ENCOUNTERS WITH INFINITY

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Questions about the ontological status of the objects about which mathematicians reason have been with us since ancient times. In my talk I will emphasize the role of mathematical practice in expanding the realm of mathematical discourse. I will begin with the example of Gödel's struggles with the philosophical consequences of his two main discoveries: the inevitability of undecidability and the consistency of the continuum hypothesis.

I will then present a number of revealing examples from the history of mathematics. The solution in terms of radicals of cubic equations seemed to force practitioners to work with square roots of negative numbers although these were thought to be impossible. Torricelli considered the region bounded above by a rectangular hyperbola, below by one of its asymptotes, and to the left by a perpendicular from the hyperbola to that asymptote. He was able to show that while that region is of infinite extent and has an infinite area, the solid formed by revolving it about the asymptote has a finite volume. This provided a shock to the world of 17th century mathematics, contradicting what Aristotle had taught about infinity.

Leibniz's infinitesimal calculus yielded useful answers although reasoning with his infinitesimals seemed to lead to contradictions. By assuming that the relationships between the zeros and the coefficients of a polynomial would hold as well for an infinite power series, Euler was able to obtain the sum of the series $\sum_{n=1}^{\infty} 1/n^2$. In solving a partial differential equation for heat conduction, Fourier used trigonometric series with a quite unjustified expansive freedom. This led Dirichlet to the modern notion of a function as an arbitrary mapping.

Cantor's investigation of uniqueness theorems for trigonometric series led him to develop his transfinite ordinal numbers. Contemporary set theorists were able to resolve hitherto intractable problems concerning the hierarchy of projective sets by invoking the determinacy of projective sets as a new axiom. More recently, assumptions about the hierarchy of large cardinals was used to prove this axiom.

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