FACULTY OF PHYSICS

Semiclassical Classification of Periodic Orbits in Quantum Many-Body Systems

Daniel Waltner

with Maram Akila, Boris Gutkin, Petr Braun, Thomas Guhr

Dynamics in Siberia,

Novosibirsk, 2018

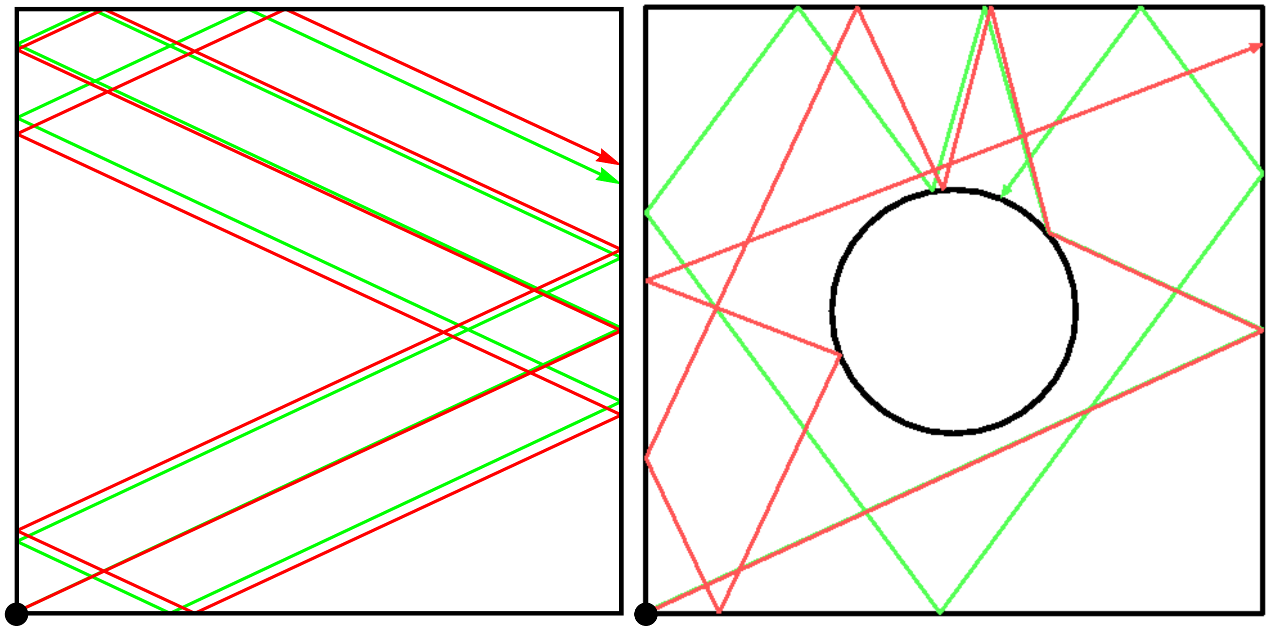
Outline

- Introduction to quantum chaos and semiclassics
- Semiclassical connection for the short-time behavior of a quantum many-body system
- Establish a quantum evolution of reduced dimension
- Impact of collective dynamics on the quantum spectrum

Classical Chaos

Fundamental aspect:

rectangular billiard Sinai billiard



Classical dynamics: chaotic and integrable dynamics

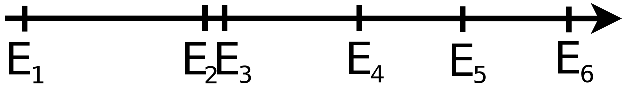
Quantum chaos: impact of the classical dynamics on the quantum system?

Quantum physics: notion of orbit undefined (Heisenberg uncertainty principle)

Quantum Chaos

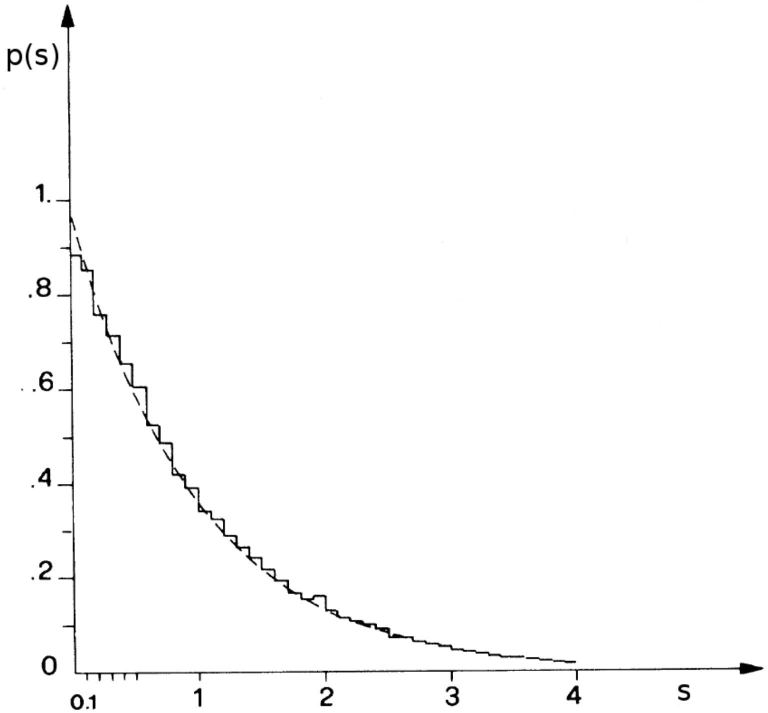
Statistical properties of the quantum spectrum:

Spectrum:



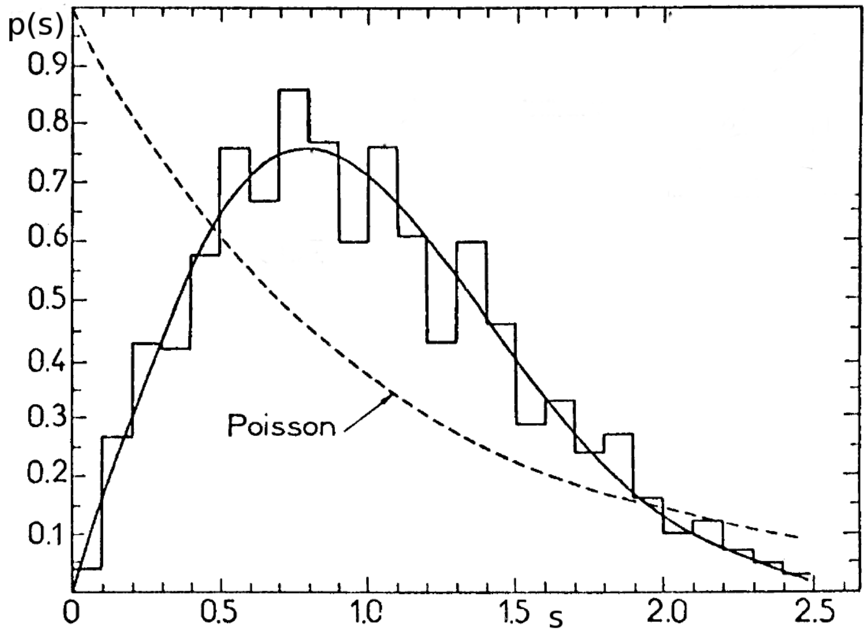
Nearest neighbor spacing distribution:

s: distance between adjacent levels on scale of mean level spacing



rectangular billiard

Casati, Chirikov, Guarneri, Phys. Rev. Lett. (1985).

