{2} x_2(t_1) \rightarrow \min$$



It is found 3 extrema for PC time 381 s.



N	Functional values	Extreme points	
		x_1	x_2
1	-2.980856	3.46114	12.88400
2	-2.472074	1.06257	7.06928
3	-0.81988	-2.00836	-2.37696

Arsenal of means
Transformation of Herneth-Valentine

$$|u(t)| \leq 1 \qquad u(t) = \sin(v(t))$$

$$\frac{\partial I}{\partial v} = \frac{\partial I}{\partial u}\cos(v(t))$$

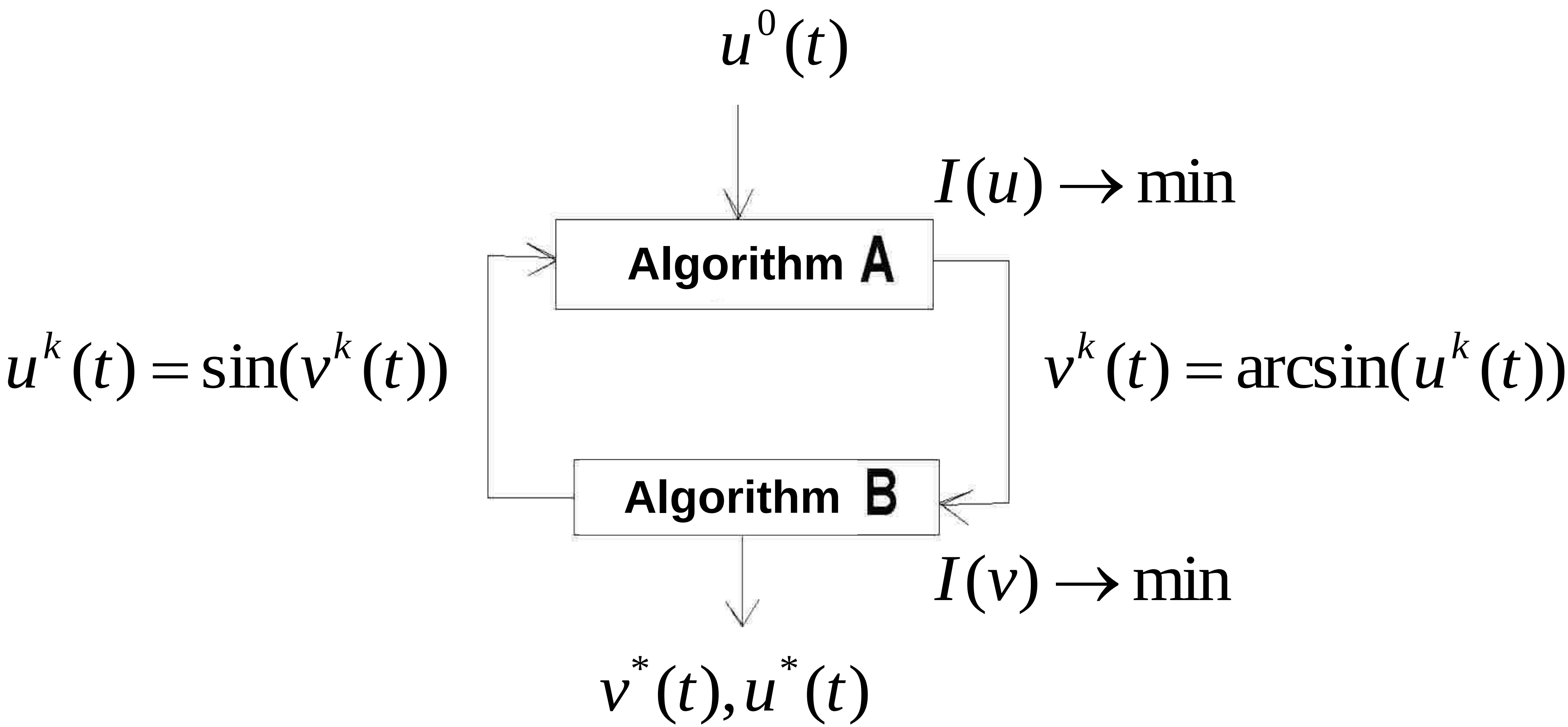
$$u(t) \to 1 \Rightarrow \frac{\partial I}{\partial v} \to 0$$

Gernet Nadezhda Nikolaevna (18.04.1877 - 1943)

Gernet N.N. On a simple problem of the calculus of variations. St. Petersburg, type. Yu.N.Erlikh, 1913.

Arsenal of funds

The scheme of the algorithm on the Hernet-Valentine transformation



Arsenal of means
Pre-optimization analysis

Purpose: to study the properties of an optimized functional in the entire "variable space"

- Verification of the program statement of the problem
- Estimating the cost of solving a Cauchy problem
- Estimation of the quality of gradient approximation algorithms
- An estimate of the degree of convexity of the functional
- Estimation of the Lipschitz constants of the functional and its gradient
- Evaluation of the quality of sampling and integration

Arsenal of means

Post-optimization analysis

Purpose: to study the properties of an optimized functional in the neighborhood of an record solution

- System Quality Discretization Study
- Verification of integration accuracy
- Estimation of the accuracy of the necessary optimality condition
- "Coordinate" Estimation of Derivatives of a Functional of High Degrees
- Assessment of the level of gully extremum
- Estimating the sensitivity of the extremum

Arsenal of means
Methods without gradients. Closers

- Method of coordinate descent
- The Powell-Brent method
- Parthan method
- Hook-Jeeves Method
- …

Arsenal of means. The method of continuation with respect to a parameter (homotopy)

- Parameterization of the right parts
- Parameterization of the initial state
- Parameterization of time
- Parameterization of direct constraints
- Reduction to a sequence of extremal problems using the solution of the previous one as the initial approximation in the following

Arsenal of means
Decomposition

- Splitting the set of optimized variables into two subsets and sequential optimization by subsets
- Split the time interval into sub-intervals
- …

Arsenal of means
Limitation of variations

- Restriction of variation

$$u^k(t) + \delta u^k(t), \quad \left\| \delta u^k(t) \right\| \le \Delta$$

- Parameterization of direct constraints
- Proxy-method
- …

Arsenal of means. Immersion ("expansion principle")

- Removing or weakening (temporary) part of the restrictions
- Expansion of the set of permissible controls and penalty for exceeding the permissible limits
- …

Arsenal of means. Accounting for box restrictions

- Constructive accounting of parallelepiped constraints
- Method of gradient projection
- Gernet-Valentine's Methodology
- …

Arsenal of means. Accounting for terminal restrictions

- Method of external penalty functions
- The method of the modified Lagrange function
- Linearization method
- The method of exact differentiable penalty functions

Arsenal of means. Accounting for phase and mixed constraints

- Method of external penalty functions
- The method of the modified Lagrange function
- The reduced gradient method
- The method of parameterization of constraints

Classification of nonconvex OCPs

- Low extremal
- Medium-extreme
- Multiextreme

Test problem 03

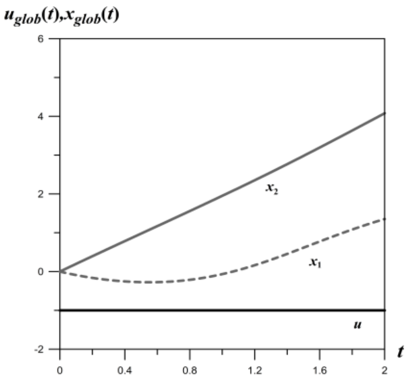
$$\dot{x}_1 = x_2 - x_1^2 + u_1$$

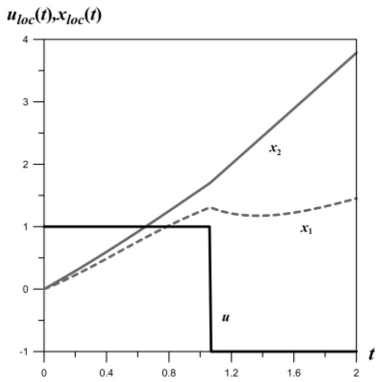
$$\dot{x}_2 = \sqrt{3 - u_1 + \sin x_1}$$

