UNIQUENESS OF THE NUMERICAL RANGE OF TRUNCATED SHIFTS BY L. KÉRCHY

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According to old and, maybe, forgotten tradition, I will discuss the paper of another mathematician, namely,

L. Kérchy, *Uniqueness of the numerical range of truncated shifts*, Acta Sci. Math. (Szeged), **83** (2017), 243–261.

Let T be an operator on a Hilbert space $\mathcal{H}$. The numerical range $W(T)$ of T is the set $W(T) = \{(Tx, x) \ : \ x \in \mathcal{H}, \ \ \|x\| = 1\}$.

Let ϑ be an inner function. Put $\mathcal{K}_\vartheta = H^2 \ominus \vartheta H^2$ and $T_\vartheta = P_{K_\vartheta} z|_{K_\vartheta}$. The following question arises.

Let ϑ and η are two inner function. Does $W(T_\vartheta) = W(T_\eta)$ imply $\vartheta = \eta$ (up to unimodular constant)?

By using well-known properties of the numerical range, it is easy to construct two infinite Blaschke products ϑ and η such that $W(T_\vartheta) = W(T_\eta)$ and $\vartheta \neq \eta$. On the other hand, the answer to the above question is positive, if ϑ and η are finite Blaschke products (H.-L. Gau and P. Y. Wu) or meromorphic inner functions (that is, have one singularity on the unit circle) (I. Chalendar, P. Gorkin and J. R. Partington). In the paper under discussion, the case of two singularities is considered. It is proved that $W(T_\vartheta) = W(T_\eta)$ if and only if $\tau_{z\vartheta} = \tau_{z\eta}$, where τ_θ is the frequency function of the argument of an inner function θ. A positive answer to the above question is given in some particular cases, but the question remains open in general.

The talk will be in Russian.