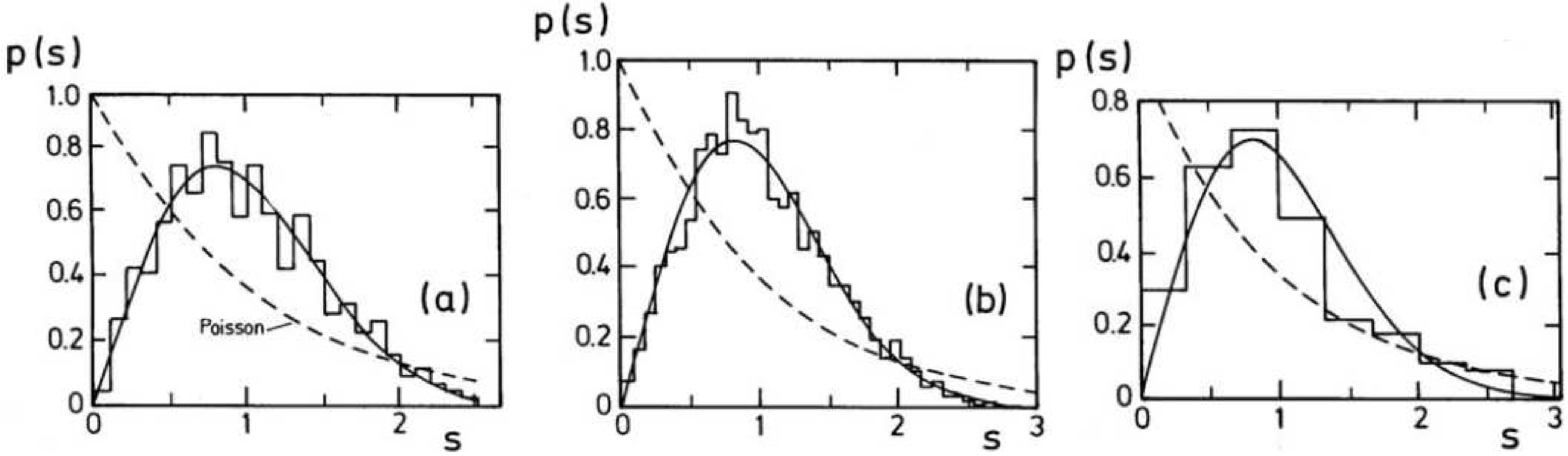


Sinai billiard

Bohigas, Giannoni, Schmit, Phys. Rev. Lett. (1984).

BGS-conjecture

Nearest neighbor spacing distribution for other chaotic systems:



Sinai billiard | Hydrogen atom in strong magnetic field | NO_2 molecule

Bohigas-Giannoni-Schmit conjecture:

Wigner distribution describes nearest neighbor distribution of classically chaotic systems Bohigas, Giannoni, Schmit, Phys. Rev. Lett. (1984).

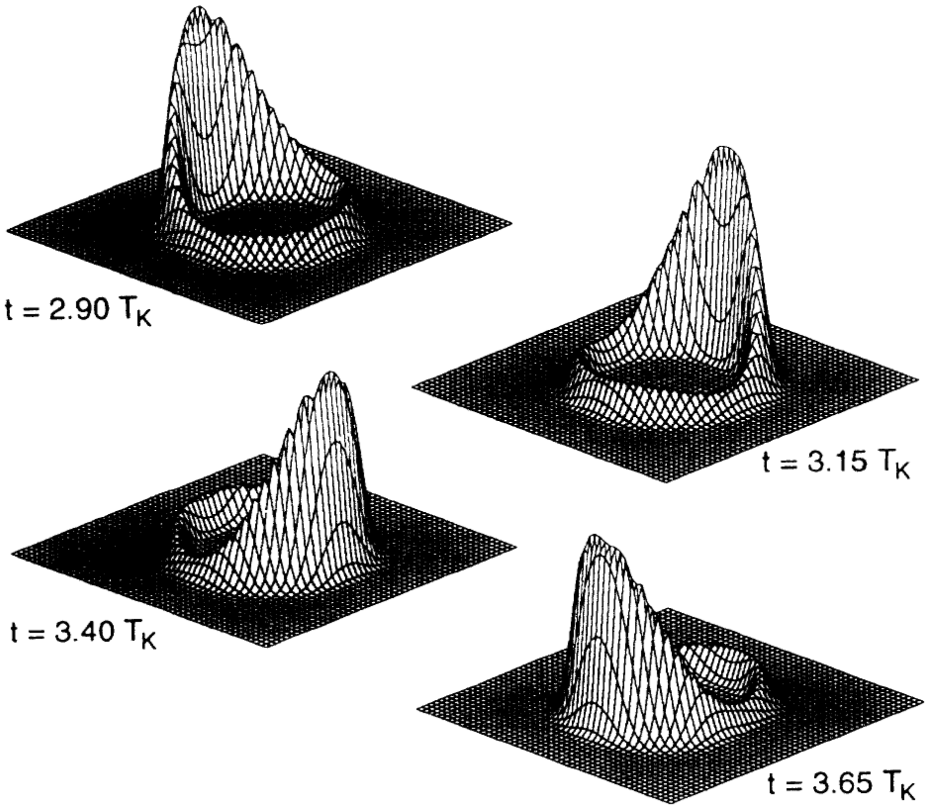
$\Rightarrow$ Universality

Semiclassics

Traces of classical orbits in the quantum system

Example:

Highly excited states in hydrogen atom (Rydberg states):



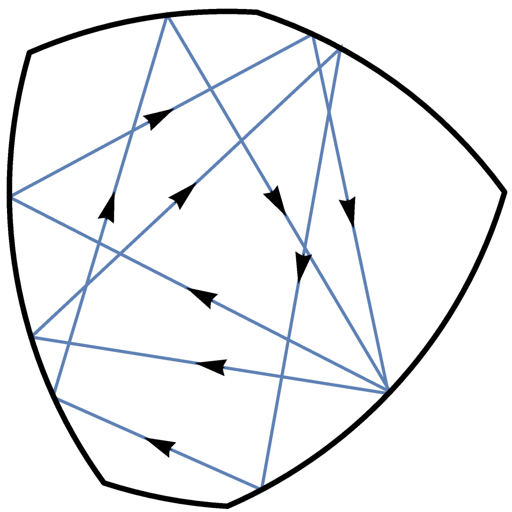
Gaeta, Noel, Stroud, Phys. Rev. Lett. (1994).

Semiclassics

Semiclassics combines classical (orbits) and quantum elements (interference)

Example: Gutzwiller trace formula for chaotic systems:

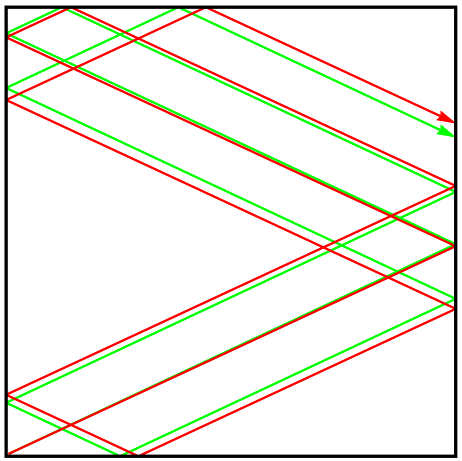
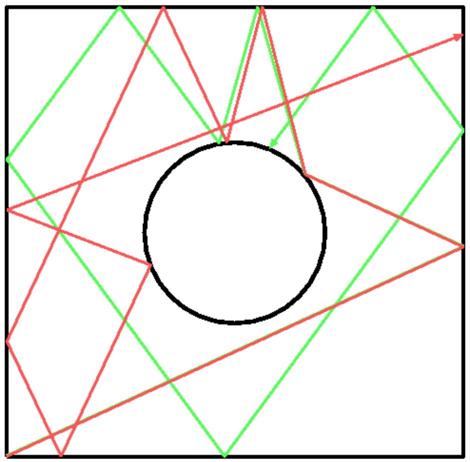
$$\rho(E) = \underbrace{\sum_{n=1}^{\infty} \delta(E - E_n)}_{\text{quantum level density}} \sim \overline{\rho}(E) + \frac{1}{\hbar}\mathrm{Re} \underbrace{\sum_{\gamma} A_\gamma \mathrm{e}^{iS_\gamma/\hbar}}_{\text{sum over classical periodic orbits with action } S_\gamma \text{ and stability coefficient } A_\gamma}, \qquad \hbar \ll S_\gamma$$



Gutzwiller, J. Math. Phys. (1971).