$$I(u) = x_1(t_1) \to \min$$

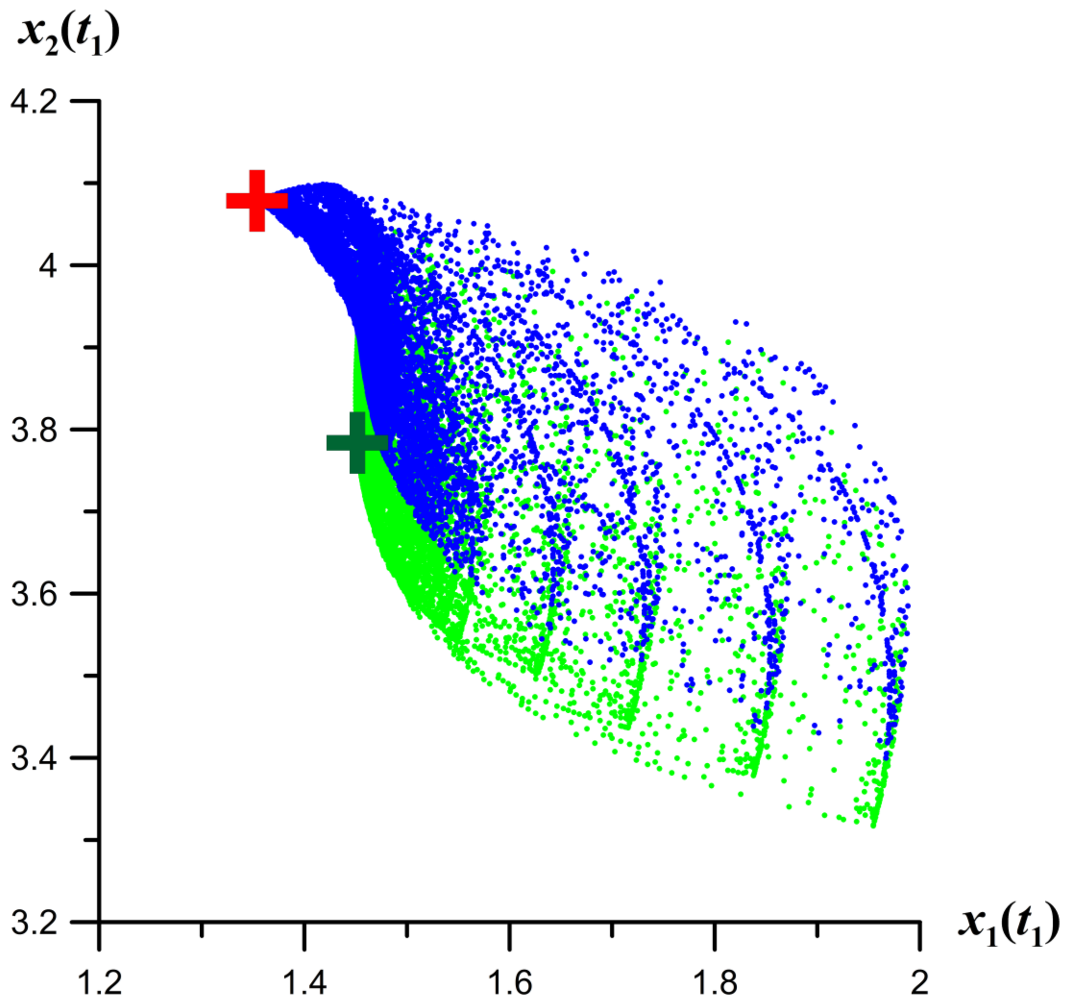
$$x_1(t_0) = x_2(t_0) = 0, \ |u| \le 1, \ t \in [0,2]$$

It is found 2 extrema for the total processor time: 5481 s.





N	Functional value	Extreme points		V	t
		X1	X2		
1	1.35391	1.3539	4.0785	0.525	7
2	1.45163	1.4516	3.7838	0.475	15

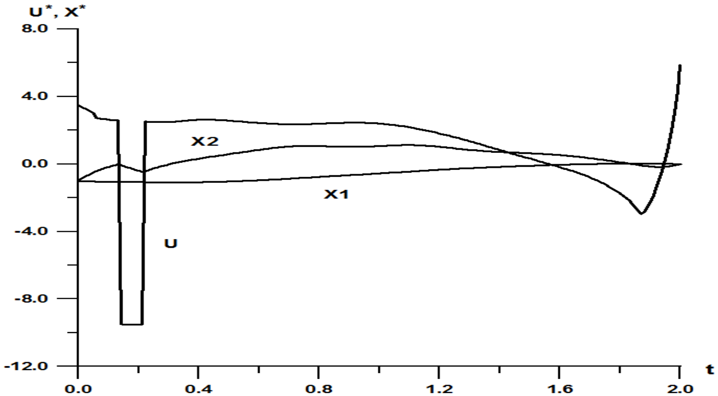


Test problem 04

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -2x_1 - 3x_2 - \frac{2}{\pi}\arctan(5x_2)\cdot e^{-x_2} + u_1 - \sin(15x_1)$$

$$x(0) = (-1,-1), |u| \le 10,\ t \in T = [0,2]$$

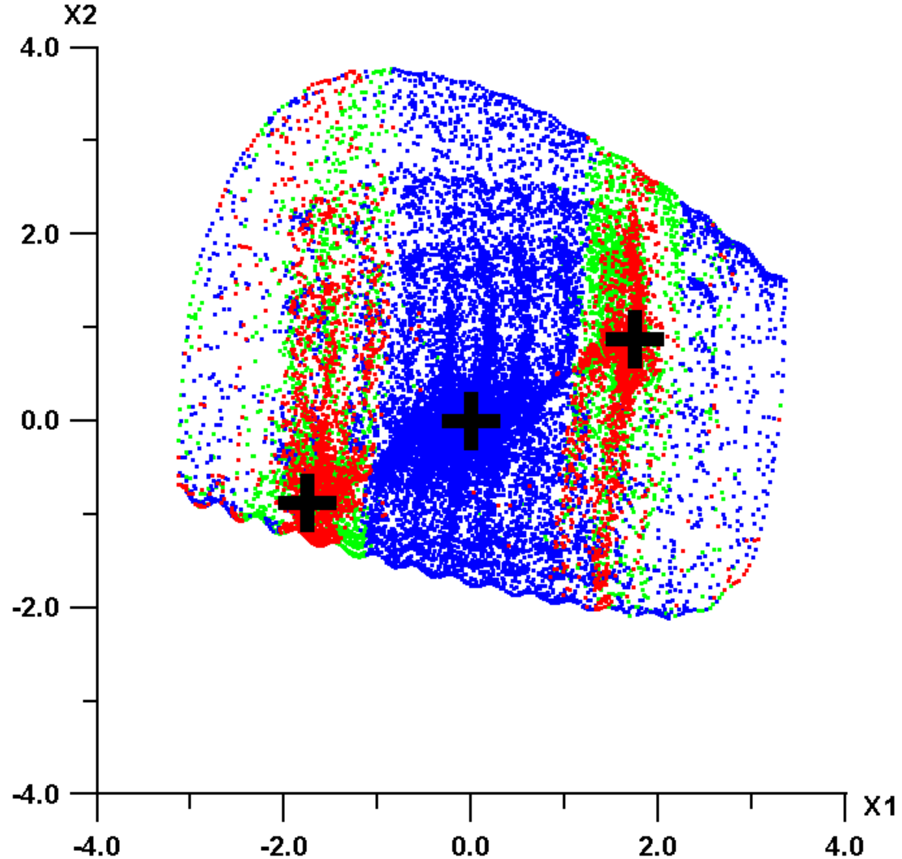
$$I(u) = 2x_1^2 - 1.05x_1^4 + 1/6\,x^6 - x_1x_2 + x_2^2 + 1 \rightarrow \min$$



Iterations – 5542, time – 3717 s.

It is found 3 extrema:

N	Functional value	Extreme points		V
		X1	X2	
1	1.29864	-1.7476	-0.8738	0.167
2	1.29864	1.74755	0.87378	0.333
3	1.0	0.00002	0.00001	0.500



Arsenal of means
Curvilinear search

Test problem 05

$$\dot{x}_1 = x_2$$

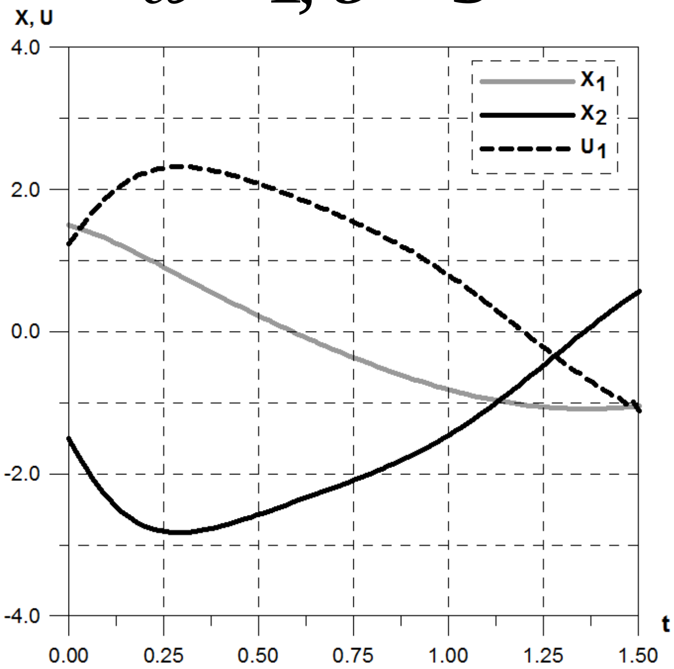
$$\dot{x}_2 = -\omega^2 x_1 - \varepsilon x_1^3 + u_1$$

