THE MECHANIZATION OF MATHEMATICS

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The phrase "formal methods" is used to describe a body of methods in computer science for specifying, developing, and verifying complex hardware and software systems. The word "formal" indicates the use of formal languages to write assertions, define objects, and specify constraints. It also indicates the use of formal semantics, that is, accounts of the meaning of a syntactic expression, which can be used to specify the desired behavior of a system or the properties of an object sought. Finally, the word "formal" suggests the use of formal rules of inference, which can be used to verify claims or guide a search.

Such methods hold great promise for mathematical discovery and verification of mathematics as well. In this talk, I will survey some applications, including verifying mathematical proofs, verifying the correctness of mathematical computation, searching for mathematical objects, and storing and communicating mathematical results.

Interactive theorem proving involves the use of computational proof assistants to construct formal proofs of mathematical claims, using the axioms and rules of a formal foundation that is implemented by the system. The user of such an assistant generally has a proof in mind and works interactively with the system to transform it into a formal derivation. Proofs are presented to the system using a specialized proof language, much like a programming language. I will discuss the current state of the field, and some recent milestone formalizations.

One place for formal verification is especially useful is in the case of mathematical proofs that rely on substantial uses of computation, where the associated code is subtle and susceptible to error. I will discuss various strategies that are employed to make such computational results more reliable.

The use of formal search methods to establish theorems of core mathematics is less common, but nonetheless I will discuss a few notable successes to date, as well as prospects for the future.

Finally, I will briefly discuss projects like the *Formal Abstracts* project, which aim to provide digital infrastructure to support mathematical activity.

References

[1] Jeremy Avigad. The mechanization of mathematics. To appear in *Notices of the American Mathematical Society*.

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