Quantum Chaos

Connection between classical and quantum mechanics:

Integrable Motion	Chaotic Motion
	
Einstein-Brillouin Keller quantization:	Gutzwiller trace formula:
$J_i = \frac{1}{2\pi} \oint_{\gamma_i} \mathbf{p} d\mathbf{q} = n_i \hbar$	$\rho(E) = \sum_n \delta(E - E_n)$ $\sim \overline{\rho}(E) + \frac{1}{\hbar}\mathrm{Re} \sum_\gamma A_\gamma \mathrm{e}^{iS_\gamma/\hbar}$

Motivation

Semiclassical connection for a single particle:

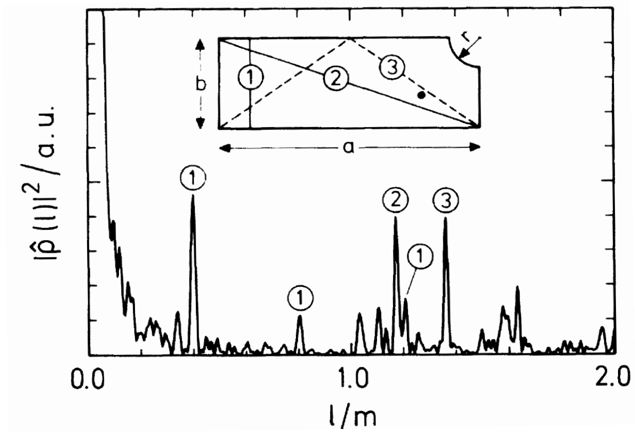
Gutzwiller trace formula:

$$\rho(E) = \underbrace{\sum_n \delta(E - E_n)}_{\text{quantum level density}} \sim \overline{\rho}(E) + \frac{1}{\hbar}\mathrm{Re}\underbrace{\sum_\gamma A_\gamma \mathrm{e}^{iS_\gamma/\hbar}}_{\text{sum over periodic orbits with action } S_\gamma \text{ and stability coefficient } A_\gamma}$$

Single-particle systems:

- Billiards: $S_\gamma = \hbar k l_\gamma$: Fourier-transform with respect to k:
  Spectrum of the classical orbits $\delta(l - l_\gamma)$

  Stöckmann, Stein (1990)



- Kicked top: Fourier-transform with respect to spin quantum number s Kuś, Haake, Delande (1993)

Kicked Top

Hamiltonian: $$\hat{H}(t) = \frac{4J\,(\hat{s}_z)^2}{(s+1/2)^2} + \frac{2\mathbf{B}\cdot\hat{\mathbf{s}}}{(s+1/2)} \sum_{n=-\infty}^{\infty} \delta(t-n)$$

Kick part of kicked top:

Quantum

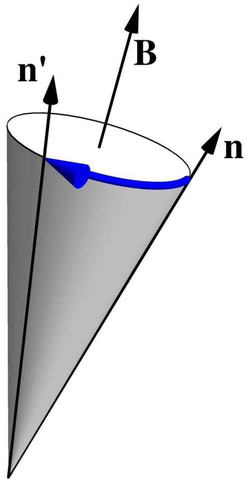
$$\hat{H}_K = \frac{2\mathbf{B}\cdot\hat{\mathbf{s}}}{s+1/2}$$

$$\hat{U}_K = \exp\left(-i(s+1/2)\hat{H}_K\right)$$

with

- magnetic field $\mathbf{B} = (B^x, 0, B^z)$
- spin vector $\hat{\mathbf{s}} = (\hat{s}_x, \hat{s}_y, \hat{s}_z)$
- spin quantum number s

Classical



$$\mathbf{n}(t+1) = R_{\mathbf{B}}(2|\mathbf{B}|)\mathbf{n}(t)$$

- unit vector $\mathbf{n}(t)$
- rotation around $\mathbf{B}$ with angle $2|\mathbf{B}|$: $R_{\mathbf{B}}(2|\mathbf{B}|)$

Kicked Top

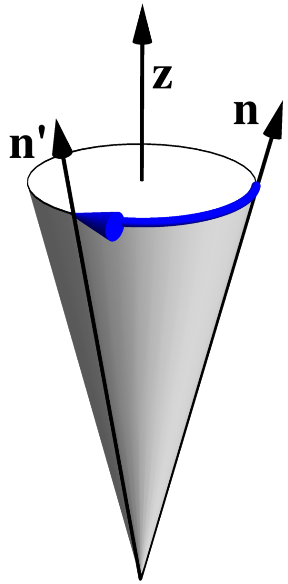
"Ising" part of kicked top:

Quantum

$$\hat{H}_I = \frac{4J(\hat{s}_z)^2}{(s+1/2)^2}$$

$$\hat{U}_I = \exp\left(-i(s+1/2)\hat{H}_I\right)$$

Classical



with

- "Ising" coupling J
- spin vector $\hat{\mathbf{s}} = (\hat{s}_x, \hat{s}_y, \hat{s}_z)$
- spin quantum number s

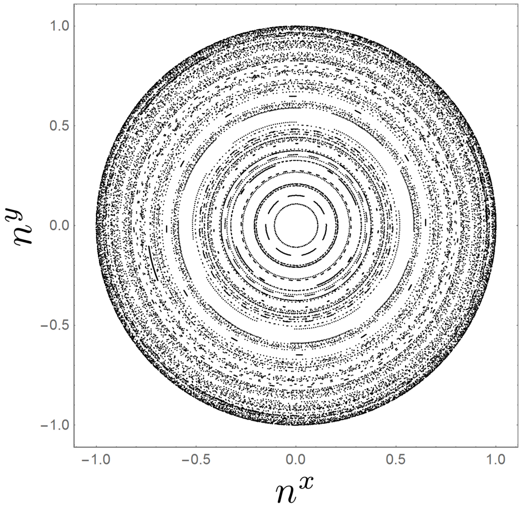
$$\mathbf{n}(t+1) = R_{\mathbf{z}}(8Jn^z)\mathbf{n}(t)$$

- unit vector $\mathbf{n}(t)$
- rotation around $\mathbf{z}$ with angle $8Jn^z$: $R_{\mathbf{z}}(8Jn^z)$

Kicked Top - Classical Dynamics

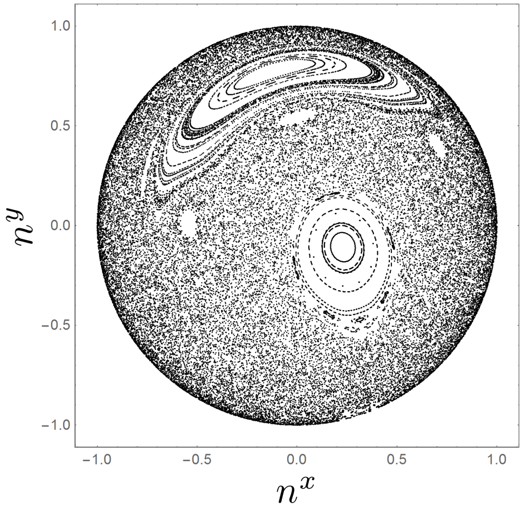
Combination of kick and Ising part: $\hat{U} = \hat{U}_I \hat{U}_K$

Parameters: $\tan\beta = B^x/B^z$, $|\mathbf{B}| = 1.27$, $J = 0.7$



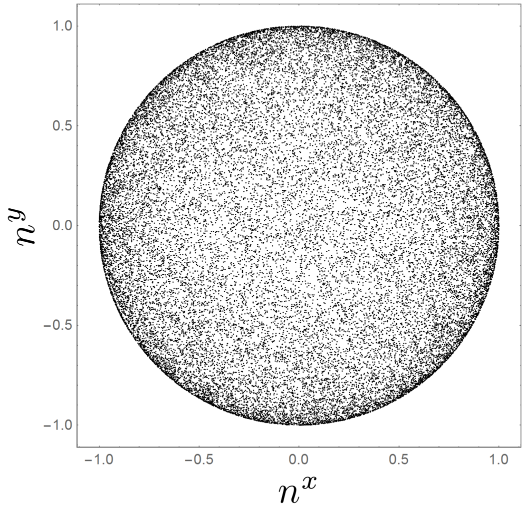
$\beta = 0$

regular



$\beta = 0.2$

mixed



$\beta = \pi/4$

chaotic

Classical Motion

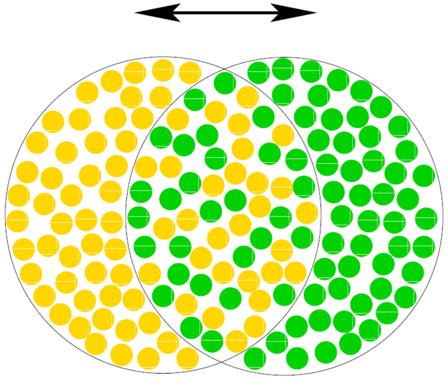
Many-particle systems: relative motion of particles provides additional degree of freedom