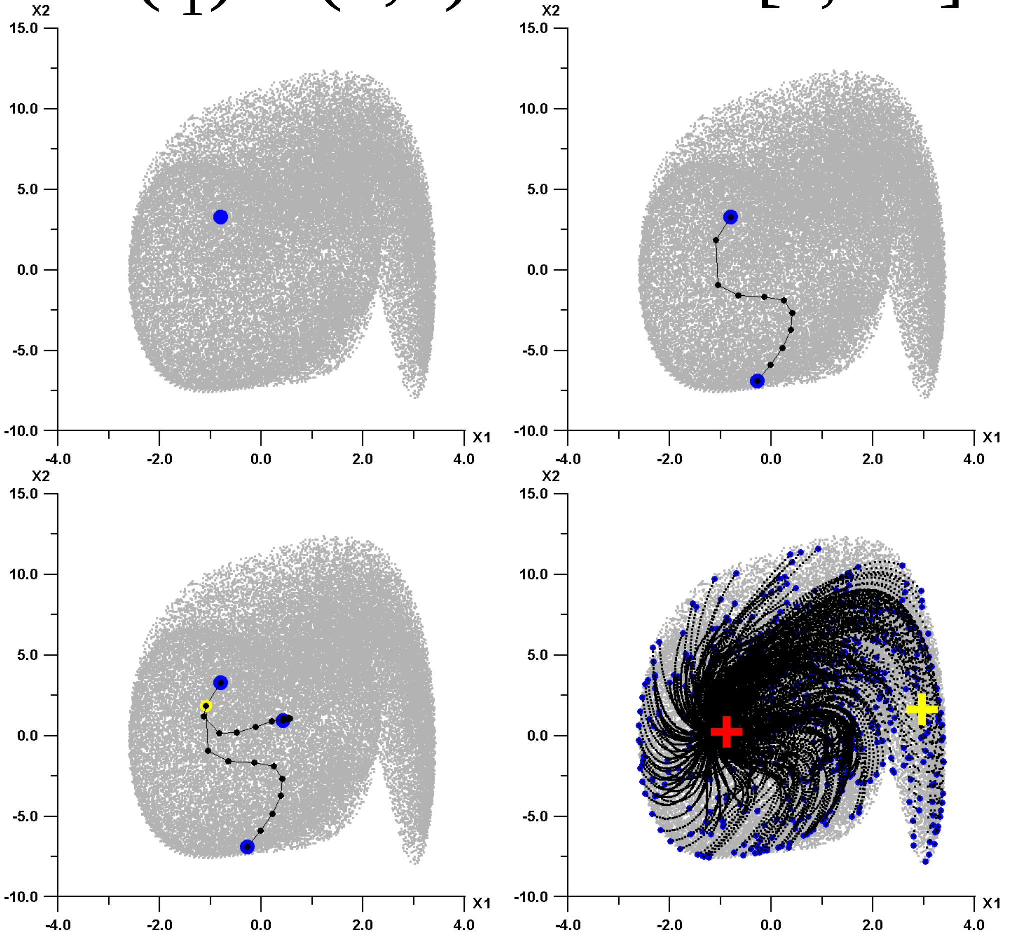
$$I(u) = 0.5\int_{t_0}^{t_1} u^2 dt \to \min$$

$$\omega = 1, \varepsilon = 3$$

$$x(0) = (1.5, -1.5) \qquad |u_1(t)| \le 10$$

$$x(t_1) = (0,0) \qquad t \in [0, 1.5]$$





Curvilinear search algorithm

Linear variation of control

$$u^k(\alpha,t) = \alpha\left(\bar{u}^1(t) - u_{rec}(t)\right) + u_{rec}(t), \quad \alpha \in [0,1]$$

Quadratic variation of control

$$u^k(\alpha,t) = \alpha^2\left(\frac{\bar{u}^1(t) + \bar{u}^2(t)}{2} - u_{rec}(t)\right) + \alpha\frac{\bar{u}^2(t) - \bar{u}^1(t)}{2} + u_{rec}(t), \quad \alpha \in [-1,1]$$

Cubic variation of control

$$u^k(\alpha,t) = \alpha^3\left(\frac{-\bar{u}^1(t) - 3\bar{u}^2(t) + \bar{u}^3(t)}{6} + 0.5\cdot u_{rec}(t)\right) + \alpha^2\left(\frac{\bar{u}^1(t) + u^2(t)}{2} - u_{rec}(t)\right) +$$

$$+\alpha\left(\frac{-2\bar{u}^1(t) + 6\bar{u}^2(t) - \bar{u}^3(t)}{6} - 0.5\cdot u_{rec}(t)\right) + u_{rec}(t), \quad \alpha \in [-1,2]$$

$u^k(\alpha,t)$ is projected onto the allowable area:

If $u^k(\alpha,t) < \underline{u}$, it is $u^k(\alpha,t) = \underline{u},\ t \in T = [t_0, t_1]$

If $u^k(\alpha,t) > \bar{u}$, it is $u^k(\alpha,t) = \bar{u},\ t \in T = [t_0, t_1]$

Arsenal of means

Expansion

The method of Expantion by Gamkrelidze

$$\dot{y} = \sum_{j=1}^{l} \alpha_j(t) f(y(t), v_j(t), t)$$

$$\alpha_j(t) \geq 0, \quad j = \overline{1,l} \qquad \sum_{j=1}^{l} \alpha_j(t) = 1,$$

$$v_j(t) \in U, \quad j = \overline{1,l} \qquad t \in T$$

$$I(u) = \varphi(x(t_1)) \to \min$$

Test problems 06

$$\dot{x}_1 = x_2$$

$$x_1(t_0) = 5,\ x_2(t_0) = 0,\quad u \in [-1,1],\ t \in [0,7]$$

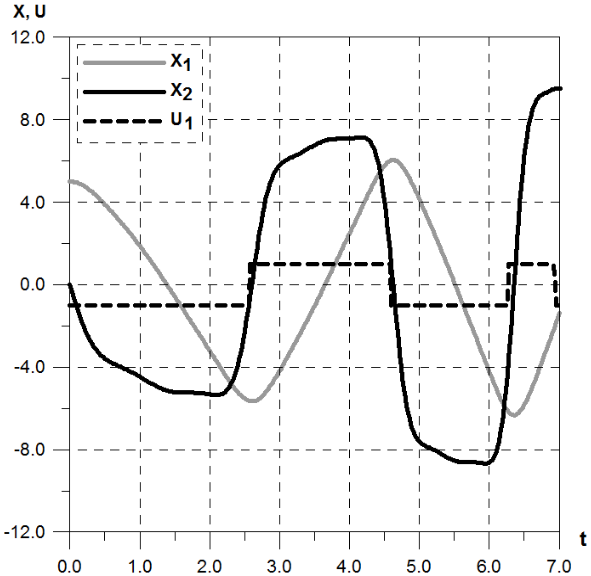
$$\dot{x}_2 = u_1 - x_1 + \frac{x_1^3}{6} - \frac{x_1^5}{120}$$

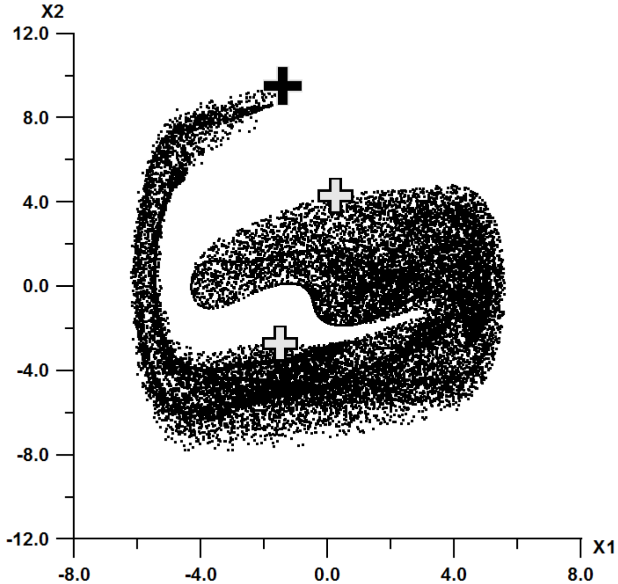
$$I(u) = (x_1 + 1)^2 + (x_2 - 9.5)^2 \to \min$$