Nuclear physics:

Coherent (collective) motion	Incoherent single particle motion

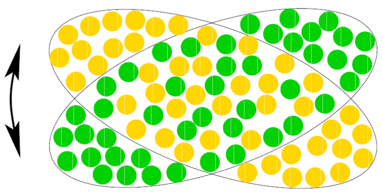
Giant-Dipole Resonance:

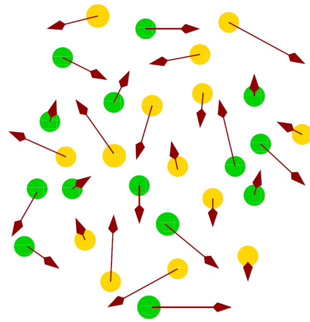
Baldwin, Klaiber (1947)



Scissor Mode:

Bohle, et al. (1984)



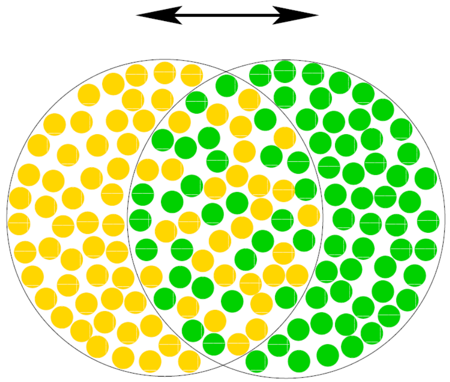


⇒ Description by effective models

Aims

- Quantum many-body systems: identify classical periodic orbits in non-integrable system and their impact on the quantum spectrum

- Replace effective degrees of freedom by microscopic degrees of freedom



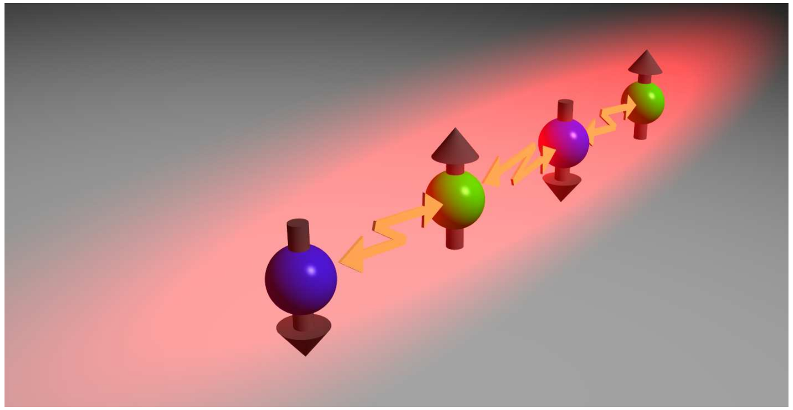
- Understand short time collective motion

- Start with easily accessible system

Kicked Spin Chain

N-particle quantum system: kicked spin chain consisting of N coupled spin-s-particles:

$$\hat{H}(t) = \underbrace{\sum_{n=1}^{N} \frac{4J\hat{s}^z_{n+1}\hat{s}^z_n}{(s+1/2)^2}}_{\text{nearest neighbor Ising interaction}} + \underbrace{\frac{2}{s+1/2}\sum_{n=1}^{N}\mathbf{B}\cdot\hat{\mathbf{s}}_n}_{\text{local kick part}} \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$



Periodic boundary conditions: $\hat{\mathbf{s}}_{N+1} = \hat{\mathbf{s}}_1$

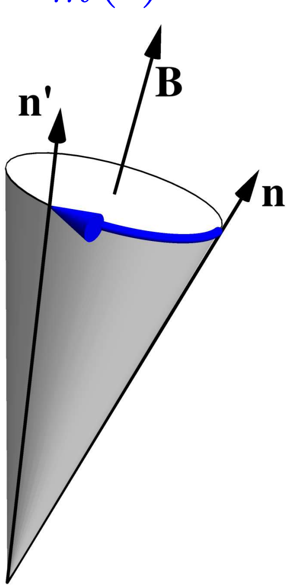
Experiments for $s = 1/2$ in groups of Bloch (Munich), Greiner (Harvard), Jochim (Heidelberg), Monroe (Maryland)

Classical Dynamics

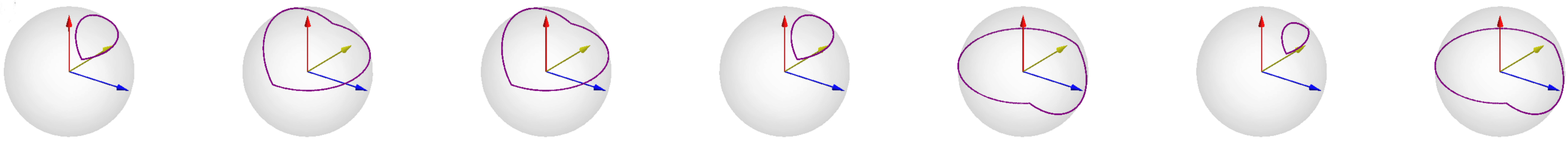
Classical state in kicked spin chain represented as unit vector $\mathbf{n}_m(t)$ on the Bloch sphere for spin m with dynamics:

Rotation: $\mathbf{n}_m(t+T) = \left(R_{\mathbf{z}}(4J\chi_m)R_{\mathbf{B}}(2|\mathbf{B}|)\right)^T \mathbf{n}_m(t)$

Angle: $\chi_m = n^z_{m-1} + n^z_{m+1}$



Periodic orbits for $T = 1$, $N = 7$:



Semiclassics

Trace formula for isolated orbits:

$$\mathrm{Tr}\hat{U}^T \sim \sum_{\gamma(T)} A_\gamma \mathrm{e}^{isS_\gamma}$$

with periodic orbits $\gamma(T)$, stability prefactor A_γ, action S_γ

Limit: Large spin quantum number $s \gg 1$

Fourier-transform yields action spectrum:

$$\rho(S) \propto \sum_{s=1}^{s_{\mathrm{cut}}} \mathrm{e}^{-isS} \mathrm{Tr}\hat{U}^T \sim \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

Waltner, Braun, Akila, Guhr, J. Phys. A (2017).

Duality Relation

Problem specific for many-body system:

$$\rho(S) \propto \sum_{s=1}^{s_{\rm cut}} \mathrm{e}^{-isS} \mathrm{Tr} \underbrace{\hat{U}^T}_{(2s+1)^N} \sim \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

-dimensional

Example: $s_{\rm cut} = 10$, $N = 20 \to (2s+1)^N = 2.8 \cdot 10^{26}$

Solution: Duality of propagations in time and particle directions

Gutkin, Osipov, Nonlinearity (2016); Akila, Waltner, Gutkin, Guhr, J. Phys. A (2016).

Duality Relation

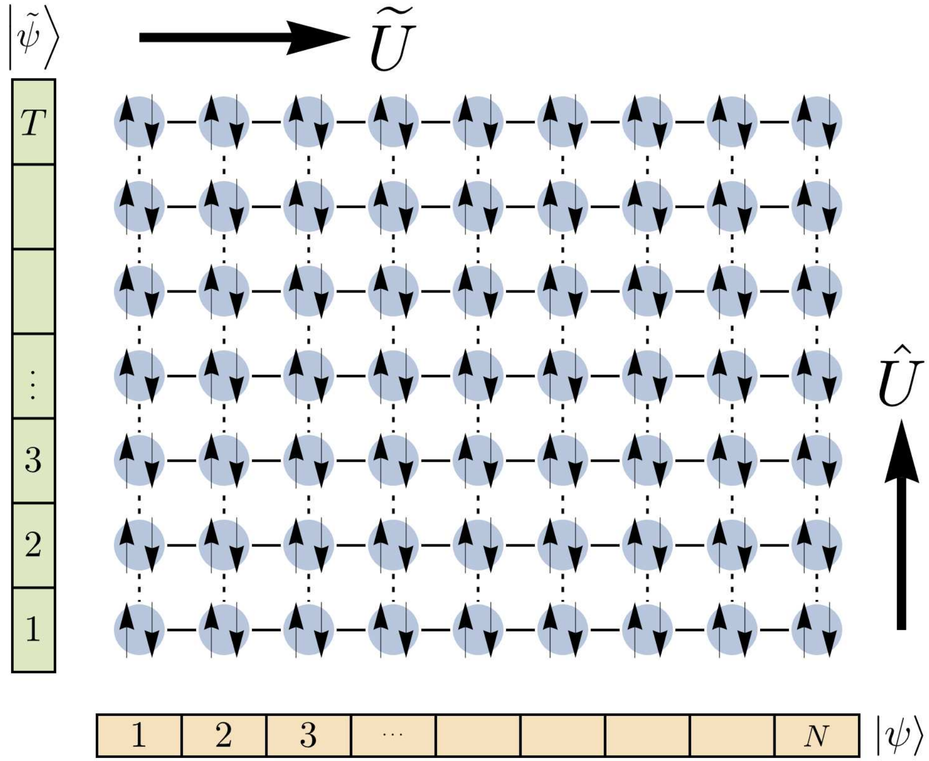
Aim: Reduce dimension of $\hat{U}^T$

Time evolution: $\hat{U}$

$|\psi(t+1)\rangle = \hat{U}\,|\psi(t)\rangle$

Particle evolution: $\widetilde{U}$

$\left|\widetilde{\psi}(n+1)\right\rangle = \widetilde{U}\left|\widetilde{\psi}(n)\right\rangle$



Duality:

$$\mathrm{Tr}\hat{U}^T = \mathrm{Tr}\widetilde{U}^N$$

with non-unitary operator $\widetilde{U}$

Dimension:

$$\dim\widetilde{U} = (2s+1)^T \times (2s+1)^T$$

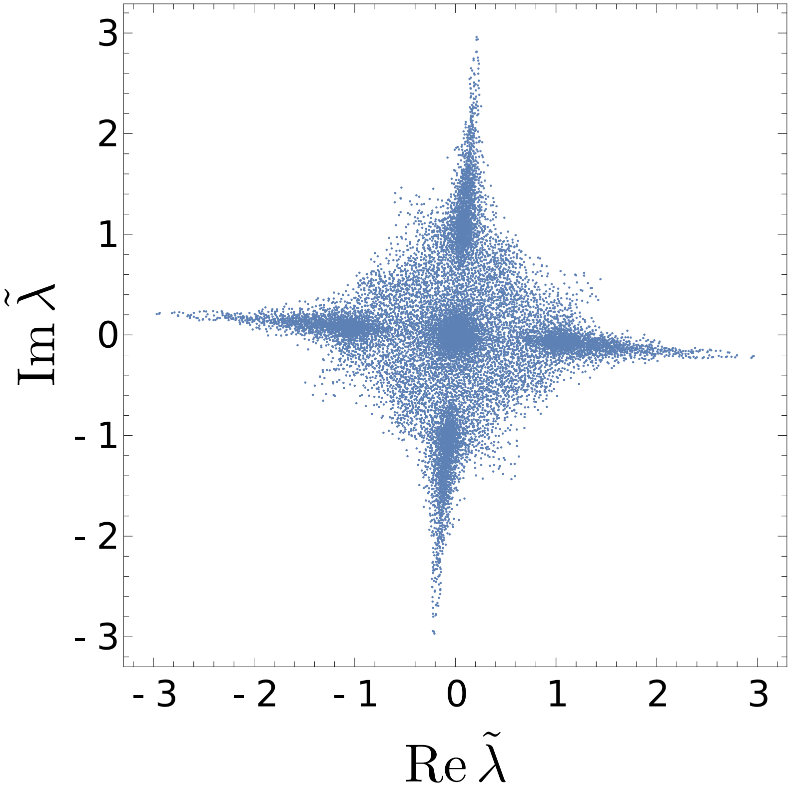
$\Rightarrow$ Dimensional reduction achieved for short times

Dual Operator

Generality of this approach:

- Interaction part diagonal in some basis
- Kick part acts locally

Eigenvalues $\tilde{\lambda}$ of $\widetilde{U}$ for $J = 0.6$, $B^x = B^z = 0.9$, $s = 100$, $T = 2$:



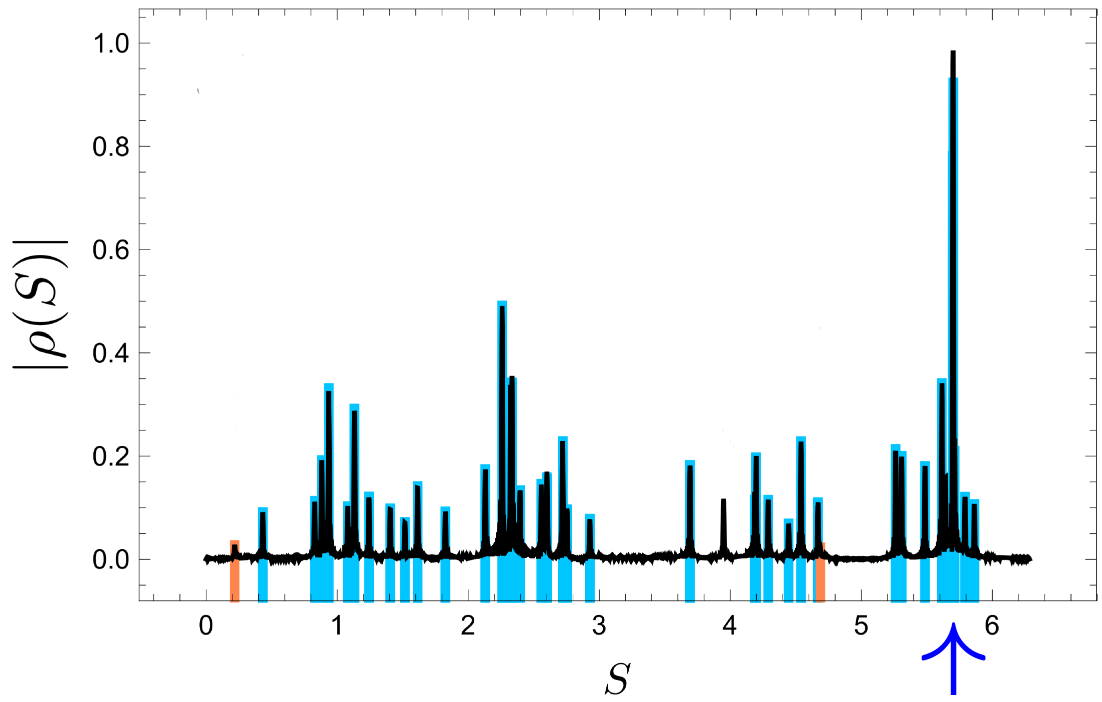
Akila, Waltner, Gutkin, Guhr, J. Phys. A (2016).