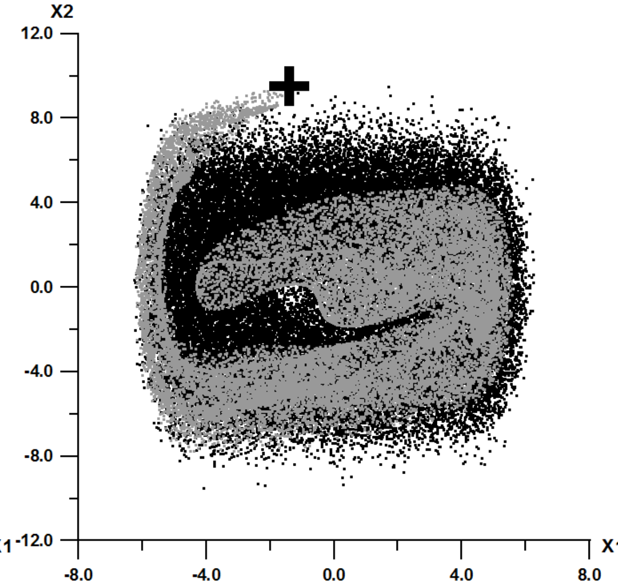
Optimal trajectories and control

A reachable set and extreme points

The convex reachable set and the global extremum







Arsenal of means

Stochastic coverage of reachable set

Test problems 07

$$\dot{x}_1 = e^{\sin x_2}$$

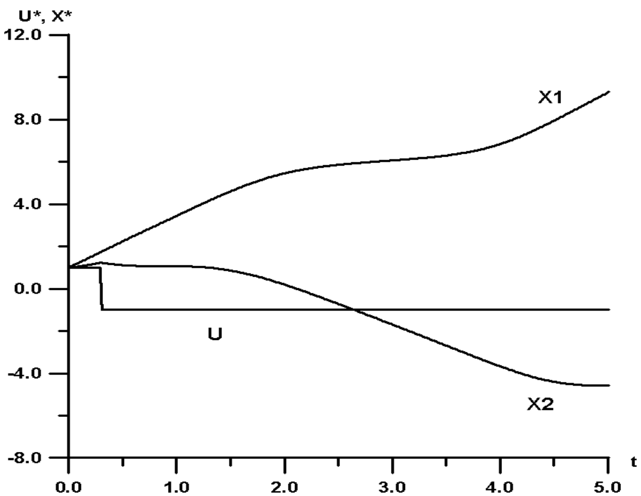
$$\dot{x}_2 = u - \cos x_1$$

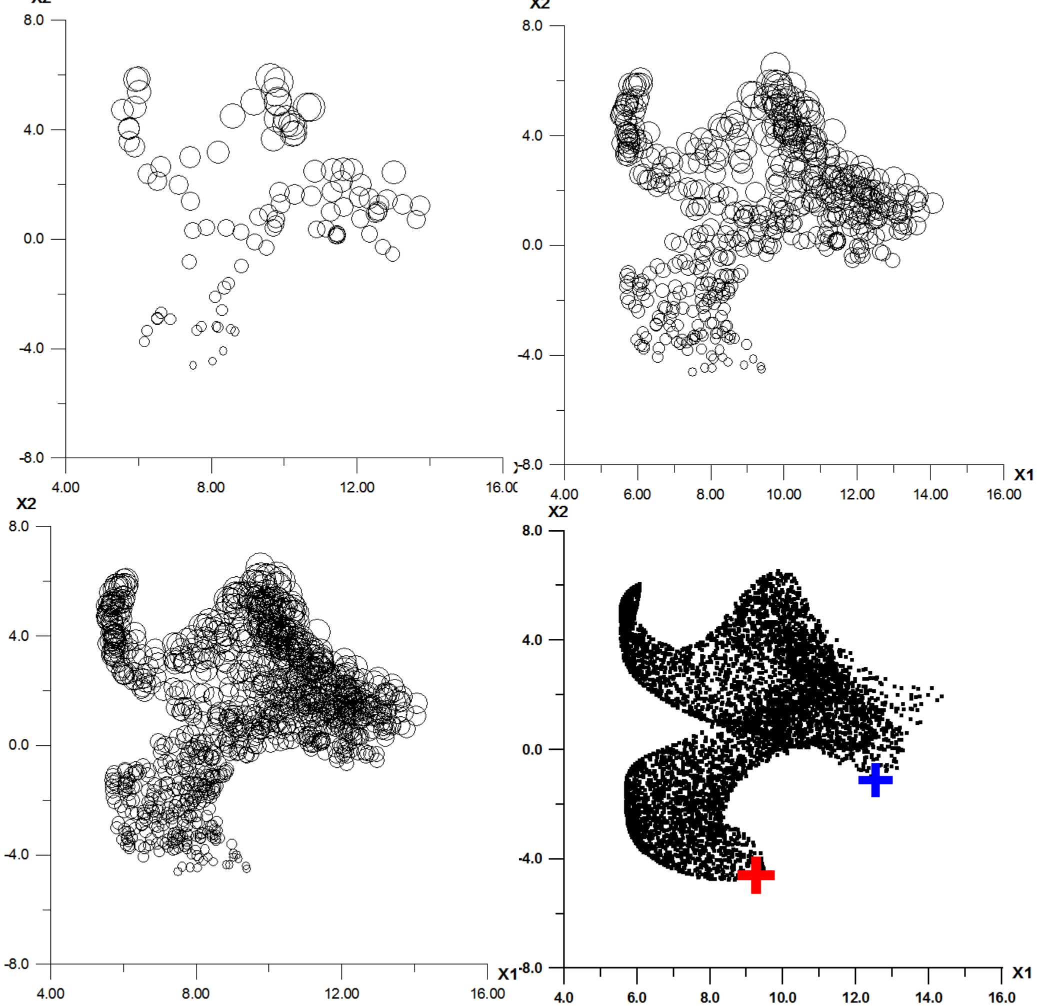
$$x(0) = (1, 1)$$

$$|u_1(t)| \le 1, \quad t \in [0, 5]$$

$$I(u) = x_1(t_1) \cdot x_2(t_1) \to \min$$

N	Functional	V
1	-42.46964	0.992
2	-13.98914	0.08





Algorithm of random coverings

Radius of the ball

$$R^j = (I^j - I_{REC} + \varepsilon_\varphi)/(K_s \cdot L), \quad j = \overline{1, M}$$

Estimation of the Lipschitz constant

$$L^j = K_s \cdot \left| I^j - I^i \right| / \left\| x^j(t_1) - x^i(t_1) \right\|$$

$$j = \overline{1, M} \qquad i = \overline{1, M}$$

Arsenal of means
Pontryagin's method

Pontryagin's method

$$\dot{x} = f(x, \bar{u}(\alpha, t), t) \qquad x(t_0) = x^0$$

$$\dot{\psi} = \frac{\partial H(\psi, x, \bar{u}(\alpha, t), t)}{\partial x} \qquad \psi(t_0) = \psi^0(\alpha)$$

$$\bar{u}(\alpha, t) = \arg\max H(\psi(t), x(t), v, t)$$

$$u_l \le v \le u_g \qquad t \in [t_0, t_1]$$

$$I_0(u(\alpha)) \to \min, \quad \alpha \in [0, 2\pi]$$

Pontryagin's method

$$n = 2$$

$$\psi^1(t_0) = \sin(\alpha)$$
$$\psi^2(t_0) = \cos(\alpha)$$

$$I_0(u(\alpha)) \to \min, \quad \alpha \in [0, 2\pi]$$

$$n = 3$$

$$\psi^1(t_0) = \sin(\alpha_1)\cos(\alpha_2)$$
$$\psi^2(t_0) = \cos(\alpha_1)\cos(\alpha_2)$$
$$\psi^3(t_0) = \sin(\alpha_2)$$