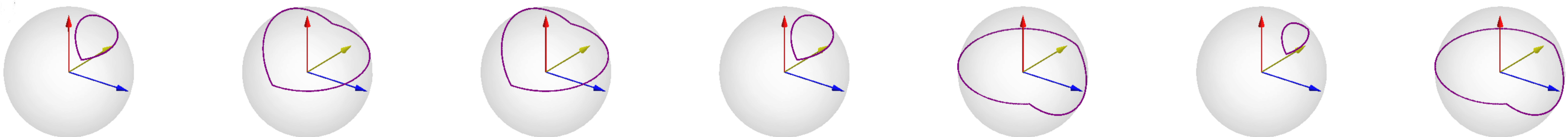
Action Spectrum $T = 1$

Periodic orbits in spin chain identified

$$\rho(S) \propto \sum_{s=1}^{s_{\mathrm{cut}}} \mathrm{e}^{-isS} \mathrm{Tr} \widetilde{U}^N \sim \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

Parameters: $N = 7$, $J = 0.75$, $B^x = B^z = 0.9$



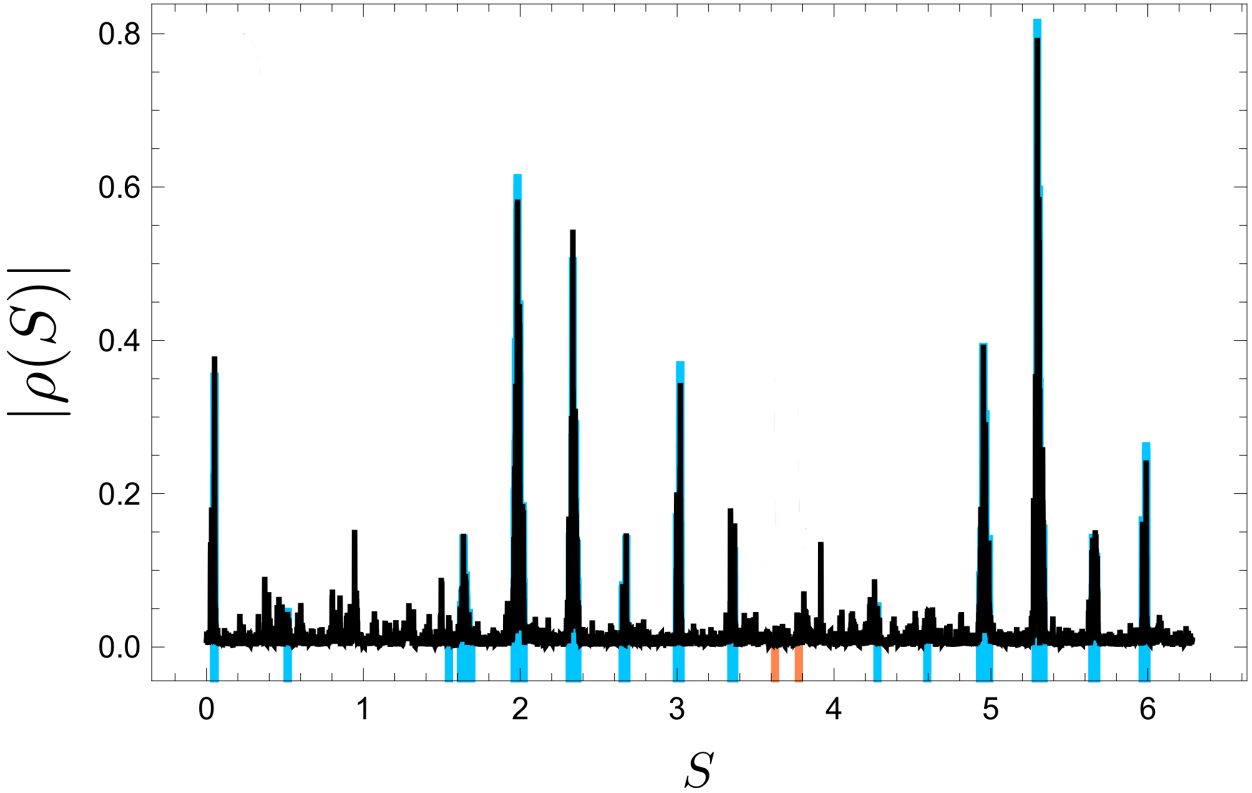


Akila, Waltner, Gutkin, Braun, Guhr, Phys. Rev. Lett. (2017).

Action Spectrum $T = 1$

Larger N: number of orbits competes with resolution

Parameters: $N = 19$, $J = 0.7$, $B^x = B^z = 0.9$,

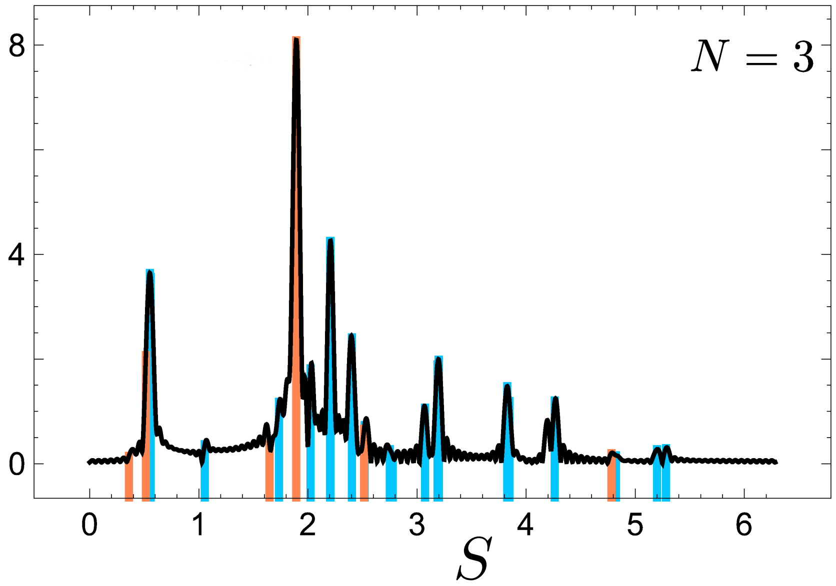


Akila, Waltner, Gutkin, Braun, Guhr, Phys. Rev. Lett. (2017).

Dominance of Collectivity

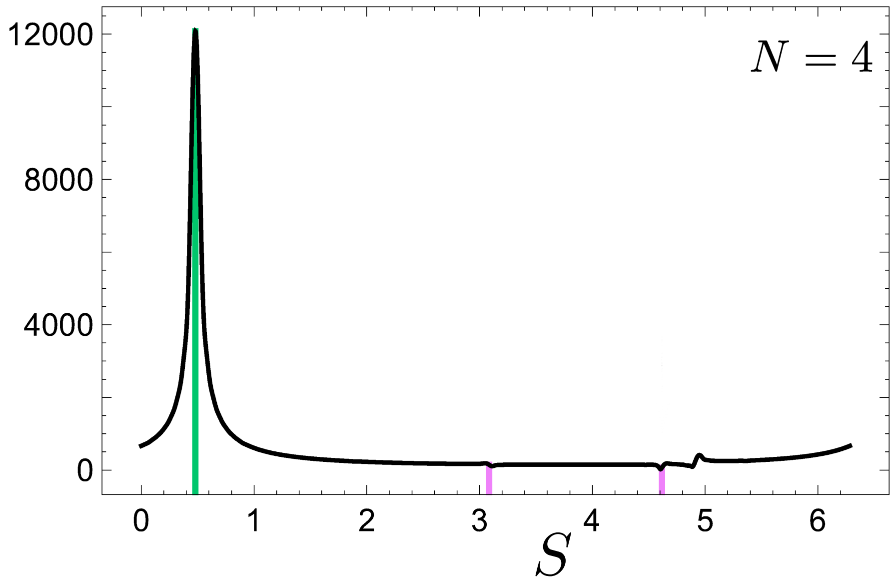
Parameters: $T = 2$, $J = 0.7$, $B^x = B^z = 0.9$