$$I_0(u(\alpha_1, \alpha_2)) \to \min$$

$$\alpha_1 \in [0, 2\pi] \quad \alpha_2 \in [0, \pi]$$

Test problems 08

$$\dot{x}_1 = x_2 + \cos x_1 - x_2 u$$

$$\dot{x}_2 = u - x_1 \sin x_2$$

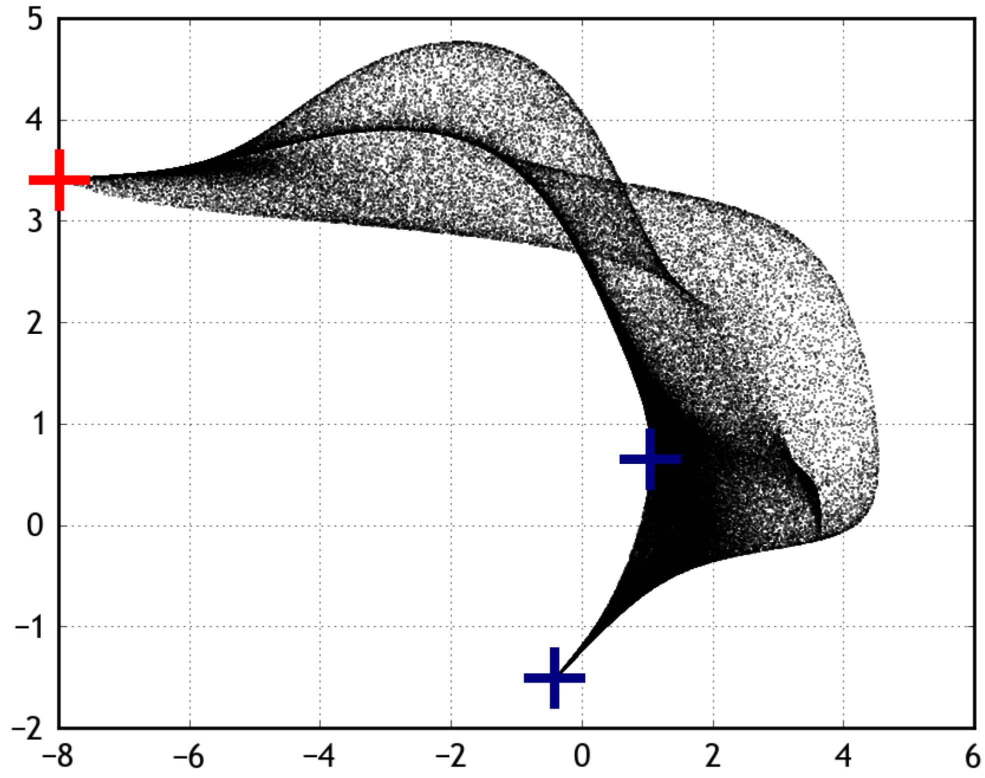
$$I(u) = x_1(t_1) \to \min$$

$$x(0) = (0,1) \quad t \in [0, 3]$$

$$u(t) \in [-1, 2]$$

3 extrema found
for CPU time 782 s.

N	Functional value	Extreme points	
		x_1	x_2
1	-0.372071	-0.372071	-1.49005
2	-7.969326	-7.969326	3.411416
3	1.056718	1.056718	0.690339



Open problems
The problem of automatic solution

- Development of an adaptive algorithm capable in automatic (not automated!) Mode to reliably find local extremums

- The probability of success in the automatic mode is 20%, in the interactive mode - about 100%

Open problems
The problem of multi-extremality

- The solution of "medium-extremal" problems
- Solving problems with "narrow" global extrema
- …

Open problems
The problem of degeneracy

- The degeneracy problem is also acute in finite-dimensional optimization
- The available techniques are associated, as a rule, with numerical-analytical approaches
- The problem of detection of degeneracy
- The problem of developing stable and well-adapted algorithms
- …

Test problems 09

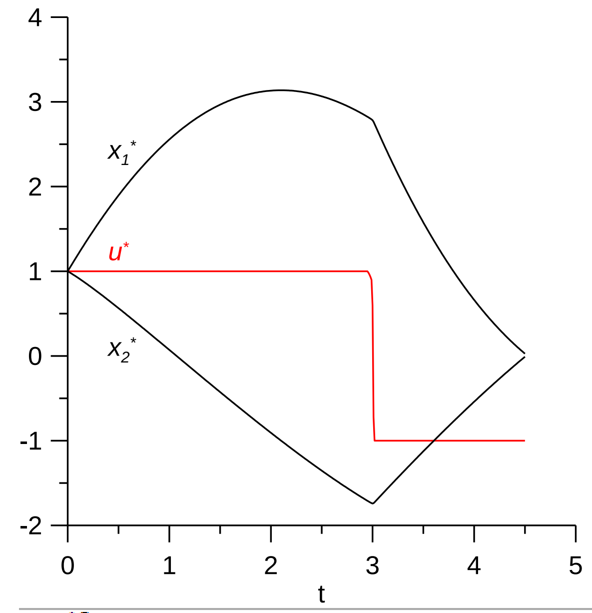
$$\dot{x}_1 = p(x_2 + u);$$

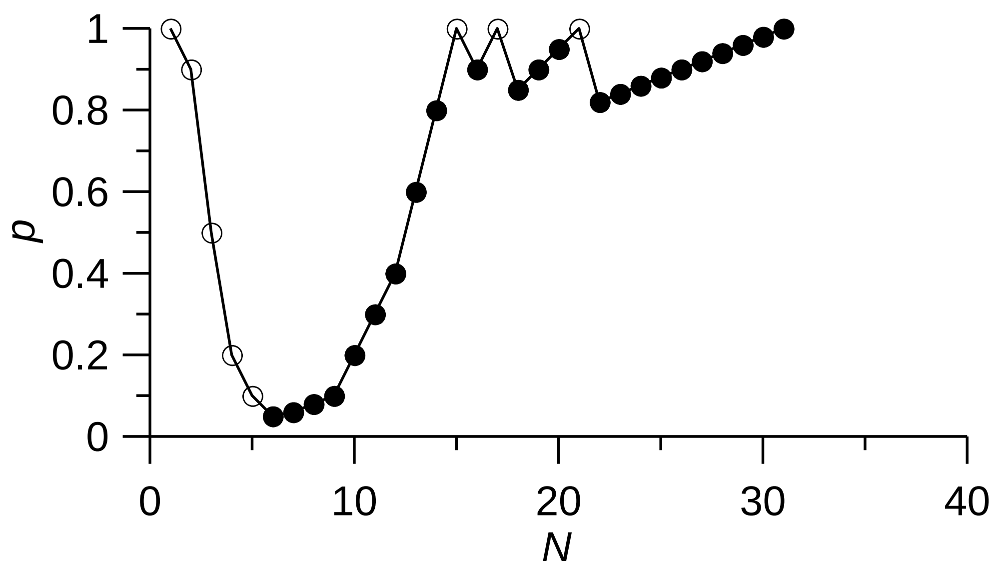
$$\dot{x}_2 = p\left(x_1 - u - \sqrt{x_1^2 - 0.5x_2^2}\right);$$

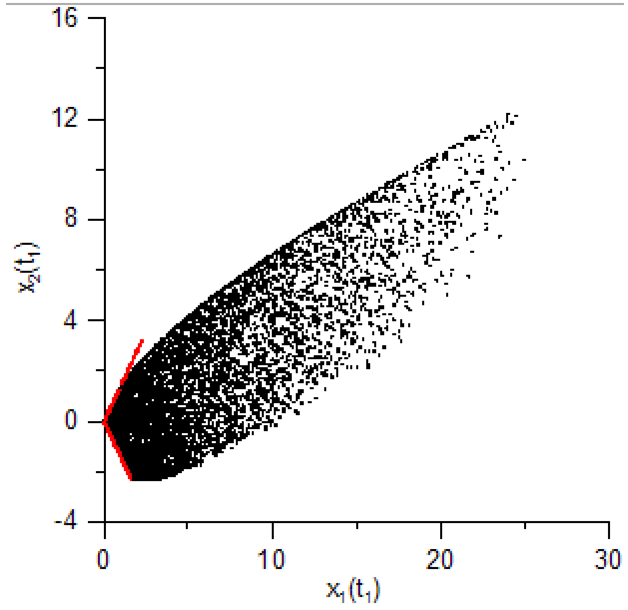
$$x(0) = (1,\ 1),\ t \in [0,\ 4.5];$$

$$x_1^2(4.5) + x_2^2(4.5) \to \min;$$

$$p \in (0,1],\ |u| \le 1,\ u^0(t) = 1.$$







Open problems
The problem of synthesis of optimal control

- Synthesis of optimal control - development of control search methods with feedback
- The problem of creating an extreme control system
- Real-Time Problem

Test problem 01

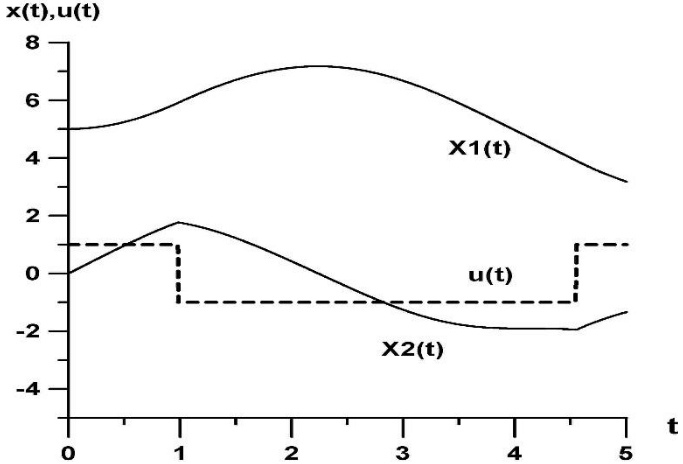
$$\dot{x}_1 = x_2 \qquad x(0) = (5, 0) \quad t \in [0, 5]$$