$N = 3$



Incoherent motion

$N = 4$

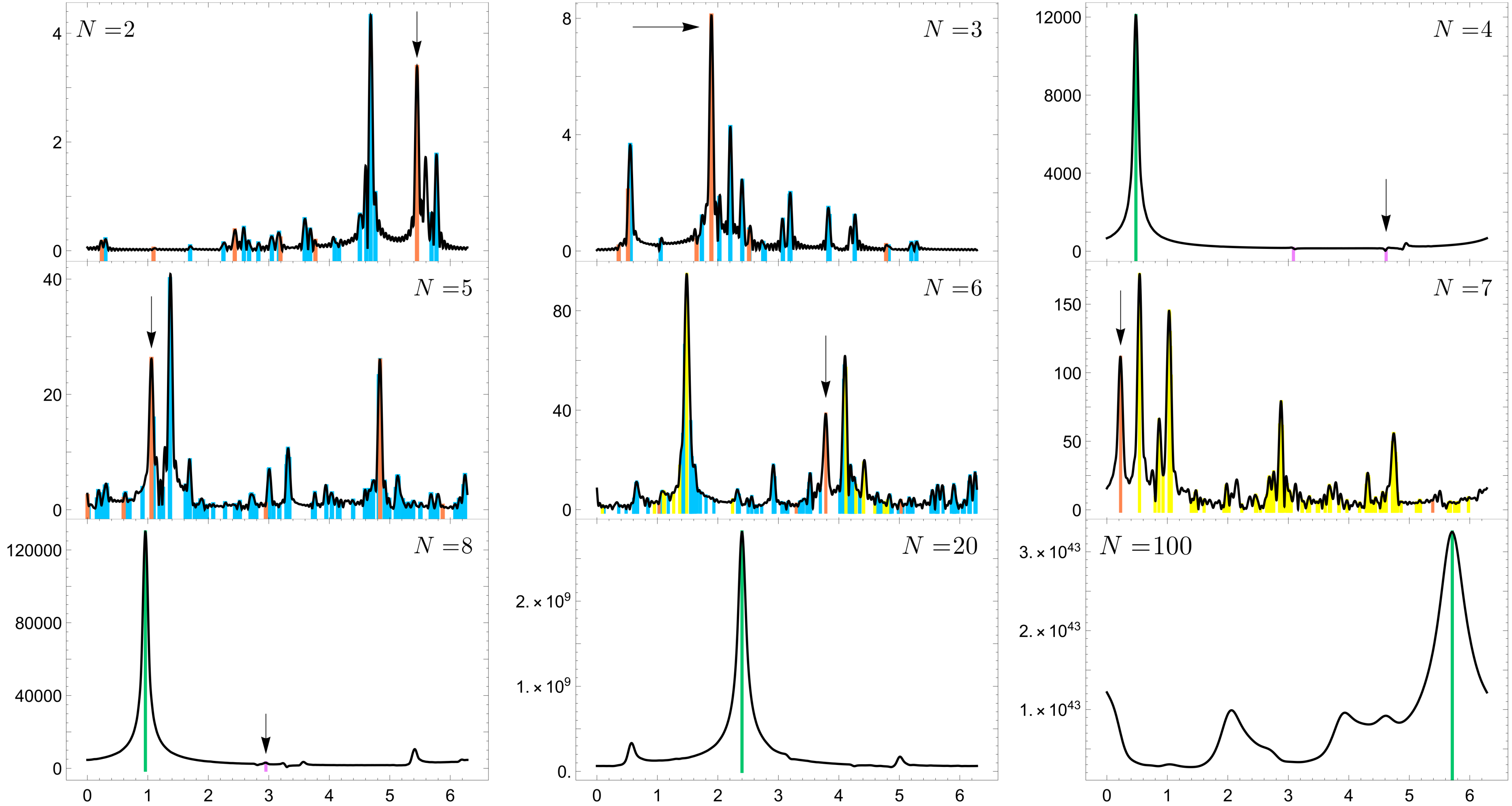


Collective motion

Akila, Waltner, Gutkin, Braun, Guhr, Phys. Rev. Lett. (2017).

Dominance of Collectivity

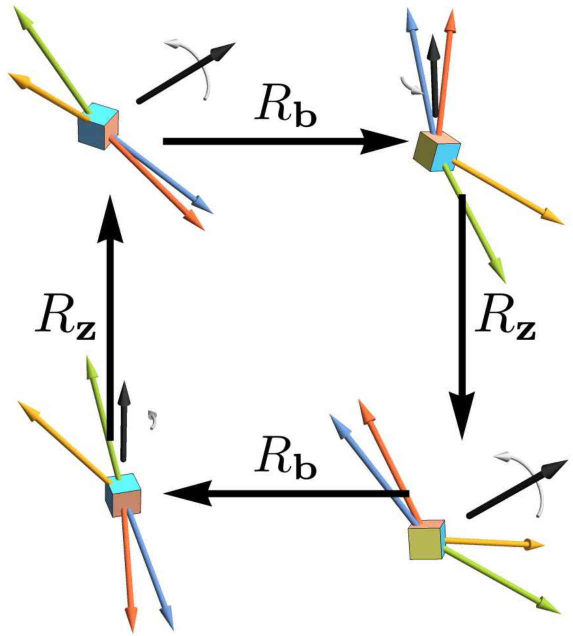
Observation generalizes to $N = 4k$ ($k \in \mathbb{N}$):

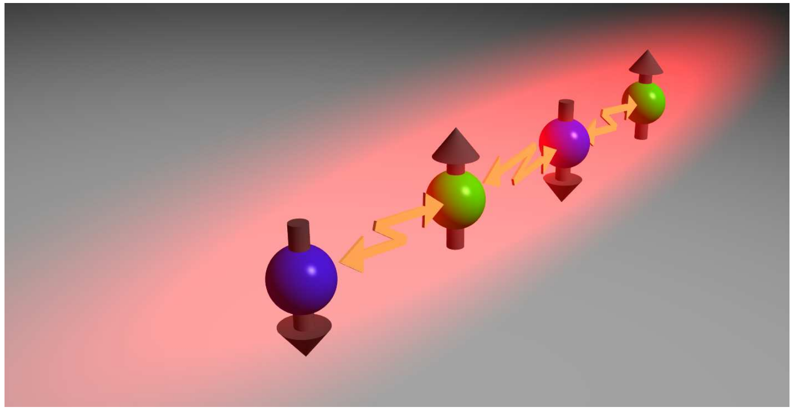


Classical Collective Dynamics

4-dimensional manifold of (nonisolated) periodic orbits for $N = 4$ with equal actions:

spins perform solid body rotation





blue spins influenced by
the green and vice versa
Condition $(R_{\mathbf{z}}(4J\chi)R_{\mathbf{B}}(2|\mathbf{B}|))^2 = \mathbb{1}$
imposes 4 restrictions

Akila, Waltner, Gutkin, Braun, Guhr, Phys. Rev. Lett. (2017).

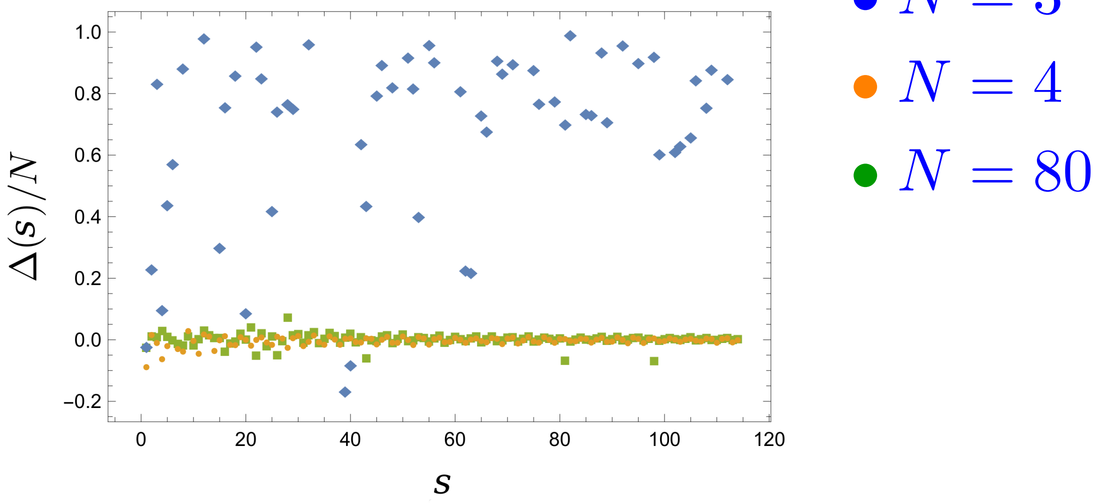
Dominance of Collectivity

Orbits on manifold dominate spectrum for specific s

$$\mathrm{Tr}\hat{U}^T \sim \sum_{\gamma(T)} A_\gamma \mathrm{e}^{isS_\gamma} \approx A_{\mathrm{man}} \mathrm{e}^{isS_{\mathrm{man}}}$$