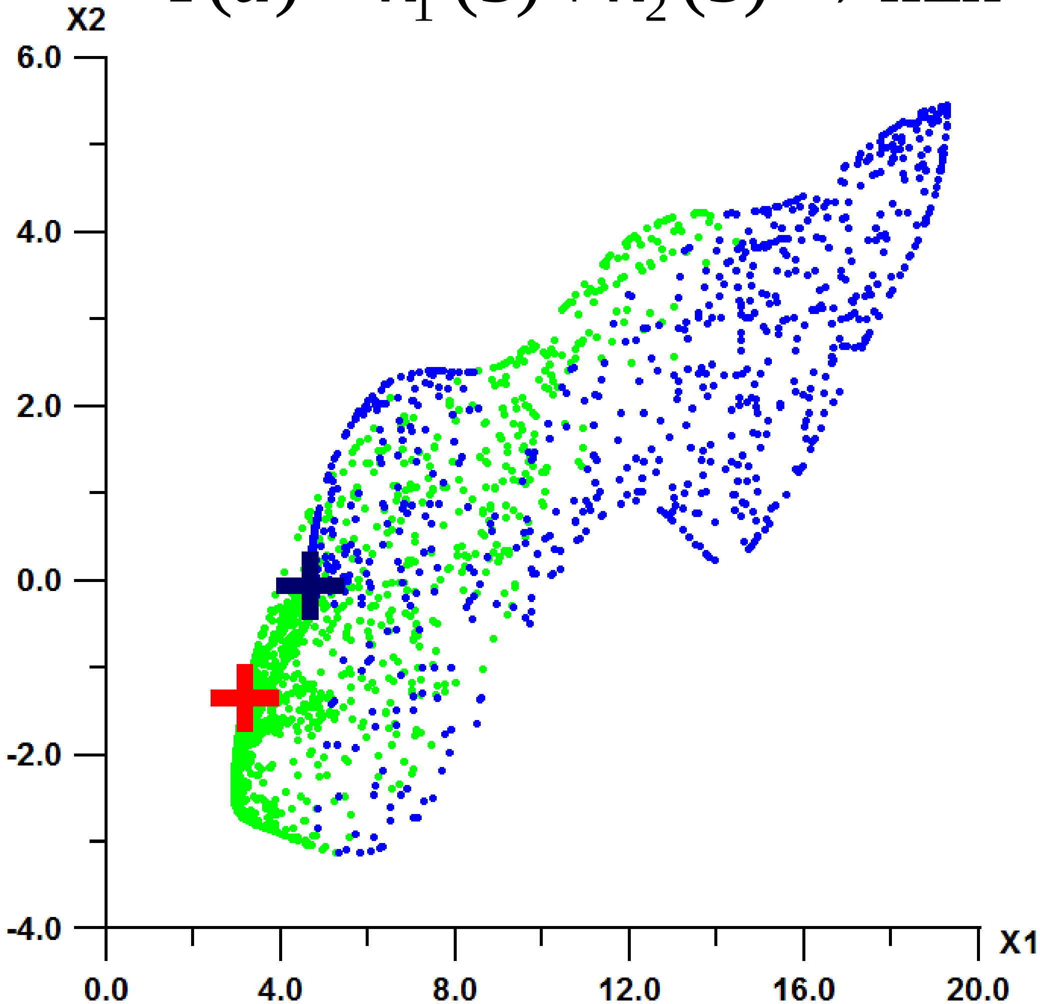
$$\dot{x}_2 = u_1 - \sin x_1$$

$$I(u) = x_1^2(5) + x_2^2(5) \to \min$$

ITERATION 1000
CP TIME 7591 сек.
STOP KRITERIA -216.8
CAUCHY PROBLEMS 186015

N	Functional	Volume
1	1.190817e+01	0.457
2	2.182900e+01	0.543





Open problems
The problem of parallelization

- Using parallel computing systems to find a local extreme

- Parallel integration

- …

Open problems
Control of trajectory beams

- Control in conditions of uncertainty
(in the system both control and perturbation)

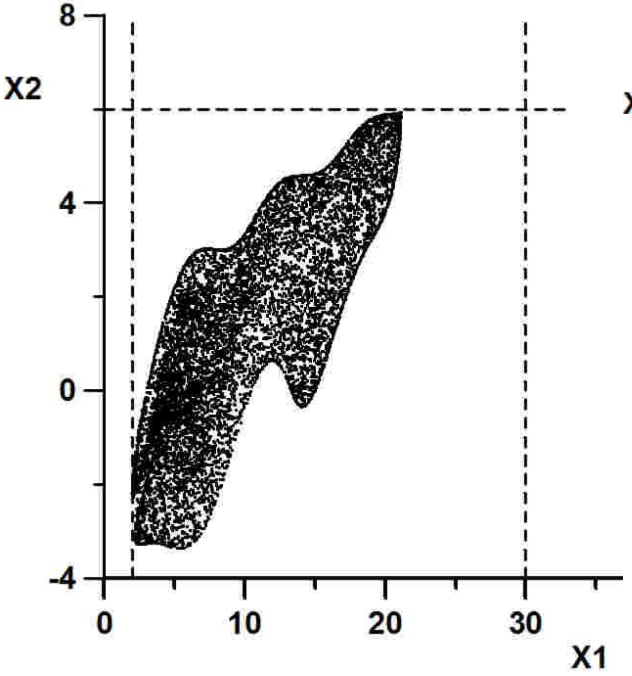
- The problem of formulating the “nuclear problem”

- Example: rationing impacts

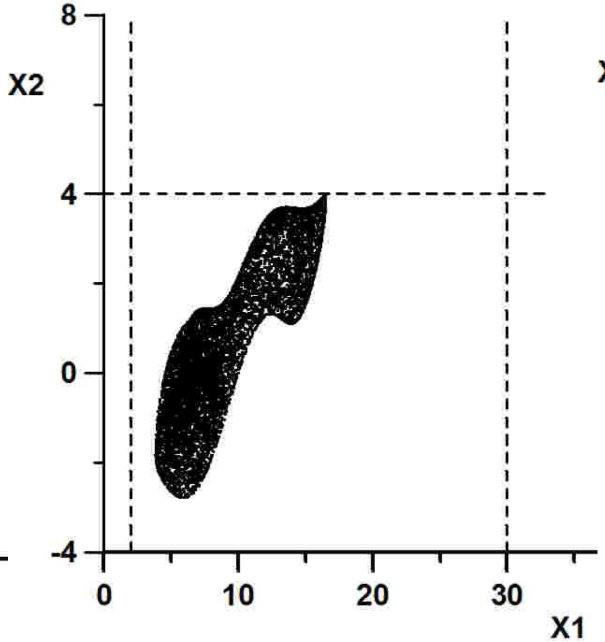
Test problem 09

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u_1 - \sin(x_1)$$

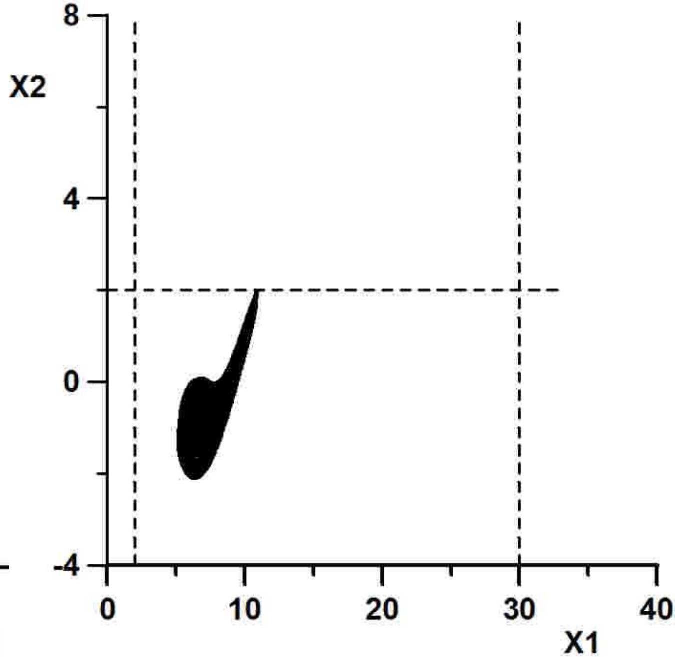
$$x_1(t_0) = 5,\ x_2(t_0) = 0,\quad |u(t)| \le \alpha,\ \ t \in [0,5]$$



$$2 \le x_1(t_0) \le 30$$
$$-4 \le x_2(t_0) \le 6$$
$$|u(t)| \le \alpha^* = 1.145529$$



$$2 \le x_1(t_0) \le 30$$
$$-4 \le x_2(t_0) \le 4$$
$$|u(t)| \le \alpha^* = 0.783166$$



$$2 \le x_1(t_0) \le 30$$
$$-4 \le x_2(t_0) \le 2$$
$$|u(t)| \le \alpha^* = 0.406571$$