Difference of the phase:

$$\Delta(s) = \mathrm{Im}\mathrm{Log}\mathrm{Tr}U^T - sS_{\mathrm{man}}$$



$\Rightarrow$ $\mathrm{Tr}\hat{U}^T$ dominated by a type of collective motion

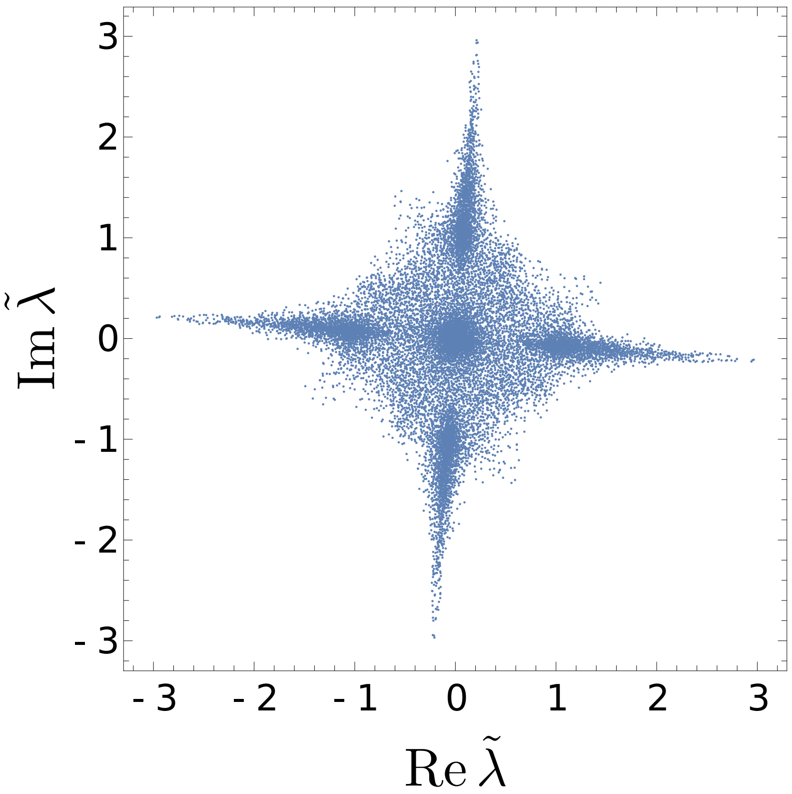
Akila, Waltner, Gutkin, Braun, Guhr, Phys. Rev. Lett. (2017)

Dual Operator and Collectivity

Compute $\rho(S)$ with duality:

$$\rho(S) \propto \sum_{s=1}^{s_{\mathrm{cut}}} \mathrm{e}^{-isS} \mathrm{Tr} U^T = \sum_{s=1}^{s_{\mathrm{cut}}} \mathrm{e}^{-isS} \mathrm{Tr} \widetilde{U}^N$$

Eigenvalues $\tilde{\lambda}$ of the dual operator: $T = 2$, $J = 0.6$, $B^x = B^z = 0.9$, $s = 100$



Relation to Dual Operator

For $N \gg 1$:

$\mathrm{Tr}\widetilde{U}^N$ approximated by 4 eigenvalues with largest magnitude $\lambda_{\max,n}$:

$$\mathrm{Tr}\widetilde{U}^N \approx \sum_{n=1}^{4} \lambda_{\max,n}^N$$

with the eigenvalues $\lambda_{\max,n} = a\mathrm{e}^{i\varphi_n}$ with

$$\varphi_n = \varphi + \pi n/2$$

$\Rightarrow$ Contribution from $\lambda_{\max,n}$ to $\mathrm{Tr}\widetilde{U}^N$:

- cancels for $N \neq 4k$ due to n-dependent phase
- dominates for $N = 4k$

$\Rightarrow$ Large contributions to $\mathrm{Tr}\widetilde{U}^N$ only for $N = 4k$

Peak heights

Action spectrum:

$$\rho(S) \propto \sum_{s=1}^{s_{\text{cut}}} \mathrm{e}^{-isS} \mathrm{Tr} \widetilde{U}^N \sim \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

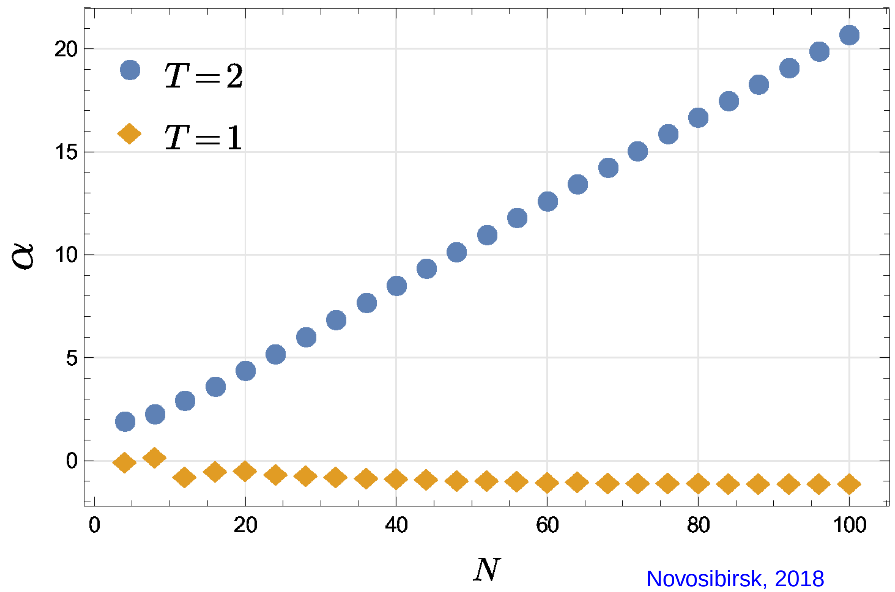
Two cases:

- For isolated orbits: A_γ predictable by trace formula, independent of s_{cut}
- For orbit families (manifolds): scaling of $|\rho(S_\gamma)| \sim (s_{\text{cut}})^\alpha$ induces

$$A_\gamma \sim s^\alpha$$

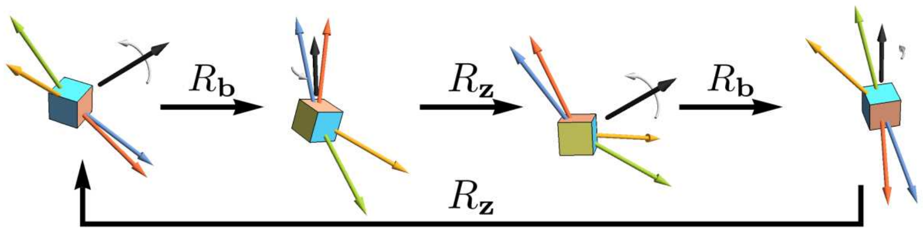
For manifold we find:

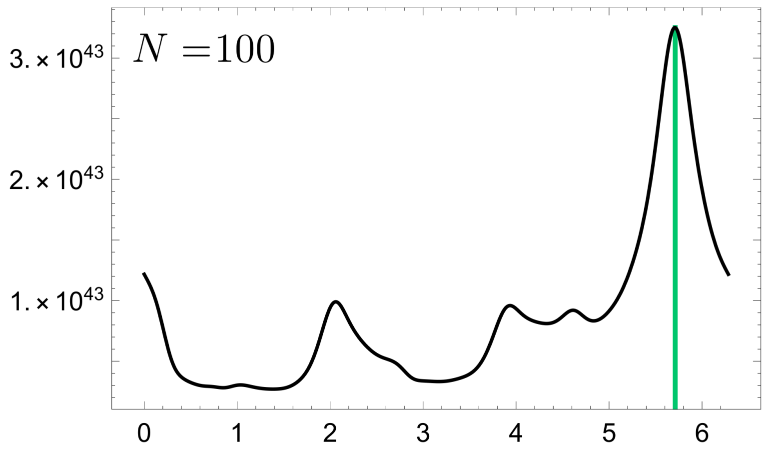
$$\alpha \propto N$$



Conclusions

- Established method to compute classical orbits in quantum many-particle system and identified impact on quantum spectrum for a spin chain
- Duality reduces dimension of $\hat{U}^T$ by an exchange of N and T
- Collective dynamics dominates the quantum spectrum





Conclusions

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Thank you for your attention!