The practice of numerical solution of optimal control problems

- two limbs: sight and mind

- optimal control problems
  can be solved

Functional-differential equations

- Systems with deviating argument

- Systems with variable type

- The Ritz method (1909)

The statement
Initial-boundary value problems

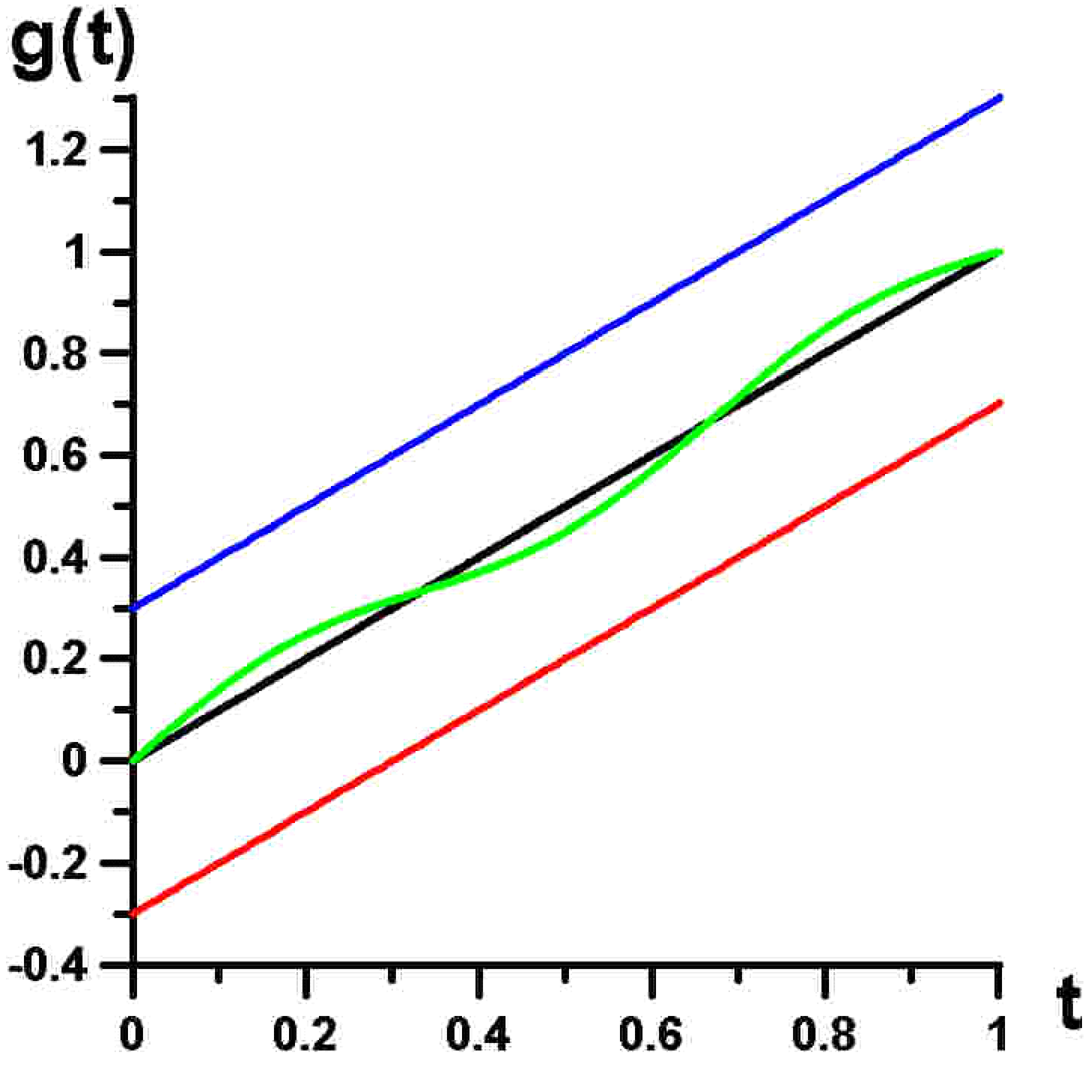
$$\dot{x}(t) = f(t, x(q_1(t)), \ldots, x(q_s(t))), \ t \in B \qquad (1)$$

$$\dot{x} = \varphi(t), \ t \notin B \qquad (2)$$

$$x(\bar{t}\,) = \bar{x}, \ \bar{x} \in R^n, \bar{t} \in R \qquad (3)$$

Here $B = [t_0, t_1]$; $R = [t_0, +\infty]$, functions $q_j(t), j = \overline{1, s}$ – are homeomorphisms of the line preserving the orientation. All the emerging features for functional-differential equations of pointwise type are a consequence of the loss of the Lipschitz constant by those functions that it performs for ordinary ones.

Homeomorphisms



The statement of optimization problems

$$0 = F_i(x(g(t)), \dot{x}(g(t)), w), i = \overline{1, \dim\_ n},\ t \in [t_0, t_1]$$

$$wl_j \le w_j \le wg_j,\ j = \overline{1, \dim\_ r}$$

On the extended interval of variation of the independent variable $t \in [t_N, t_K], t_N \le t_0, t_K \ge t_1$ outside the basic interval, the values of the derivatives of the phase variables are defined as follows

$$\dot{x}_i^L = h_i^L(t, w), t \in [t_N, t_0],\ \dot{x}_i^R = h_i^R(t, w), t \in [t_1, t_K],$$

$$xl_i \le x_i(g(t)) \le xg_i, i = \overline{1, \dim\_ n}$$

The functions of homeomorphisms $g_i(s), i = \overline{1, \dim\_ s}$ are defined on the interval and must satisfy the monotonicity conditions

$$\frac{dg}{ds} > 0, i = \overline{1, \dim\_ s}$$

The statement of optimization problems

The boundary conditions are given by the functionals

$$K_j(x(g(\tau_j)), \dot{x}(g(\tau_j)), w) = 0, j = \overline{1, \dim\_l}, \tau_j \in [t_0, t_1]$$

The optimization problem consists in finding the values of trajectories and control parameters - a pair $\{x(t), w\}$ that allows to achieve a minimum of functional

$$K_0(x(g(\tau_0)), \dot{x}(g(\tau_0)), w) \to \min, \tau_0 \in [t_0, t_1]$$

To solve the initial problem it is constructed a convolution of the residual functional and the objective functional

$$I(x(t), w) = \sum_{i=1}^{\dim\_n} v_i^N \int_{t_0}^{t_1} F_i^2(x(g(t)), \dot{x}(g(t)), w)dt +$$

$$\sum_{i=1}^{\dim\_n} v_i^N \int_{t_N}^{t_0} [\dot{x}_i(g(t)) - h_i^L(t, w)]^2 dt + \sum_{i=1}^{\dim\_n} v_i^N \int_{t_1}^{t_K} [\dot{x}_i(g(t)) - h_i^R(t, w)]^2 dt +$$

$$\sum_{j=1}^{\dim\_l} v_j^K K_j^2(x(g(\tau_j)), \dot{x}(g(\tau_j), w) + K_0(x(g(\tau_0)), \dot{x}(\tau\ ), w)$$

where $v_i^N, i = \overline{1, \dim\_n}$ and $v_j^K, j = \overline{1, \dim\_l}$ are weight coefficients for residuals and boundary conditions.

Test problem 10

$$\dot{x}_1(t) = x_2(t), \quad t \in [0,2] \qquad \dot{x}_2(t) = \frac{1}{2}x_1(t-1) - \frac{1}{2}x_1(t \cdot t)$$

$$x_1(0) = 0$$

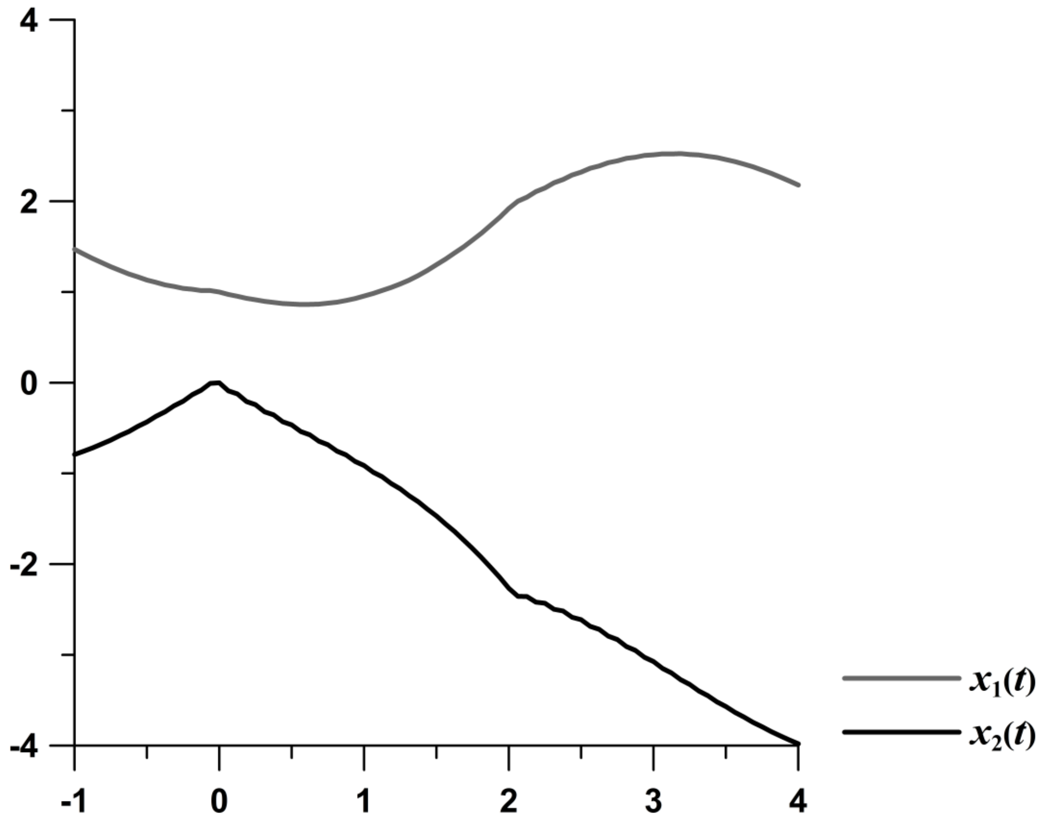
$$x_2(t_0) = 1$$

$$\dot{x}_1(t) = \cos(t)$$

$$\dot{x}_2(t) = \sin(t), t \in [2,4]$$

$$\dot{x}_1(t) = \cos(t)$$

$$\dot{x}_2(t) = \sin(t), t \in [-1,0]$$



Test problem 11

$$\dot{x}(t) = -0.5\pi\left(x(t-1) - x(t+1)\right),$$

$$t \in [4, 6]$$

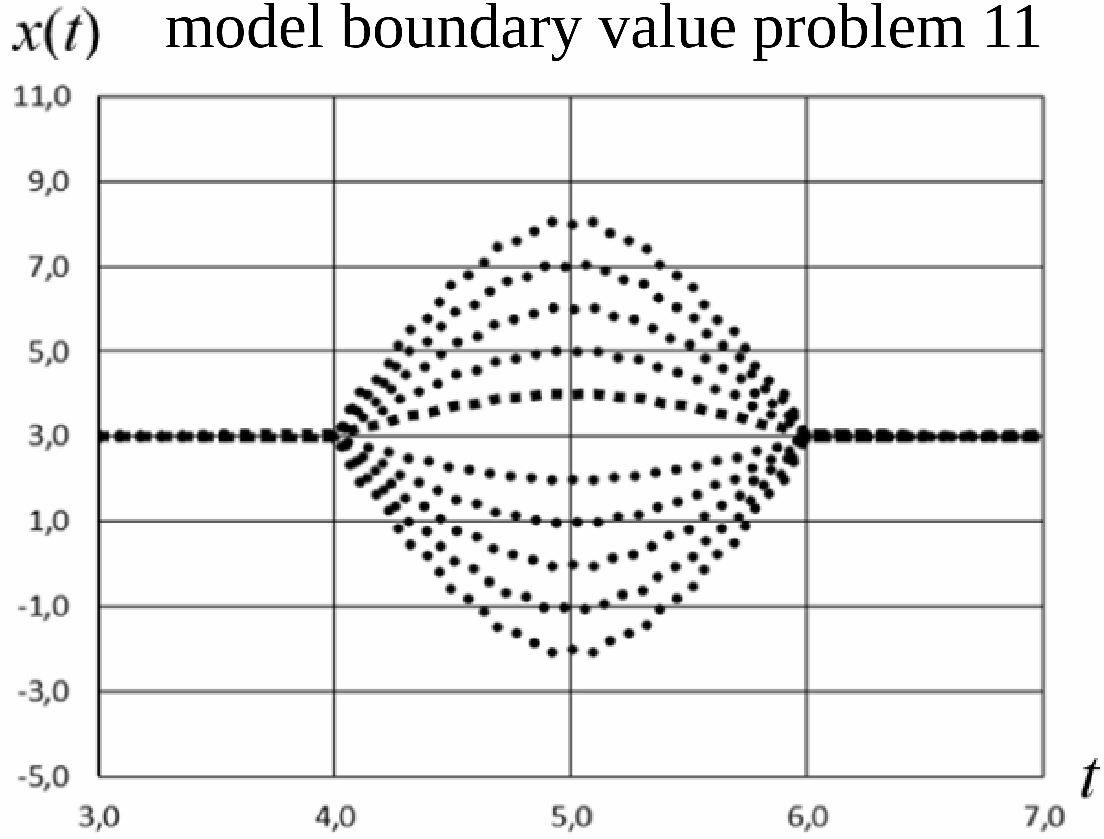
$$\dot{x}(t) \equiv 0,$$

$$t \in R \setminus [4,6]$$

$$x(4) = a, \quad \alpha \in R$$

Sets of solutions

model boundary value problem 11



Total Summary: What is needed to solve applied OCP?

- High communication skills
- Sum of technologies
- Software package for "nuclear" tasks
- Minimal mathematical skills
- Perseverance
- COMMON SENSE

Thanks you for attention!

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Areas of application

- Flight dynamics
- Space navigation
- Mechanics
- Robotics
- Electrical power engineering
- Chemistry
- Materials science
- Quantum physics
- Economics
- Ecology
- Geography
- Medicine
- Criminal science
- Materials science
- Seismology
- …

Applied problems
Flight dynamics

- Investigation of critical aircraft dynamics in general flight maneuver and sortie modes
- Task of optimal maneuvers synthesis in the frame “aircraft against radar”
- Problem of landing a heavy aircraft (“Buran”) to the maximum range

- Chelyabinsk higher military school of navigators

- Ramenskoe instrument-making design office

- State research Institute of aviation systems

Applied problems
Flight dynamics

- Computation of aircraft maneuver for protection against missiles attacking from the rear hemisphere
- Optimization of spatial maneuvers of the helicopter and modes for engine failure

- *Sukhoi experimental design office*
- *Gromov flight research institute*
- *Kamov helicopter factory*

Applied problems
Space navigation

- Task of orbital spacecraft orientation
- Problem of the spacecraft landing to the Earth, Moon, Mars

- Central specialized design bureau “Progress”

- Rocket-space corporation “Energy”

Applied problems
Electrical power engineering

- Optimization of operation modes of electric networks with direct current elements

- Melentiev energy systems institute

Applied problems
Robotics

- Optimization of anthropogenic robot movement
- Analysis of spherical robot dynamics

- Irkutsk state transport university

- Kyushu university, Fukuoka, Japan

Applied problems
Chemistry

- Model identification and search of oscillating heterogeneous catalytic reactions
- Software developing for analyzing electrocardiogram data of experimental animals

- *Boreskov catalysis institute, Novosibirsk, Russia*
- *Vorozhtsov Novosibirsk institute of organic chemistry*

Applied problems
Materials science

- Optimization problem of composite structures
- Calculations for designing space engines of new type

- Design Technological Institute of Digital Techniques, Novosibirsk

Applied problems
Quantum physics

- Calculation of the basic operations of the quantum computer
- Identification of distributed dynamic models of electrons spin epitaxy
- Modeling of strained heterostructures in quantum dots “silicon-germanium”

- Rzhanov institute of semiconductor physics, Novosibirsk

Applied problems
Ecology

- Modeling of lands desertification processes for the steppe regions of Kalmykia and Mongolia
- Optimization problem forest utilization on the territory of Irkutsk region
- Modelling of biotransformation processes of organic substances in forest ecosystems of the Baikal region

- Sochava institute of geography, Irkutsk

- Siberian institute of plant physiology and biochemistry, Irkutsk

Applied problems
Medicine

- Evaluation of medico-social factors of fertility and oncological diseases of the Irkutsk region population
- Investigation of socially important environmental health problems of the Baikal and Arctic regions population

- East-Siberian institute of medico-ecological research, Angarsk

- Irkutsk diagnostic and treatment center

- Irkutsk state medical university

Applied problems
Biology

- Identification of significant factors in biomolecular networks

- Mongolian national institute

Applied problems
Criminal science

- Optimization problem of anti drug addiction and crime investment programs
- Modeling of youth crime processes in the Irkutsk region

- *Vienna University of Technology, Austria*

- *East-Siberian institute of the Ministry of internal Affairs of Russia*

Applied problems
Economics

- Calculation of investment programs of Kabansky region in Buryatia
- Modeling market equilibrium for the grain market of Mongolia

- *Mongolian national institute*

- *Sochava institute of geography, Irkutsk*

Applied problems
Seismology

- Modeling and estimation of seismic resistance of buildings

- Institute of earth crust, Irkutsk